Heralded Single-Phonon Preparation, Storage, and Readout in Cavity Optomechanics

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We show how to use the radiation pressure optomechanical coupling between a mechanical oscillator and an optical cavity field to generate in a heralded way a single quantum of mechanical motion (a Fock state). Starting with the oscillator close to its ground state, a laser pumping the upper motional sideband produces correlated photon-phonon pairs via optomechanical parametric down-conversion. Subsequent detection of a single scattered Stokes photon projects the macroscopic oscillator into a single-phonon Fock state. The nonclassical nature of this mechanical state can be demonstrated by applying a readout laser on the lower sideband to map the phononic state to a photonic mode and performing an autocorrelation measurement. Our approach proves the relevance of cavity optomechanics as an enabling quantum technology.

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Introduction.—Cavity optomechanical systems consist of a mechanical oscillator with resonant frequency $\Omega_m/2\pi$ coupled to an electromagnetic cavity mode with resonant frequency $\omega_c/2\pi$ [1] [Fig. 1(a)]. The radiation pressure optomechanical coupling can be used to either amplify [2] or cool [3–8] the mechanical degree of freedom. This has enabled the preparation of mechanical oscillators in the quantum regime [9–12] and the quantum coherent coupling between light and mechanical degrees of freedom [13,14]. Likewise, the optomechanical interaction allows for the readout of mechanical motion with an imprecision below that at the standard quantum limit [15,16]. In addition, optomechanically induced transparency [17] can be utilized for slowing or advancing electromagnetic signals [18,19], for coherent transfer between two optical wavelengths [20], between the microwave and optical domains [21,22], and for information storage and retrieval in long-lived oscillations [14,23,24].

In the context of quantum information, continuous-variable schemes [25] such as optomechanical squeezing [26,27] and entanglement [28] in the quadrature operators have been demonstrated in recent experiments. Yet there are many advantages to using discrete variables, for which heralded probabilistic protocols can exhibit very high fidelity and loss resilience [29]. Moreover, on a fundamental level, studying quantized energy eigenstates of macroscopic objects may allow new tests of quantum mechanics [30] and of the nature of entanglement [31–33]. The first step toward this goal is to generate single-phonon Fock states in long-lived mechanical oscillators.

One possible route is to break the harmonicity of the system’s eigenstates by reaching the single-photon strong coupling regime [34–43], or to use the nonlinearity resulting from coupling to two-level systems [44,45]. However, the former requires $g_0/\sqrt{\kappa\Omega_m} \gtrsim 1$, where $g_0$ is the single-photon optomechanical coupling rate (see below) and $\kappa$ is the total cavity energy decay rate—a regime far from state-of-the-art experiments, where $g_0/\kappa \sim 10^{-3}$ [13,46]. If multiple optical modes are introduced, a nonconventional photon blockade regime can be used to relax the constraint on the coupling strength [47,48] and has recently been considered for conditional preparation of nonclassical states [49,50]. Projective measurements have also been proposed by Vanner et al. to realize phonon addition and subtraction operations for general quantum state orthogonalization [51].

In this Letter, we present an approach based on single-photon detection to generate a single-phonon Fock state in a heralded way and then convert it into a single photon, in the experimentally relevant weak-coupling and resolved-sideband [6,7] regime of a single-mode optomechanical system [Figs. 1(a)–1(d)]. Starting with the mechanical mode close to its ground state (mean phonon number $\bar{n}_0 \ll 1$), a write laser pulse, tuned to the upper motional sideband of the optical cavity, is used to amplify [52] the mechanical motion and generate (with low probability) a correlated photon-phonon pair via optomechanical parametric down-conversion. The scattered photon—referred to as the Stokes photon in the following—is spectrally filtered from the pump and detected by a photon-counting module, thereby projecting the mechanical oscillator (from its weak coherent state) into a single-phonon Fock state while heralding the success of the procedure. To verify the nonclassical nature of the heralded mechanical state, the mechanical excitation is coherently mapped onto the optical cavity field by applying a readout laser tuned to the lower mechanical sideband (corresponding to resolved sideband cooling [8]), and the statistics of these anti-Stokes photons is analyzed in an autocorrelation ($g^{(2)}$).
In the limit where the write (amplifying) and readout (cooling) pulses are shorter than the mechanical decoherence time, and for a small enough initial phonon occupancy ($\bar{n}_0 \ll 1$), the twofold coincidence probability vanishes ($g^{(2)} \to 0$) [Fig. 1(d)], demonstrating the heralded creation of a single-phonon Fock state and its successful up-conversion into a single cavity photon.

