Recovering Social Networks from Outcome Data: Identification and an Application to Tax Competition

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Solari Lecture



Networks are Everywhere

- Social and economic networks mediate many aspects of individual choice and outcomes:
 - *Development:* technology adoption, insurance.
 - Peer Effects: learning, delinquency, consumption.
 - IO: buyer-supplier networks, strategic interactions. 💽
 - Macro, Finance and Trade: contagion, gravity equations.
 - Political Economy: yardstick competition.
 - More examples: Jackson [2009], de Paula [forthcoming].

But . . .

- Network information are **not** available in most datasets.
- When available, usually imperfect:
 - Self-reported data (censoring, \neq econ int $\Rightarrow \neq$ ties);
 - Postulated (e.g., classroom, zip code).
- Hence, empirical analysis of network effects may be challenging.
- Existing models are **conditioned** on postulated network.
- Potential for misspecification.



This Project

- We study identification of the unobserved networks and parameters of interest in a social interactions model ... (spatial model with *unobserved* neighbourhood matrix)
- ... under standard network "intransitivity" hypothesis ...
- ... and explore estimation strategies.
 - *N* individuals $\Rightarrow O(N^2)$ parameters to estimate.
 - High-dimensional model techniques.
 - Consistency and asymptotic distribution.

The Model

- Many interdependent outcomes are mediated by connections ("networks").
- A popular representation follows the "linear-in-means" specification suggested in Manski [1993]. For example,

$$y_{it} = \alpha_t + \rho_0 \sum_{j=1}^{N} W_{0,ij} y_{jt} + \beta_0 x_{it} + \gamma_0 \sum_{j=1}^{N} W_{0,ij} x_{jt} + \epsilon_{it}$$

$$\Leftrightarrow$$

$$\mathbf{y}_{t,N\times 1} = \alpha_t \mathbf{1}_{N\times 1} + \rho_0 W_{0,N\times N} \mathbf{y}_{t,N\times 1} + \beta_0 \mathbf{x}_{t,N\times 1} + \gamma_0 W_{0,N\times N} \mathbf{x}_{t,N\times 1} + \epsilon_{t,N\times 1}$$
with $\mathbb{E}(\epsilon_{it} | \mathbf{x}_t, \alpha_t) = \mathbf{0}$.

- Customary to assume $W_0 \mathbf{1} = \mathbf{1}$ and $|\rho_0| < \mathbf{1}$.
- Here we do *not* observe W_0 .

A Motivating Example

 Besley and Case [AER, 1995]: "Incumbent Behavior: Vote-Seeking, Tax-Setting, and Yardstick Competition"

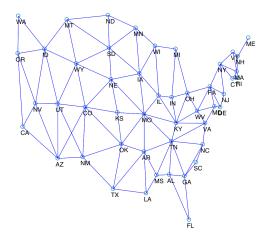
"This paper develops a model of the political economy of tax-setting in a multijurisdictional world, where voters' choices and incumbent behavior are determined simultaneously. Voters are assumed to make comparisons between jurisdictions to overcome political agency problems. This forces incumbents in to a (yardstick) competition in which they care about what other incumbents are doing."

From data on state tax liabilities from 1962 until 1988, the authors estimate (essentially):

$$\Delta \tau_{it} = \alpha_t + \rho_0 \sum_{j=1}^N W_{0,ij} \Delta \tau_{jt} + \beta_0 x_{it} + \gamma_0 \sum_{j=1}^N W_{0,ij} x_{jt} + \epsilon_{it}$$

Neighbouring states are geographically adjacent ones.

In other words...



Could there be relevant, non-adjacent states? Do all adjacent states matter?



(Some) Literature

- 1. Spatial Econometrics, conditional on W_0 .
 - Kelejian and Prucha [1998, 1999], Lee [2004], Lee, Liu and Lin [2010] and Anselin [2010].

2. Identification.

- ... conditional on W₀: Manski [1993], Bramoullé, Djebbari and Fortin [2009], De Giorgi, Pellizzari and Redaelli [2010];
- ► ... not conditional on W₀: Rose [2015], see also Blume, Brock, Durlauf and Jayaraman [2015].

3. Estimating W_0 .

- Lam and Souza [various].
- Manresa [2015], Rose [2015], Gautier and Rose [2016].

