Quantum Repeaters with Photon Pair Sources and Multimode Memories

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We propose a quantum repeater protocol which builds on the well-known Duan-Lukin-Cirac-Zoller (DLCZ) protocol [L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, Nature (London) 414, 413 (2001)], but which uses photon pair sources in combination with memories that allow to store a large number of temporal modes. We suggest to realize such multimode memories based on the principle of photon echo, using solids doped with rare-earth-metal ions. The use of multimode memories promises a speedup in entanglement generation by several orders of magnitude and a significant reduction in stability requirements compared to the DLCZ protocol.

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The distribution of entanglement over long distances is an important challenge in quantum information. It would extend the range for tests of Bell’s inequalities, quantum key distribution, and quantum networks. The direct distribution of entangled states is limited by transmission losses. For example, 1000 km of standard telecommunications optical fiber have a transmission of order $10^{-20}$. To distribute entanglement over such distances, quantum repeaters [1] are likely to be required. Implementations of quantum repeaters have been proposed in various systems [2–5]. A basic element of all protocols is the creation of entanglement between neighboring nodes A and B, typically conditional on the outcome of a measurement, e.g., the detection of one or more photons at a station between two nodes. In order to profit from a nested repeater protocol [1], the entanglement connection operations creating entanglement between non-neighboring nodes can only be performed once one knows the relevant measurement outcomes. This requires a communication time of order $L_0/c$, where $L_0$ is the distance between A and B. Conventional repeater protocols are limited to a single entanglement generation attempt per elementary link per time interval $L_0/c$. Here we propose to overcome this limitation using a scheme that combines photon pair sources and memories that can store a large number of distinguishable temporal modes. We also show that such memories could be realized based on the principle of photon echo, using solids doped with rare-earth-metal ions.

Our scheme is inspired by the Duan-Lukin-Cirac-Zoller (DLCZ) protocol [2], which uses Raman transitions in atomic ensembles that lead to nonclassical correlations between atomic excitations and emitted photons [6]. The basic procedure for entanglement creation between two remote locations A and B in our protocol requires one memory and one source of photon pairs at each location, denoted $M_{A(B)}$ and $S_{A(B)}$, respectively. The two sources are coherently excited such that each has a small probability $p/2$ of creating a pair, corresponding to a state

$$\left[ 1 + \frac{p}{2} (e^{i\phi_a} a^\dagger a^\dagger + e^{i\phi_b} b^\dagger b^\dagger) + O(p) \right]|0\rangle. \quad (1)$$

Here $a$ and $a'$ ($b$ and $b'$) are the two modes corresponding to $S_A$ ($S_B$), $\phi_A$ ($\phi_B$) is the phase of the pump laser at location A (B), and $|0\rangle$ is the vacuum state. The $O(p)$ term introduces errors in the protocol, leading to the requirement that $p$ has to be kept small cf. below. The photons in modes $a$ and $b$ are stored in the local memories $M_A$ and $M_B$. The modes $a'$ and $b'$ are coupled into fibers and combined on a beam splitter at a station between A and B. The modes after the beam splitter are $\tilde{a} = \frac{1}{\sqrt{2}} (a' e^{-i\phi_A} + b' e^{-i\phi_B})$, $\tilde{b} = \frac{1}{\sqrt{2}} (a' e^{-i\phi_A} - b' e^{-i\phi_B})$, where $\chi_{AB}$ are the phases acquired by the photons on their way to the central station. Detection of a single photon in $\tilde{a}$, for example, creates a state $|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}} (a^\dagger e^{i\phi_A} + b^\dagger e^{i\phi_B})|0\rangle$ [neglecting $O(p)$ corrections], with $\theta_{AB} = \phi_A + \chi_{AB}$, where $a$ and $b$ are now stored in the memories. This can be rewritten as an entangled state of the two memories,$$|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B + e^{i\theta_{AB}} |0\rangle_A |1\rangle_B), \quad (2)$$

where $|0\rangle_A$ denotes the empty state of $M_{A(B)}$. $|1\rangle_A$ denotes the state storing a single photon, and $\theta_{AB} = \theta_B - \theta_A$.