**Principle.**—We consider the optical and mechanical modes (represented by bosonic annihilation operators $\hat{a}$ and $\hat{b}$, respectively) of an optomechanical cavity driven by a laser on the lower or upper mechanical sideband, corresponding to the angular frequencies $\omega_\pm = \omega_c \pm \Omega_m$ [Fig. 1(b)]. The Hamiltonian is a sum of three terms, $\hat{H} = \hat{H}_0 + \hat{H}_{\text{OM}} + \hat{H}_{\text{dr},\pm}$, describing the uncoupled systems, $\hat{H}_0 = \hbar \omega_c \hat{a}^\dagger \hat{a} + \hbar \Omega_m \hat{b}^\dagger \hat{b}$, the optomechanical interaction, $\hat{H}_{\text{OM}} = -\hbar g_0 \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b})$, and the laser driving, $\hat{H}_{\text{dr},\pm} = \hbar (\gamma \pm \epsilon) \hat{a}^\dagger \hat{a} + \hbar \bar{n}_w (\pm \epsilon) \hat{b}^\dagger \hat{b}$, where $|\bar{n}_w| = \sqrt{\bar{n} \bar{n}_w}$ is the incoming photon flux for a laser power $P_\pm$. As detailed in Ref. [55], after switching to the interaction picture with respect to $\hat{H}_0$ and taking the weak-coupling ($g_0 \ll \kappa$) and resolved-sideband ($\kappa \ll \Omega_m$) limits, we obtain the linearized Langevin equations during the write (amplification) pulse,

$$\frac{d\hat{a}}{dt} = \frac{i}{\hbar} [\hat{H}_{\text{BSB}}, \hat{a}] - \frac{\kappa}{2} \hat{a} + \sqrt{\kappa} \hat{a}_{\text{in}}, \quad (1a)$$

$$\frac{d\hat{b}}{dt} = \frac{i}{\hbar} [\hat{H}_{\text{BSB}}, \hat{b}] - \gamma \hat{b} + \sqrt{\gamma} \hat{b}_{\text{in}}, \quad (1b)$$

(where BSB is blue sideband) with $\gamma$ the energy decay rate of the mechanical oscillator. $\hat{H}_{\text{BSB}} = -\hbar g_0 \hat{a}^\dagger \hat{b}^\dagger + \text{H.c.}$ is the effective optomechanical coupling rate enhanced by the intracavity photon number $\bar{n}_w = (|\sqrt{\bar{n}^2} + \kappa^2/2|)^2 = \kappa P_\pm / \hbar \Omega_m (\Omega_m^2 + \kappa^2/4)$ at the laser frequency. For simplicity, we consider the optical cavity to be overcoupled, i.e., the total cavity decay rate is dominated by the external in- or out-coupling rate $\kappa_{\text{ext}}$, so that $\kappa \approx \kappa_{\text{ext}}$. The operator $\hat{a}_{\text{in}}$ represents the vacuum noise entering the optical cavity, and $\hat{b}_{\text{in}}$ is the thermal noise from a phonon bath at temperature $T_{\text{bath}}$ and mean occupancy $\bar{n}_b \approx (k_B T_{\text{bath}} / \hbar \Omega_m)$.

In a first simplified treatment, we neglect the decay of the mechanical oscillator, which is a valid approximation if the pulse sequence is shorter than the thermal decoherence time $(\gamma \bar{n}_b)^{-1}$. Since in our scheme $g_0 \ll \kappa$, we can adiabatically eliminate the cavity mode in Eqs. (1a) and (1b), $\hat{a}_{\text{w}}(t) = (2/\kappa) (ig_0 \bar{n}_w \hat{b}^\dagger + \sqrt{\kappa} \hat{a}_{\text{w},\text{in}})$.

Using the input-output relations [56] $\hat{a}_{\text{w},\text{out}} = -\hat{a}_{\text{w},\text{in}} + \sqrt{\kappa} \hat{a}_w$ (the subscript w refers to the operators during the write pulse), we obtain the coupled optomechanical equations

$$\dot{\hat{a}}_{\text{w},\text{out}} = \hat{a}_{\text{w},\text{in}} + i \sqrt{2g_0 \bar{n}_w} \hat{a}_w, \quad (2a)$$

$$\frac{d\hat{b}_w}{dt} = g_0 \hat{b}_w + i \sqrt{2g_0 \bar{n}_w} \hat{a}_{\text{w},\text{in}}, \quad (2b)$$

where $g_0 \equiv (2g_0^2/\kappa)$. Introducing the temporal modes [57] for the cavity driven by a write pulse of duration $T_w$, $\hat{A}_{\text{w},\text{in(out)}}(T_w) = [(\pm 2g_0)/(1 - e^{\mp 2g_0 T_w})]^{-1/2} \int_0^{T_w} e^{\mp 2g_0 t} \hat{a}_{\text{in(out)}}(t) dt$, we can write the solutions of Eqs. (2a) and (2b) as $U^a \hat{A}_{\text{w},\text{in}}U$ and $U^b \hat{b}_w(0)U$, where the propagator $U$ is given by [55].