Identification (Known W_0)

▶ Manski [1993] and the "reflection problem." $(W_{0,ij} = (N-1)^{-1} \text{ if } i \neq j, W_{0,ii} = 0)$





Identification (Known W_0)

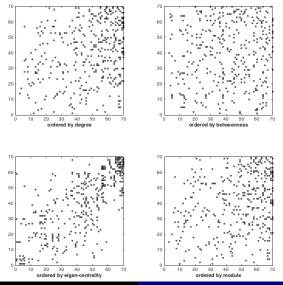
• Potential avenue: "exclusion restrictions" in W_0 .

If $\rho_0\beta_0 + \gamma_0 \neq 0$ and I, W_0 , W_0^2 are linearly independent, $(\rho_0, \beta_0, \gamma_0)$ is point-identified. (Assuming $\alpha_t = 0$.) (Bramoullé, Djebbari and Fortin [2009])

Linear independence valid generally. In fact,

 $\sum_{j=1}^{N} W_{0,ij} = 1$ and **I**, W_0 , W_0^2 linearly dependent $\Rightarrow W_0$ block diagonal with blocks of the same size and nonzero entries are $(N_l - 1)^{-1}$. (Blume, Brock, Durlauf and Jayaraman [2015])

Figure: High School Friendship Network



Áureo de Paula

Identifying Social Connections

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What if W_0 is unknown?

 "If researchers do not know how individuals form reference groups and perceive reference-group outcomes, then it is reasonable to ask whether observed behavior can be used to infer these unknowns" (Manski [1993])



Identification

► The model has reduced-form (assuming, for simplicity that $\alpha_t = 0$)

$$\mathbf{y}_t = \Pi_0 \mathbf{x}_t + \mathbf{v}_t$$

where

$$\Pi_0 = (\mathbf{I} - \rho_0 W_0)^{-1} (\beta_0 \mathbf{I} + \gamma_0 W_0)$$

• If $(\rho_0, \beta_0, \gamma_0)$ were known, W_0 would be identified:

$$W_0 = (\Pi_0 - \beta_0 \mathbf{I})(\rho_0 \Pi_0 + \gamma_0 \mathbf{I})^{-1}$$

• In practice, $(\rho_0, \beta_0, \gamma_0)$ is not known.



Identification

- Further assumptions are necessary to identify $\theta_0 = (\rho_0, \beta_0, \gamma_0, W_0).$
- ► Take, for example, θ_0 and θ such that $\beta_0 = \beta = 1$, $\rho_0 = 0.5$, $\rho = 1.5$, $\gamma_0 = 0.5$, $\gamma = -2.5$,

$$W_0 = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0.5 \\ 0.5 & 0 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0.5 & 0 \end{bmatrix} W = \begin{bmatrix} 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \end{bmatrix}$$

- Then $(I \rho_0 W_0)^{-1} (\beta_0 I + \rho_0 W_0) = (I \rho W)^{-1} (\beta I + \rho W).$
- (Notice that I, W_0 and W_0^2 are LI and so are I, W and W^2 !)

But . . .

If the spectral radius of ρ₀W₀ is less than one, then an eigenvector of W₀ is also an eigenvector of Π₀.

Take the reduced-form parameter matrix:

$$\Pi_{0} = (I + \rho_{0} W_{0} + \rho_{0}^{2} W_{0}^{2} + \cdots) (\beta_{0} I + \gamma_{0} W_{0})$$

= $\beta_{0} I + (\rho_{0} \beta_{0} + \gamma_{0}) W_{0} + \rho_{0} (\rho_{0} \beta_{0} + \gamma_{0}) W_{0}^{2} + \cdots$

Postmultiplying by v_i , an eigenvector of W_0 ,

$$\Pi_0 \mathbf{v}_j = \frac{\beta_0 + \gamma_0 \lambda_{j,0}}{1 - \rho_0 \lambda_{j,0}} \mathbf{v}_j$$

► If *W*₀ is nonnegative and irreducible, e.g., only one eigenvector can be chosen to have positive entries.



Local Identification

• Can the model identify $\theta_0 = (\rho_0, \beta_0, \gamma_0, W_0)$?