This entanglement can be extended via entanglement swapping as in Ref. [2]. Starting from entangled states $|\Phi_{AB}\rangle = \frac{1}{\sqrt{2}} (a^\dagger + e^{i\theta_{AB}} b^\dagger)|0\rangle$ between memories $M_A$ and $M_B$, and $|\Phi_{CD}\rangle = \frac{1}{\sqrt{2}} (e^\dagger + e^{i\theta_{CD}} d^\dagger)|0\rangle$ between $M_C$ and $M_D$, one can create an entangled state between $M_A$ and $M_B$ by converting the memory excitations of $M_B$ and $M_D$ back into propagating photonic modes and combining these modes on a beam splitter. Detection of a single photon after the beam splitter, e.g., in the mode $\frac{1}{\sqrt{2}} (b + c)$, will create an entangled state of the same type between $M_A$ and $M_B$, namely $\frac{1}{\sqrt{2}} (a^\dagger + e^{i\theta_{CD}} d^\dagger)|0\rangle$. In this way, it is possible to establish entanglement between more distant memories, which can be used for quantum communication as follows [2].

Suppose that location A (Z) contains a pair of memories $M_{A_1}$ and $M_{A_2}$ ($M_{Z_1}$ and $M_{Z_2}$), and that entanglement has been established between $M_{A_1}$ and $M_{Z_1}$, and between $M_{A_2}$
and $M_{Z2}$, i.e., that we have a state $\frac{1}{\sqrt{2}}(a_1^{+} + e^{i\theta}z_1^{+})(a_2^{+} + e^{i\phi}z_2^{+})|0\rangle$. The projection of this state onto the subspace with one memory excitation in each location is

$$|\Psi_{AZ}| = \frac{1}{\sqrt{2}}(a_1^{+}z_2^{+} + e^{i(\theta_1-\theta_2)}a_2^{+}z_1^{+})|0\rangle,$$

which is analogous to conventional polarization or time-bin entangled states. The required projection can be performed post-selectively by converting the memory excitations back into photons and counting the number of photons in each location. Measurements in arbitrary bases are possible by combining modes $a_1$ and $a_2$ (and also $z_1$ and $z_2$) on beam splitters with appropriate transmission coefficients and phases.

The repeater scheme described above is attractive because it requires only pair sources, photon memories, and linear optical components. The reliance on a single detection for the elementary entanglement creation makes it less sensitive to fiber losses than schemes based on coincident two-photon detections [7]. The price to pay is the requirement of phase stability cf. below.

The time required for a successful creation of an entangled state of the form Eq. (3) is given by

$$T_{\text{tot}} = \frac{L_0}{c} \left[ \frac{1}{P_0P_1\cdots P_nP_{pr}} \left( \frac{2}{\sqrt{2}} \right)^{n+1} \right].$$

Figure 1. Quantum repeater scheme using pair sources and multimode memories. (a) The sources $S_A$ and $S_B$ can each emit a photon pair into a sequence of time bins. The detection of a single photon behind the beam splitter at the central station projects the memories $M_A$ and $M_B$ into an entangled state Eq. (2). (b) If entangled states have been established between the $m$th time bin in $M_A$ and $M_B$, an entangled state between the $n$th time bin in $M_A$ and the $r$th time bin in $M_B$ can be created by reconverting the memory modes into photonic modes and combining them on a beam splitter. (c) Useful entanglement can be created between two pairs of distant memories as in Ref. [2]. Again the appropriate time bins have to be combined on beam splitters.
same factor, in particular, schemes based on coincident two-photon detection [7], because the speedup occurs at the most basic level, that of elementary entanglement generation. There is no obvious equivalent to the use of MMMs as described above within the Raman-transition based approach of Ref. [2], since all stored modes would be retrieved at the same time, when the relevant control beam is turned on. Other forms of multiplexing (spatial, frequency) can be applied to both approaches [8].