![FIG. 1 (color online).](image-url)
term is the single-phonon Fock state resonator initially in its ground state (with a Hanbury-Brown onto the anti-Stokes photons and subsequently measured states [Fig. 1(c)]. The phonon statistics can thus be mapped explicitly in Ref. [55]), can be close to 1.

For an oscillator initially in a thermal state characterized by the density matrix $\rho_b(0) = (1 - p)\sum_{n=0}^\infty p^n |n\rangle \langle n|$, with $p = \langle \hat{n}_0 | (1 + \hat{n}_0) \rangle$, the state of the optomechanical system at the end of the write pulse is $
abla_{A,b}(T_w) = U(T_w) |\Omega_A\rangle \otimes |\Omega_b\rangle \rho_b(0) U(T_w)$. The conditional mechanical state upon detection of a single photon in mode $\hat{A}_{\text{out}}$ is obtained by applying the projection operator $|1\rangle_A \otimes |1\rangle_b$, tracing out the optical mode and normalizing,

$$
\rho_{b,\text{cond}}(T_w) = \frac{\text{tr}_A\{|1\rangle_A \otimes |1\rangle_b \rho_b(T_w)\}}{\text{tr}_A\{|1\rangle_A \otimes |1\rangle_b \rho_b(T_w)\}} = (1 - \bar{p})^2 \sum_{n=0}^\infty \bar{p}^n (n + 1 |n + 1\rangle |n + 1\rangle\langle n + 1|), \tag{4}
$$

where $\bar{p} = p e^{-\bar{\gamma}T_w}$. For a small gain parameter ($\bar{\gamma}_w T_w \ll 1$), which is essential to maximize the probability of successful single-phonon heralding (see Ref. [55]), and a resonator initially in its ground state ($p \ll 1$), the dominant term is the single-phonon Fock state $|1\rangle_b$.

In the readout step, driving the lower sideband at $\omega_r$ leads to the beam-splitter interaction $\hat{H}_{\text{RSB}} = -\hbar g_b \hat{A}^\dagger \hat{b} + \text{H.c.}$ with $g_b = \gamma_b \sqrt{\bar{\gamma}}$ and $\bar{n}_t$, the intracavity photon number at the red sideband (RSB) replacing $\hat{H}_{\text{RSB}}$ in Eqs. (1a) and (1b), which coherently swaps the optical and mechanical states [Fig. 1(c)]. The phonon statistics can thus be mapped onto the anti-Stokes photons and subsequently measured with a Hanbury-Brown–Twiss setup [Fig. 1(a)] [53]. Following similar steps as above, we compute the zero-delay second-order autocorrelation of the anti-Stokes photons during the readout pulse, $g_{\text{cond}}^{(2)}(0) = \langle \langle \hat{A}^\dagger_{\text{out}} \hat{A}^\dagger_{\text{out}} \hat{A}_{\text{out}} \hat{A}_{\text{out}} \rangle \rangle_c / \langle \hat{A}^\dagger_{\text{out}} \hat{A}_{\text{out}} \rangle^2_c$, where the expectation value is taken on the postselected mechanical state, Eq. (4). We find

$$
g_{\text{cond}}^{(2)}(0) = \frac{2\bar{p}(2 + \bar{p})}{(1 + \bar{p})^2} \approx 4\bar{n}_0, \tag{5}
$$

where the last approximation is valid in the limit $\bar{n}_0 \ll 1$ and $\bar{\gamma}_w T_w \ll 1$. This result shows that the twofold coincidence probability vanishes linearly with $\bar{n}_0$ and proves the non-classical nature of the heralded phonon state. In Fig. 1(e), we plot Eq. (5) along with the results obtained when multiple photon emission is taken into account (see Ref. [55]) for different values of the gain parameter $\bar{\gamma}_w T_w$. We note that for sufficient readout laser power the internal phonon-to-photon conversion efficiency, approximated by $1 - e^{-2\bar{\gamma}_w T_w}$, in the limit $p \ll 1$ ($\bar{\gamma}_w T_w$ are given explicitly in Ref. [55]), can be close to 1.