Assume:

(A1) $(W_0)_{ii} = 0, i = 1, ..., N$ (no self-links);

- (A2) $\sum_{j=1}^{N} |(W_0)_{ij}| \le 1$ for every i = 1, ..., N and $|\rho_0| < 1$;
- (A3) There is *i* such that $\sum_{i=1}^{N} (W_0)_{ij} = 1$ (normalization);
- (A4) There are *I* and *k* such that $(W_0^2)_{II} \neq (W_0^2)_{kk} \iff I, W_0, W_0^2$ LI as in Bramoullé, Djebbari and Fortin [2009]);

(A5) $\beta_0 \rho_0 + \gamma_0 \neq 0$ (social effects do not cancel).

 Under (A1)-(A5) (ρ₀, β₀, γ₀, W₀) is locally identified. (Application of Rothenberg [1971].)

 Under (possibly strong) conditions it is straightforward to obtain global identification.

Under Assumptions (A1) and (A3), if ρ₀ = 0, then (γ₀, β₀, W₀) is globally identified.
 (As in, e.g., Manresa [2015].)

Under Assumptions (A1)-(A3) and (A5), if γ₀ = 0, then (ρ₀, β₀, W₀) is globally identified. (γ₀ = 0 ⇒ exclusion restrictions.)

- It is nevertheless possible to strengthen local identification conclusions obtained previously.
- Assume (A1)-(A5). {θ : Π(θ) = Π(θ₀)} is finite.
 (This obtains as Π(θ) is a proper mapping.)
- Let $\Theta_+ = \{\theta \in \Theta : \rho\beta + \gamma > 0\}$. Then we can state that:

Assume (A1)-(A5), then for every $\theta \in \Theta_+$ we have that $\Pi(\theta) = \Pi(\theta_0) \Rightarrow \theta = \theta_0$. That is, θ_0 is globally identified with respect to the set Θ_+ .



This uses the following result:

Suppose the function $\Pi(\cdot)$ is continuous, proper and locally invertible with a connected image. Then the cardinality of $\Pi^{-1}(\{\overline{\Pi}\})$ is constant for any $\overline{\Pi}$ in the image of $\Pi(\cdot)$. (see, e.g., Ambrosetti and Prodi [1995], p.46)

- We show that the mapping Π : Θ₊ → ℝ^{N×N} is proper with connected image, and non-singular Jacobian at any point.
- This implies that the cardinality of the pre-image of {Π(θ)} is finite and constant.
- Take θ ∈ Θ₊ such that γ = 0. The cardinality of Π⁻¹({Π(θ)}) is one for such θ and the result follows.

Since an analogous result holds for $\Theta_{-} = \{\theta \in \Theta \text{ such that } \rho\beta + \gamma < 0\}$, we can state that:

Assume (A1)-(A5). The identified set contains at most two elements.

Furthermore, if $\rho_0 > 0$ and $(W_0)_{ij} \ge 0$ one is able to sign $\rho_0\beta_0 + \gamma_0$ and obtain that:

Assume (A1)-(A5), $\rho_0 > 0$ and $(W_0)_{ij} \ge 0$. Then θ_0 is globally identified.

Finally, if W₀ is non-negative and irreducible, one is also able to sign ρ₀β₀ + γ₀!

Assume (A1)-(A5). $(W_0)_{ij} \ge 0$ and W_0 irreducible. Then θ_0 is globally identified if W_0 has at least two real eigenvalues or $|\rho_0| \le \sqrt{2}/2$.

A Few Remarks

- v_j is an eigenvector of Π₀ and W₀: eigencentralities are identified even when W₀ is not.
- Row-sum normalization of W₀ implies that row-sum of Π is constant: testable hypothesis.
- ► We also allow for individual and time specific effects.
- ► Analysis extends to multivariate x_{i,t}. The reduced-form model is

$$\mathbf{y}_t = \sum_{s=1}^k \Pi_{0,s} \mathbf{x}_{t,s} + \mathbf{v}_t$$

where $\mathbf{x}_{t,s}$ refers to the *s*-th column of \mathbf{x}_t and

$$\Pi_{\mathbf{0},\boldsymbol{s}} = (\mathbf{I} - \rho_{\mathbf{0}} W_{\mathbf{0}})^{-1} (\beta_{\mathbf{0},\boldsymbol{s}} + \gamma_{\mathbf{0},\boldsymbol{s}} W_{\mathbf{0}}).$$

Estimation Strategies

• Π has N^2 parameters, and possibly $NT \ll N^2$.

- Feasible if W or Π are sparse. (e.g., Atalay et al. [2011] < 1%; Carvalho [2014] ≈ 3%; AddHealth ≈ 2%).</p>
- ► Sparsity on *W* or Π?
- Explore the relation between structural- and reduced-form sparsities (in paper).



