Purification of Single-Photon Entanglement

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Single-photon entanglement is a simple form of entanglement that exists between two spatial modes sharing a single photon. Despite its elementary form, it provides a resource as useful as polarization-entangled photons and it can be used for quantum teleportation and entanglement swapping operations. Here, we report the first experiment where single-photon entanglement is purified with a simple linear-optics based protocol. In addition to its conceptual interest, this result might find applications in long distance quantum communication based on quantum repeaters.

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Entanglement purification provides a fascinating conceptual viewpoint to gain insight into the properties of entanglement. It can be used for the quantification of entanglement in bipartite systems [1]. It may also be useful in practical applications, e.g., in the context of long distance quantum communication where the direct transmission of photons through an optical fiber is limited by losses and the no-cloning theorem. This can be overcome using quantum repeaters [2], which require the creation of entanglement over short links, the storage of entangled states within these links, and entanglement swapping operations to distribute entangled states over longer distances. In practice, these operations introduce errors, limiting the number of links that can be used. While the most immediate goal of outperforming the direct transmission may not need purification, the entanglement distribution within future quantum networks requires a larger number of links, necessitating several purification operations [3].

Initial proposals by Bennett et al. [4] and Deutsch et al. [5] for entanglement purification were expressed in terms of quantum gates. For practical applications, e.g., in the frame of quantum repeaters, it is important to keep implementations as simple as possible. For example, the protocol presented in Ref. [6] and implemented as reported in Ref. [7] requires linear optical elements only, and can easily be integrated into quantum repeater architectures. However, this last proposal is suited to the purification of polarization-entangled pairs of photons whereas many attractive quantum repeater protocols [8–10] and related experiments [11,12] use single-photon entanglement, i.e., entanglement of the form $|1⟩_A|0⟩_B + |0⟩_A|1⟩_B$, where two modes $A$ and $B$ share a single photon. First, these repeaters are rather simple: they require significantly fewer resources than other protocols and are thus less sensitive to memory and photon detector inefficiencies [3]. Furthermore, these quantum repeaters are efficient since they offer high entanglement distribution rates when combined with temporal multiplexing [9]. The main drawback of protocols based on single-photon detections is that, unlike protocols based on two-photon detections, they are interferometrically sensitive to path length fluctuations [13] that are at the origin of phase errors. An active stabilization system, such as the one reported in [14], would be required. However, remaining path length fluctuations and additional phase noise coming from unfaithful quantum memories and imperfections in entanglement swapping operations would likely require purification. Here, we report the first experimental implementation of a protocol for phase-error purification of single-photon entanglement based on linear optics.

The principle of purification for phase errors (see Ref. [15] for details) can be illustrated as follows. Alice and Bob, two protagonists located at remote locations $A$ and $B$ respectively, wish to share a maximally entangled state $ψ^{ab} = \frac{1}{\sqrt{2}} (|1⟩_A|0⟩_B + |0⟩_A|1⟩_B) = \frac{1}{\sqrt{2}} (a^\dagger + b^\dagger)|0⟩$, but due to phase errors, they share a state which has an admixture of the singlet state $ψ^{a_b}$:

$$ρ_{ab} = F|ψ^{ab}⟩⟨ψ^{ab}| + (1 - F)|ψ^{a_b}⟩⟨ψ^{a_b}|. \quad (1)$$

$F$ is the fidelity of the shared state: if $F = 1/2$, the phase information is lost and no entanglement is left while in the case where $F = 1$, the state is maximally entangled. Note that these phase errors are the most important. The empty component $|0⟩_A|0⟩_B$ does not affect the fidelity of the distributed state since the final step of single-photon protocols postselects the cases where there was a photon in the output state [8]. The multiphoton components $|1⟩_A|1⟩_B$ can be greatly reduced using specific architectures [10].