We now discuss how to realize the elements of our proposal in practice. Photon pair sources with the required properties (sufficiently high $p$, appropriate bandwidth cf. below) can be realized both with parametric down-conversion [9] and with atomic ensembles [6,10]. Several approaches to the realization of photon memories have been proposed and studied experimentally, including electromagnetically induced transparency [11], off-resonant interactions [12], and photon echo [13]. The echo approach lends itself naturally to the storage of many temporal modes. Storage and retrieval of up to 1760-pulse sequences has been demonstrated [14]. The temporal information is stored in the relative phases of atomic excitations at different frequencies. Photon echoes based on controlled reversible inhomogeneous broadening (CRIB) [15,16] allow in principle perfect reconstruction of the stored light. The method is well adapted to atomic ensembles in solids, e.g., crystals doped with rare-earth-metal ions. To implement such a memory, one has to prepare a narrow absorption line inside a wide spectral hole, using optical pumping techniques [17]. The line is artificially inhomogeneously broadened, e.g., by applying an electric field gradient. Then the light can be absorbed, e.g., a train of pulses as described above. After the absorption, the electric field is turned off, and atoms in the excited state are transferred by the preparation of the initial spectral hole (e.g., another hyperfine state). For recall, the population is transferred back to the excited state by a counterpropagating pulse, e.g., a different hyperfine state. This leads to a time reversal of the absorption. The pulse train is reemitted in inverted order, with a retrieval efficiency that is not limited by reabsorption [15]. Photons absorbed in different memories at different times can be reemitted simultaneously (as in Fig. 1) by choosing appropriate times for the sign-flip of the applied electric field.

The achievable memory efficiency is [18]

$$\eta_M(t) = (1 - e^{-\alpha_0 t \gamma_0 / \gamma})^2 \text{sinc}^2(\gamma_0 t),$$

where for simplicity we consider square spectral atomic distributions, both for the initial narrow line and the artificially broadened line. Here $\alpha_0 L$ is the optical depth of the medium before the artificial broadening, $\gamma_0$ is the initial spectral distribution width, $\gamma$ is the width after broadening, and $t$ is the time before transfer to the hyperfine state (neglecting hyperfine decoherence). The above formula is exact for all pulse shapes whose spectral support is completely inside the square atomic distribution. Otherwise there are additional losses due to spectral truncation of the pulse. The width $\gamma$ has to be large enough to allow for pulse durations significantly shorter than the interval between pulses $\Delta t$, in order to avoid errors due to pulse overlap. For truncated Gaussian pulses choosing $\gamma \Delta t = 6$ is sufficient for such errors to be negligible. On the other hand, $\gamma$ is required to be smaller than the separation between the hyperfine states used in the memory protocol and whatever state is used for shelving unwanted atoms in the preparation of the initial spectral hole (e.g., another hyperfine state). The initial width $\gamma_0$ has to fulfill $\gamma_0 > 2 \gamma_h$, where $\gamma_h$ is the homogeneous linewidth of the relevant transition; $\gamma_0$ should be chosen such as to optimize $\eta_M$, the average of Eq. (5) over all time bins, i.e., for $t$ between 0 and $N \Delta t$. One can show that $\eta_M$ can be expressed as a function of the two variables $x = \gamma_0 N \Delta t$ and $y = \alpha_0 L / N$. By adjusting $\gamma_0$ one can choose, for a given value of $y$, the value of $x$ that maximizes $\eta_M$. Then $\eta_M$ becomes a function of $y$ only, which is plotted in Fig. 2.

The storage has to be phase preserving, so as to conserve the entanglement for the states of Eqs. (2) and (3). Decoherence can affect the excited states during the absorption of the pulse train and the hyperfine ground states during the long-term storage. In rare-earth-metal ions, excited state coherence times ranging from tens of $\mu$s to 6 ms [19] and hyperfine coherence times as long as 30 s [20] have been demonstrated.

We now discuss how to achieve high values for $N$ and $\eta_M$ experimentally. Pr:Y$_2$SiO$_5$ is a very promising material for initial experiments, since excellent hyperfine coherence [20] and memory efficiencies of order 13% [16] for macroscopic light pulses have already been demonstrated. The main drawback of Pr is the small hyperfine separation (of order a few MHz), which limits the possible pulse bandwidth and thus $N$, since by definition $N \geq \frac{\Delta v}{\Delta v_{6}\text{c}} \approx \frac{\Delta v}{\Delta v_{6}}$. Neodymium has hyperfine separations of hundreds of MHz [21]. Nd also has strong absorption, e.g., Nd:YVO$_4$
with a Nd content of 10 ppm has an absorption coefficient $\alpha_0 = 100/\text{cm}$ [19] at 879 nm. Choosing $\gamma = 300 \text{ MHz}$ (which gives $\Delta t = 6/\gamma = 20 \text{ ns}$) and $\gamma_0 = 100 \text{ kHz}$, which is well compatible with $\gamma_\lambda = 10 \text{ kHz}$ as measured for Nd in [19], our above calculations show that, e.g., $N = 400$ and $\bar{\eta}_M = 0.9$ would be possible with $\alpha_0 L = 30N = 12000$, which could be achieved with a multipass configuration.