Rewrite the model as

$$y_i = x_i^\top \pi_i + v_i$$

stacking all observations for individual *i* at t = 1, ..., T.

Penalization in the reduced form (e.g., AdaLasso of Kock and Callot [2015]:

$$\tilde{\pi}_i = \operatorname*{arg\,min}_{\pi_i \in \mathbb{R}^N} \frac{1}{T} \| y_i - x_i^\top \pi_i \|_2 + 2\lambda_T \| \pi_i \|_1$$

and

$$\hat{\pi}_{i} = \operatorname*{arg\,min}_{\pi_{i} \in \mathbb{R}^{N}} \frac{1}{T} \| y_{i} - x_{i}^{\top} \pi_{i} \|_{2} + 2\lambda_{T} \sum_{\tilde{\pi}_{ij} \neq 0} \left| \frac{\pi_{ij}}{\tilde{\pi}_{ij}} \right|$$

with λ_T chosen by BIC).



- Penalization in the structural form (e.g., Adaptive Elastic Net GMM of Caner and Zhang [2014]:
- $\mathbf{x}_t \perp \epsilon_t \Rightarrow$ moment conditions.

$$\tilde{\theta} = (1 + \lambda_2 / T) \cdot \operatorname*{arg\,min}_{\theta \in \mathbb{R}^p} \left\{ g(\theta)^\top M_T g(\theta) + \lambda_1 \sum_{i,j=1}^n |w_{i,j}| + \lambda_2 \sum_{i,j=1}^n |w_{i,j}|^2 \right\}$$

and

$$\hat{\theta} = (1 + \lambda_2 / T) \cdot \argmin_{\theta \in \mathbb{R}^p} \left\{ g(\theta)^\top M_T g(\theta) + \lambda_1^* \sum_{\tilde{w}_{i,j} \neq 0} \frac{|w_{i,j}|}{|\tilde{w}_{i,j}|^{\gamma}} + \lambda_2 \sum_{i,j=1}^n |w_{i,j}|^2 \right\}$$

where $\theta = (\text{vec}(W)^{\top}, \rho, \beta, \gamma)^{\top}$ and λ_1^*, λ_1 and λ_2 chosen by BIC.)



Simulations

- Estimators: GMM Adaptive Elastic Net, Adaptive Lasso, SCAD, OLS.
- $\rho_0 = 0.3, \beta_0 = 0.4, \gamma_0 = 0.5.$
- 1,000 simulations.
- ▶ In the paper: N = 15, 30, 50. T = 50, 100, 150.

- Many versions in the paper: time and individual effects, correlated effects, other network generating processes.
- ► Here: High School Friendship (Coleman [1964]), N = 73, T = 50, 100.

Figure: High School Friendship Network

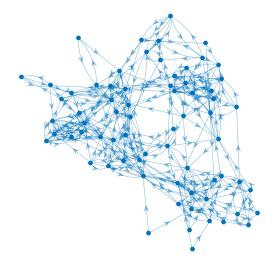
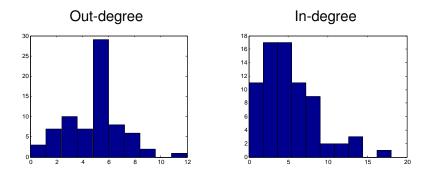




Figure: High School Friendship Network Degree Distribution

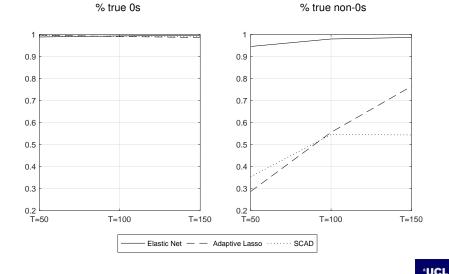


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Simulations: High School Friendships