Let us briefly recall the conditions for observing strong antibunching: (i) weak-coupling and resolved-sideband regime, $g_b \ll \kappa \ll \Omega_m$; (ii) negligible mechanical decoherence, $T_w + T_{\text{off}} \ll (\gamma \bar{n}_t)^{-1}$, and (iii) high initial occupancy of the ground state, $\bar{n}_0 \ll 1$. Because the pulse duration is bounded from below by $T_w > 1/\kappa$ (the spectral width of the pulse should be narrower than the cavity), we can recast (ii) onto the condition $\gamma \bar{n}_t \ll \kappa$. Noting that for a given bath temperature $\bar{n}_t$ on $1/\Omega_m$, this shows that the oscillator should have both a large $Q$ and a large frequency $\Omega_m$.

**Experimental feasibility.**—Many optomechanical systems have already been demonstrated that satisfy (i) and for which condition (ii) would be trivially achieved owing to the typically long mechanical decay time [58–60], but condition (iii) is challenging to meet in these systems. Here we consider a photonic crystal nanobeam resonator [11,46,61], for which the very high frequency of the confined phonon mode ($\Omega_m/2\pi = 5$ GHz) is beneficial. For a given bath temperature, fewer quanta are thermally excited, while a large $\Omega_m$ also facilitates spectral filtering of the (anti-)Stokes photons from the pump laser beam (e.g., with high-finesse Fabry-Perot filters). Moreover, the structures reported in Ref. [46] exhibit large optomechanical coupling rate $g_b/2\pi = 1$ MHz and their optical linewidth $\kappa/2\pi < 1$ GHz places them in the resolved-sideband regime. Finally, coherence times of $O((10–100) \times 10^{-6} \text{ s})$ are within reach at 4 K and below [62] as mechanical energy decay rates of $\gamma/2\pi = 7.5$ kHz have been measured at 10 K [46].

Using the parameters reported in Ref. [46] and a realistic bath temperature $T_{\text{bath}} = 1.6$ K (corresponding to $^4$He buffer gas cooling [63]), an initial occupancy of $\bar{n}_0 \approx 0.01$ can be achieved by 100 ns of sideband cooling with a peak intracavity photon number $\bar{n}_r = 10^3$ corresponding to 150 $\mu$W of peak external laser power (see Ref. [55], Sec. III). The cooling laser is switched off during the write-store sequence. Including mechanical dissipation, we integrate Eqs. (1a) and (1b) and compute $g_{\text{cond}}^{(2)}(\tau/\tau_r)$, the

![FIG. 2 (color online).](a) Conditional second-order correlation function $g_{\text{cond}}^{(2)}(\tau/\tau_r)$ at fixed $\tau_r = 1$ ns for increasing waiting time $T_{\text{off}}$ between the write and readout pulses. The bath temperature is set to 1.6 K ($\bar{n}_t = 6.4$). The transition from antibunching to bunching is a signature of the relaxation from a single-phonon Fock state to a thermal state. (b) Same-time correlation $g_{\text{cond}}^{(2)}(0)$ (i.e., two-photon emission probability) as a function of $T_{\text{off}}$ for decreasing phonon bath thermal occupancy $\bar{n}_t$ yielding coherence times up to $(\gamma \bar{n}_t)^{-1} \sim 100$ $\mu$s.](image-url)
probability for anti-Stokes photon emission at times $t_r$ and $t_r + \tau$ during the readout pulse, conditioned on the detection of a herald photon during the write pulse [see Fig. 1(d)]. In Fig. 2(a), we plot $g_{\text{cond}}^\text{inh}(\tau|t_r = 1 \text{ ns})$ for fixed write pulse parameters $T_w = 50 \text{ ns}$ and $\bar{n}_w = 0.1$, corresponding to a probability of Stokes emission $\sim 2\bar{\gamma}_w T_w (1 + \bar{n}_0) \sim 10^{-3}$ per pulse. For waiting times between the write and readout pulses shorter than the decoherence time of the mechanics, $T_{\text{off}} < (\bar{\gamma}_n \bar{n}_\text{inh})^{-1} \approx 20 \mu s$, we observe clear antibunching, a signature of successful conversion of the phonon Fock state into a single photon.

Beyond verifying the nonclassical state of the macroscopic oscillator, our results also suggest a new tool for the on-demand generation of single photons [64–66]. Within a time window $\sim (\bar{\gamma}_n \bar{n}_\text{inh})^{-1}$, the heralded Fock state is stored in the mechanical oscillator and can be retrieved on demand by applying the readout pulse.