Suppose that Alice and Bob share two copies of the state described by (1), $ρ_{a'b'}$, with fidelity $F_1$ and $ρ_{a'b'}$, with fidelity $F_2$ (see Fig. 1). Alice and Bob both perform unitary transformations on their modes $a_1, a_2$ and $b_1, b_2$ respectively: Alice combines the two modes $a_1, a_2$ on a beam splitter with an intensity transmission of 85% and Bob uses a beam splitter with an intensity transmission of 15%. The detection of a single photon by Alice in mode $d_a$ (or by Bob in mode $d_b$), projects the modes $a, b$ on a mixed state.
close to divides them by a factor of 2, i.e., with fidelity $b$ order of is substantially purified. As an example, if errors are of the on polarization entanglement squares the errors, i.e., Note also that the previously mentioned proposal [6] based that provided both indistinguishable photons and high demands a good overlap of the temporal, spectral, spatial and polarization modes of the photons. Thus, the successful construction and operation of an experimental setup that provided both indistinguishable photons and high visibility single-photon interference allows us, for the first time, to demonstrate the purification of single-photon entanglement.

The experimental setup is shown in Fig. 2. A Ti-indiffused 7 μm wide waveguide in 25 mm long periodically poled ($\Lambda = 9.15 \, \mu$m) lithium niobate (Ti:PPLN) operated at 43 °C—monomode around 1.5 μm wavelength—creates degenerate photon pairs through the process of spontaneous parametric down conversion. The periodicity of $\Lambda$ has been chosen to obtain “type-II” quasi-phase-matching for orthogonally polarized signal and idler photons. The waveguide is pumped by a continuous-wave single-mode external cavity diode laser at 780 nm (Toptica DL100). After the waveguide, the remaining light from the pump laser is blocked by a silicon filter. The signal and idler photons, with a spectral width of 3 nm (full width at half maximum) centered at 1560 nm, both pass through the same narrowband filter with a bandwidth of 1.3 nm reducing their spectral distinguishability. The photons are then separated with a polarizing beam splitter (PBS) and coupled into single-mode optical fibers. Each photon is sent through a 50/50 coupler (BS1 and BS2) to prepare the two-mode entangled states $\rho_{a_1b_1}$ and $\rho_{a_2b_2}$. These states are then distributed between Alice and Bob. They each receive two modes, one from each entangled state, and combine them using couplers BS4 and BS5, respectively. These last two couplers are manual
variable-ratio evanescent wave couplers (Canadian Instrumentation & Research Ltd).

Finally, for all measurements we use two single-photon detectors: the heralding detector is a free-running InGaAs/InP avalanche photodiode with homemade electronics [16]. Depending on the measurement, this detector is at positions $D_1$, $D_2$, or $D_3$ in Fig. 2. The quantum efficiency is 10.2% with 1.3 kHz of dark counts for a temperature of $-50^\circ C$ and a bias voltage of $-48.5$ V. Its effective efficiency decreases as the number of singles increases, since the detection rate is limited by a dead time of 31 $\mu s$. The triggered detector is an InGaAs avalanche photodiode (idQuantique, id200) working in gated mode of using the auxiliary measurement interferometer (depicted in Fig. 2). After performing these adjustments, we determined the initial fidelity of $\rho_{a_1 b_1}$ as $F_1 = (97.8 \pm 0.2)\%$ while $\rho_{a_2 b_2}$ has a fidelity $F_2 = (97.7 \pm 0.2)\%$. The residual 2% is mainly due to path length instabilities (see the supplementary information in Ref. [18]).

This purification protocol works for a wide range of fidelities $F_1$ and $F_2$, but it is at fidelities close to $F_1 = F_2 = 76\%$ where the fidelity increase is greatest [15]. To reduce the initial fidelities, we generate noise in a controlled and reproducible way with two additional circular piezos ($\phi_1$ for state $\rho_{a_1 b_1}$ and $\phi_2$ for state $\rho_{a_2 b_2}$) that vibrate at a frequency much higher than the integration time of the measurement. This noise is independently generated for each piezo. We chose a function that reproduces the Gaussian phase-noise distribution in a fiber, as observed in real world networks [13]. Interference fringes measured in one of the entangled states and the reduction of the fidelity due to the noise generation are shown in Fig. 3(a). After applying the noise, the fidelities reduce to $F_1 = (75.1 \pm 0.8)\%$ and $F_2 = (75.0 \pm 0.7)\%$.