	Ø	EN	\mathbf{AL}	\mathbf{SC}	OLS		Ø	EN	\mathbf{AL}	\mathbf{SC}	OLS
		n =	= 73 , T =	= 50		-		n =	73 , T =	= 100	
$mse(\hat{\Pi})$	0.000 (0.000)	0.083 (0.188)	$\begin{array}{c} 0.356 \\ \scriptscriptstyle (0.133) \end{array}$	0.331 (0.127)	_		0.000 (0.000)	0.064 (0.163)	0.244 (0.014)	0.256 (0.038)	3.447 (0.242)
$mse(\hat{W})$	0.000 (0.000)	0.082 (0.183)	0.480 (0.183)	0.682 (0.309)	_		0.000	0.047 $_{(0.118)}$	$\begin{array}{c} 0.507 \\ (0.083) \end{array}$	0.618 (0.129)	3.627 (0.637)
% true 0s	1.000 (0.000)	0.989 (0.024)	0.998 (0.001)	0.995 (0.005)	_		1.000 (0.000)	0.994 (0.016)	0.991 (0.003)	0.991 (0.004)	0.005 (0.001)
% true 1s	1.000 (0.000)	0.946 (0.122)	0.287 (0.268)	0.354 $_{(0.257)}$	_		1.000 (0.000)	0.980 (0.052)	0.556 (0.055)	0.546 (0.131)	$0.999 \\ (0.004)$
$\hat{ ho} - ho_0$	0.000 (0.000)	-0.252 $_{(0.063)}$	-0.252 (0.029)	-0.270 $_{(0.020)}$	-		0.000 (0.000)	-0.149 (0.066)	-0.258 $_{(0.025)}$	-0.265 (0.023)	0.026 (0.068)
$\hat{eta} - eta_0$	0.000	0.004 (0.013)	-0.351 (0.131)	-0.337 (0.130)	_		(0.000)	0.003 (0.009)	-0.257 $_{(0.040)}$	-0.270 (0.051)	-0.039
$\hat{\gamma} - \gamma_0$	0.000 (0.000)	$\underset{(0.234)}{0.101}$	0.013 (0.093)	-0.057 (0.088)	_		0.000 (0.000)	0.039 (0.104)	-0.053 $_{(0.082)}$	-0.127 (0.084)	0.499 (0.035)

Figure: Sparsity pattern



Yardstick Competition

Besley and Case estimate

$$\Delta \tau_{it} = \alpha_t + \rho_0 \sum_{j=1}^{N} W_{0,ij} \Delta \tau_{jt} + \beta_0 x_{it} + \gamma_0 \sum_{j=1}^{N} W_{0,ij} x_{jt} + \epsilon_{it}$$

using W_0 as the geographically neighbouring states.

We revisit the yardstick competition, estimating and identifying neighbouring states W



Yardstick Competition (B&C [1995])

- Yardstick competition applies to governors not facing term limits.
 - Compare main effects across two subsamples: governor can run for reelection and cannot run for reelection.
- Endogeneity:
 - Neighbours tax rates are endogenous.
 - IVs: neighbour's change of income per capita lagged and neighbours' change of unemployment rate lagged.
- Specification:
 - Controls: neighbors' tax change, state income per capita, state unemployment rate, proportion of young and elderly.
 - All specifications contain state fixed effects and time effects.

- Sample extension:
 - Continental US states, N = 48
 - Original B&C sample: 1962-1988, T = 26 time periods.
 - Extended sample: 1962-2015, T = 53 time periods.

Table 1: Geographic Neighbors

Dependent variable: Change in per capital income and corporate taxes Coefficient estimates, standard errors in parentheses

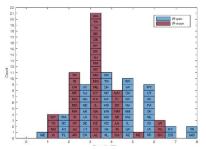
	Besley and Cas	e [1995] Sample	Extended Sample		
	(1) OLS	(2) 2SLS	(3) OLS	(4) 2SLS	
Geographic Neighbors' Tax Change (t - [t-2])	.375***	.868***	.271***	.642***	
	(.120)	(.273)	(.075)	(.152)	
Period	1962-1988	1962-1988	1962-2015	1962-2015	
First Stage (F-stat, p-value)		0.004		0.000	
Controls	Yes	Yes	Yes	Yes	
State and Year Fixed Effects	Yes	Yes	Yes	Yes	
Observations	1,296	1,248	2,592	2,544	

Table 2: Economic Neighbors

Dependent variable: Change in per capital income and corporate taxes Coefficient estimates, standard errors in parentheses