To assess the potential performance of our scheme, consider a distance $L = 1000 \text{ km}$ and a fiber attenuation of 0.2 $\text{ dB/km}$, corresponding to telecom wavelength photons. Note that the wavelengths of the photon propagating in the fiber and of the photon stored in the memory can be different in our scheme. Assume $\bar{\eta}_M = 0.9$ and photon-number-resolving detectors with efficiency $\eta_D = 0.9$. Highly efficient number-resolving detectors are being developed [22,23]. One can show that the optimal nesting level for the repeater protocol for these values is $n = 2$, corresponding to $2^n = 4$ elementary links, which gives $L_0 = 250 \text{ km}$. Using Eq. (4) and Ref. [2] one can show that the total time for creating a state of the form Eq. (3) using the scheme of Fig. 1 is

$$T_{\text{tot}} = \frac{L_0}{c} \frac{27(2 - \eta)(4 - 3\eta)}{N \gamma P_0 \eta_D \eta_B \eta_B t^4}, \quad (6)$$

where $\eta = \bar{\eta}_M \eta_D$ and $\eta_B = e^{-L_{\text{att}}/(2L_{\text{att}})}$, with $L_{\text{att}} = 22 \text{ km}$, and $c = 2 \times 10^8 \text{ m/s}$ in the fiber. One can show by explicit calculation of the errors due to double emissions that the fidelity $F$ of the final entangled state compared to the ideal maximally entangled state for a repeater with $n = 2$ levels is approximately $F = 1 - 56(1 - \eta)p$. If one wants, for example, $F = 0.9$ one therefore has to choose $p = 0.009$, which finally gives $T_{\text{tot}} = 5100/\text{N s}$. If one can achieve $\bar{\eta}_M = \eta_D = 0.95$ one finds $T_{\text{tot}} = 1200/\text{N s}$. High-efficiency MMMs as discussed above could thus reduce $T_{\text{tot}}$ for 1000 km to a few seconds.

MMMs can also help to significantly alleviate the stabilization requirements for the repeater protocol. For simplicity, let us just consider an elementary link (between locations A and B) from our above example, with $L_0 = 250 \text{ km}$. The entanglement in Eq. (3) depends on the phase difference $\theta_2 - \theta_1$, which can be rewritten as $[\theta_B(t_2) - \theta_B(t_1)] - [\theta_A(t_2) - \theta_A(t_1)]$. Here $t_1(t_2)$ is the time when the first (second) entangled state of the type of Eq. (2) is created. The phases thus have to remain very stable on the time scale given by the typical value of $t_2 - t_1$. In the case without MMMs the mean value of $t_2 - t_1 = L_0/(c P_0)$, which is of order 10 s for our above example ($L_0/c$ of order 1 ms and $P_0$ of order $10^{-4}$). Over such long time scales, both the phases of the pump lasers and the fiber lengths are expected to fluctuate significantly. Active stabilization would thus be required. With MMMs, for large values of $N$, $P_0$ can be made sufficiently large that it becomes realistic to work only with states Eq. (3) where the initial entanglement between $A_1$ and $B_1$ and between $A_2$ and $B_2$ was created in the same interval $L_0/c$ (the probability for such a double success is $P_0^2$). For our above example, $P_0$ can be made of order $10^{-4}$ for $N$ of order 10$^3$.

Working only with entangled states from the same interval increases the time $T_{\text{tot}}$ by a factor of order $1/P_0$, but it reduces the mean value of $t_2 - t_1$ to of order $N \Delta t$ (tens of $\mu$s in our example). For such short time scales, active stabilization of the laser and fiber phases may not be required.

In conclusion, the combination of photon pair sources and multimode memories should allow the realization of a quantum repeater protocol that is much faster and more robust than the protocol of Ref. [2] while retaining its attractive features, in particular, the use of linear optical elements and of single photon detections for entanglement generation.

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