To prepare the purified state, the variable couplers BS4 and BS5 are adjusted to the intensity transmissions re-

FIG. 3 (color online). Raw interference fringes observed while the phase $\Phi$ is being scanned. (a) Coincidences between detectors $D_1$ (herald) and $D_4$ corresponding to the state $\rho_{a_1 b_1}$. Initially, the fidelity is $F_2 = (97.7 \pm 0.2)\%$. When the noise generator $\Phi_2$ is switched on, the fidelity decreases to $F_2 = (75.0 \pm 0.7)\%$. (b) Coincidences between detectors $D_3$ and $D_4$ corresponding to the purified state $\rho_{a_2 b_2}$. While both noise generators are on, the fidelity is $\tilde{F} = (79.6 \pm 1.1)\%$. These values are obtained after the subtraction of accidental coincidences due to dark counts. Even when they are not subtracted, a definite purification effect can still be observed. The vertical lines mark every time the voltage ramp reaches its end, reversing the scan direction.
FIG. 4 (color online). Distribution of the fidelity measurements. Probability densities $P$ as a function of the measured fidelities for (a) state $\rho_{a_1 b_1}$, (b) state $\rho_{a_2 b_2}$ and (c) the purified state $\rho_{\tilde{a} \tilde{b}}$. The corresponding mean fidelity and standard deviation are given next to each distribution. The curves are Gaussian functions with the same mean and $\sigma$ as the sample data. The number of bins chosen is the optimal one using Scott’s principle [22]. The agreement with the histogram can be seen by the fact that the fitted curve passes through the center of most histogram bars.

quired to apply the purification protocol [15], corresponding to 85% for Alice and 15% for Bob (or vice versa). Modes $a_1$ and $a_2$ are combined by Alice to form modes $\tilde{a}$ and $d_a$, while modes $b_1$ and $b_2$ are combined by Bob to form modes $\tilde{b}$ and $d_b$. Conditioned on the detection of one photon at $d_a$ (with detector $D_3$), modes $\tilde{a}$ and $\tilde{b}$ become purified. To verify this, they are combined at coupler BS3 and detected at $D_4$. Again, because we cannot know which path the photons have taken, interference fringes are observed when the phase $\Phi$ is scanned [see Fig. 3(b)]. As before, the fidelity $\tilde{F}$ of the state $\rho_{\tilde{a} \tilde{b}}$ is deduced from the visibility of the fringes.

For each of the entangled states ($\rho_{a_1 b_1}$, $\rho_{a_2 b_2}$ and $\rho_{\tilde{a} \tilde{b}}$), measurements of several interference fringes were obtained. Using sequential sinusoidal fits of approximately two periods, we calculated the fidelities for all fringes. The resulting distributions of fidelity values are represented in Fig. 4. From each set of values, the mean fidelities $F_1$, $F_2$, and $\tilde{F}$ were calculated. The given uncertainty values are the standard deviations ($\sigma$) associated with each distribution [18].

After the implementation of the purification protocol, we obtain a state $\rho_{\tilde{a} \tilde{b}}$ with fidelity $\tilde{F} = (79.6 \pm 1.1)\%$. The improvement in the degree of entanglement, taken as the difference between $\tilde{F}$ and $F_1$, is as high as 4.5%. Note that it has been shown in Ref. [15] that the optimal theoretical value is of 5.7%. We believe that the remaining 1.2% is due to phase fluctuations of modes and to the uncertainty in the transmission of variable couplers [18]. As shown in Fig. 4, the overlap between the distributions of initial and purified fidelity values is negligible, leaving no doubt about the influence of the purification effect.

Single-photon entanglement has been at the heart of a lively debate [19,20]. Part of the controversy has been solved by mapping single-photon entanglement into two atomic ensembles and by revealing the entanglement between these ensembles [12]. Note also that entanglement between four modes sharing a single photon has been characterized by direct measurements of the optical modes [21]. Our experiment further shows that single-photon entanglement can be purified using linear optics. Looking further ahead, this simple protocol could be useful in the context of quantum repeaters.

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[18] See supplementary material at http://link.aps.org/supplemental/10.1103/PhysRevLett.104.180504 for more detailed information about the measurement procedure, the data analysis, accidental coincidences and the imperfect HOM dip visibility and fidelities.