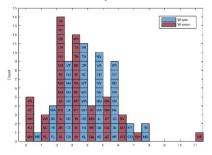
	Not Penalizing Geographic Neighbors No Exogenous Social Effects			Penalizing Geographic Neighbors No Exogenous Social Effects			Penalizing Geographic Neighbors			
									Exogenous Social Effects	
	(1) Initial	(2) OLS	(3) 2SLS	(4) Initial	(5) OLS	(6) 2SLS	(7) Initial	(8) OLS	(9) 2SLS: IVs are Characteristics of Neighbors	(10) 2SLS: IVs are Characteristics of Neighbors-of Neighbors
Economic Neighbors' Tax Change (t - [t-2])	.824	.274***	.652***	.886	.378***	.641***	.645	.145**	.332*	.608***
		(.057)	(.061)		(.061)	(.060)		(.072)	(.199)	(.220)
Period		1962-2015			1962-2015				1962-2015	
First Stage (F-stat, p-value)			.000			.000			.000	.000
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
State and Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2,952	2,952	2,544	2,952	2,952	2,544	2,952	2,952	2,544	2,592





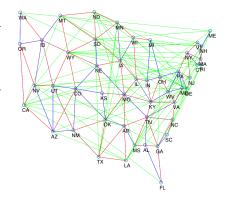
Panel A: In-degree distribution

Panel B: Out-degree distribution





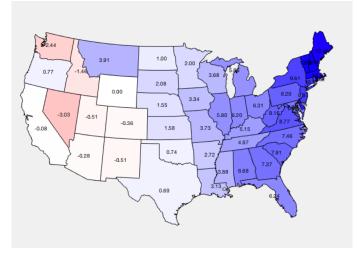
Relative to BC network	
Total number of edges	144
new edges	65
removed edges	135
Reciprocated edges	29.7%
Clustering	0.0259



green = new edges relative to B&C blue = existing edges red = removed edges

- Large discrepancies between estimated network and geo neighbours
- Fewer edges relative to Besley and Case
- Geographically dispersed US tax competition

Figure: Impulse Response Comparison



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Table 4: Predicting Links to Economic Neighbors

Columns 1-7: Linear Probability Model; Column 8: Tobit Dependent variable (Cols 1-7): = 1 if Economic Link Between States Identified Dependent variable (Col 8): =Weighted Link Between States Coefficient estimates, standard errors in parentheses

	Geography		Economic and Demographic Homophyly	Labor Mobility	Political Homophyly	Tax Havens	Tobit, Partial Avg Effects	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Geographic Neighbor	.699***		.701***	.701***	.698***	.698***	.697***	.068***
	(.030)		(.032)	(.030)	(.031)	(.031)	(.031)	(.006)
Distance		453***	008					
		(.033)	(.024)					
Distance sq.		.0949***	.003					
		(.007)	(.006)					
GDP Homophyly				2.409**	2.369*	2.296*	1.046	.322
				(1.183)	(1.186)	(1.193)	(1.150)	(.302)
Demographic Homophyly				.222	.235	.241	.256	.077
				(.226)	(.226)	(.228)	(.225)	(.067)
Net Migration					.044*	.044*	-0.032	0.001
					(.025)	(.025)	(.025)	(.002)
Political Homophyly						057	083**	025*
						(.042)	(.042)	(.014)
Tax Haven Sender							.107***	.021***
							(.024)	(.005)
Adjusted R-squared	0.427	0.152	0.427	0.428	0.429	0.429	0.440	-
Observations	2,256	2,256	2,256	2,256	2,256	2,256	2,256	2,256

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Table 5: Gubernatorial Term Limits

Dependent variable: Change in per capital income and corporate taxes Coefficient estimates, standard errors in parentheses IVs: Characteristics of Neighbors-of Neighbors

	Exogenous Social Effects								
	All Go	vernors		not Run for Re- ction	Governor Can Run tor Re- election				
	(1) OLS	(2) 2SLS	(3) OLS	(4) 2SLS	(5) OLS	(6) 2SLS			
Economic Neighbors' tax change (t - [t-2])	.145**	.608***	.016	.937*	.182**	.543**			
	(.072)	(.220)	(.105)	(.534)	(.084)	(.237)			
Period	1962	2-2015	1962	2-2015	1962	2-2015			
First Stage (F-stat, p-value)		.000		.073		.000			
Controls	Yes	Yes	Yes	Yes	Yes	Yes			
State and Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes			
Observations	2,592	2,592	640	640	1,917	1,917			

Penalizing Geographic Neighbors



Conclusion

- In this project, we study identification of social connections under standard hypothesis in the literature on social interactions.
- Sparsity inducing methods can be used for estimation (though further research is welcome!).
- Empirical application (Besley and Case [1995]).





Thank You!