Some advantageous features of the optomechanical systems considered here are that the single photons are emitted in a well-defined spatial mode and may be coupled into a single-mode fiber with high efficiency $> 90\%$ [67,68]. Operation over the entire electromagnetic wavelength range and integration into large-scale photonic circuits [69] are other appealing assets. By engineering a cavity supporting two optical modes, both coupled to the same mechanical mode, one could generate the herald photon and release the readout photon at two arbitrary wavelengths. Although the write step is intrinsically probabilistic, it is possible to achieve near-deterministic Fock state creation by employing simple feedback techniques [65,66,70].

Our scheme additionally enables precise control on the linewidth and coherence properties of the on-demand single photons [71] by tuning the strength of the readout pulse characterized by the peak intracavity photon number $\bar{n}_r$ (at the sideband $\omega_w$), as shown in Fig. 3. In the limit of weak readout laser ($\bar{n}_r \ll 1$), the anti-Stokes photon coherence time is set by the thermal coherence time of the oscillator $(\bar{\gamma}_n \bar{n}_\text{inh})^{-1}$. Increasing $\bar{n}_r$ shortens the coherence time and eventually we reach the (laser-enhanced) strong coupling regime $\bar{n}_r \gtrsim \kappa$ and observe the onset of Rabi oscillations for $\bar{n}_r \gtrsim 10^6$, corresponding to multiple phonon-photon swapping cycles within the optical cavity lifetime. This yields a remarkable range of achievable coherence times, and therefore provides a way to generate on-demand single photons with tunable linewidths from tens of kHz to hundreds of MHz, an interesting feature for envisioned quantum networks, e.g., to couple various physical realizations of nodes using photons as carriers of quantum information.

Entanglement and quantum repeaters.—The potential applications of optomechanical systems become more evident when noting the analogy with the scheme based on Raman transitions in atomic ensembles first proposed by Duan et al. [72] to achieve scalable entanglement distribution between distant nodes [Duan-Lukin-Cirac-Zoller (DLCZ) protocol]. Specifically, consider two distant optomechanical systems coherently excited by a weak laser beam, such that the probability that both systems are simultaneously excited is negligible. The resulting Stokes modes are interfered on a beam splitter [32], and the detection of a single photon projects the distant mechanical oscillators into an entangled state where they share a single delocalized phonon. Successive entanglement swapping operations can then be used to extend the entanglement over hundreds of kilometers [73].

As a quantitative example, let us estimate the average time $T_{\text{ent}}$ required to establish entanglement between two optomechanical resonators separated by 10 km of optical fiber using the DLCZ scheme. To first order in the small parameter $\bar{\gamma}_w T_w$, we have $T_{\text{ent}} = (2R_r \bar{\gamma}_w T_w \kappa)^{-1}$, where $R_r$ is the repetition rate of the experiment and $\kappa$ the overall detection efficiency of the Stokes photons. For the particular system considered here, realistic values are $R_r = 10 \text{ MHz}$ and $\kappa = 0.5 \times 0.6 \times 0.2 = 6\%$, where the three factors correspond, in this order, to the collection in a single-mode fiber, the propagation over 10 km of fibers, and the detection efficiency. Although $T_{\text{ent}}$ can be made shorter by increasing $\bar{\gamma}_w T_w$, this also increases the probability for multiple pair excitation and thereby decreases the fidelity expressed as $F = (1 - \bar{n}_0)^{-1}[(1 - 3) \times 2\bar{\gamma}_w T_w (1 - \eta)]$. Assuming a target fidelity of $F = 0.9$ [73] and $1 - \bar{n}_0 \sim 1$, we obtain $\bar{\gamma}_w T_w = 0.017$ and thus $T_{\text{ent}} = 23.5 \mu s$. Remarkably, this time is slightly shorter than the light propagation time of $\sim 50 \mu s$, which
would therefore set the lower bound on entanglement distribution time.

In summary, we have shown how to generate a single-phonon Fock state in an optomechanical resonator under the experimentally accessible weak-coupling and resolved-sideband regimes. Starting with the oscillator in its motional ground state, a write laser pulse tuned on the upper mechanical sideband creates correlated phonon-pair states. The detection of the Stokes photon heralds the successful preparation of a single-phonon Fock state in the mechanical oscillator. Finally, the nonclassical statistics of the phonon state is mapped onto the optical field by a readout pulse tuned on the lower sideband, and conditional two-photon correlations reveal antibunching. Our proposal opens promising perspectives for the use of optomechanical systems as quantum memories and on-demand single-photon sources for emerging applications in quantum information processing and communication.

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