Paulus Gerdes¹

Exploration of technologies, emerging from African cultural practices, in mathematics education

Abstract and introduction

When students, pupils, teachers or future mathematics teachers believe that 'mathematics' does not have any roots in 'their' culture, a 'cultural-psychological' blockage exists that hinders the teaching and learning of mathematical thinking, a blockage that turns the realisation of the full development of the mathematical potential of the learners impossible. The study at teacher education institutions of (future) African mathematics teachers and increase their socio-cultural self-confidence and awareness. Exploring educationally mathematical ideas embedded in and derived from technologies in various African cultural practices may contribute further to bridge the gap between 'home' and 'school' culture. Examples of the author's experience in studying and exploring these technologies and cultural practices as varied as storytelling, basket making, salt production, and mat, trap and hat weaving.

ICMI, culture and mathematics education

A milestone in the history of ICMI concerning the recognition of the socio-cultural bases for mathematics education is Ubiratan D'Ambrosio's inaugural plenary lecture at ICME5 (Adelaide, 1984) (see Carss, 1986, 1-6; D'Ambrosio, 1985). From ICME6 onwards, special meetings on ethnomathematics have been organised at ICME. The proceedings of the discussion group on ethnomathematics at ICME10 (Copenhagen, 2004) were published recently in book form (Favilli, 2007). An earlier overview of the tendencies in research on ethnomathematics and mathematics education was included in the *International Handbook of Mathematics Education* (see Gerdes, 1996b).

With the exception of South Africa, African countries generally have been weakly represented at ICME. Peter Lassa (Nigeria) and Mohamed El Tom (Sudan) organised meetings of African delegates at ICME4 (Berkeley, 1980) and ICME5 (Adelaide, 1984), respectively. The papers presented by Sam Ale (Nigeria), Norma Presmeg, John Volmink (South Africa), Siaka Bamba Kanté, Salimata Doumbia (Côte d'Ivoire) in the Fifth Day Special Programme on "Mathematics, Education, and Society" at ICME6 (Budapest, 1988) underscore the importance of cultural aspects in mathematics education in African countries, as does Gerdes' invited lecture at ICME8 (Sevilla, 1996). The realization in the last five years of two regional ICMI conferences in Africa (AFRICME) may mark an important change in African participation in ICMI activities. An important result is the publication of the study *Mathematics Teacher Education: Trends across twelve African countries* (Adler et al., 2007).

¹ Centro de Estudos Moçambicanos e de Etnociência (Centre for Mozambican Studies and Ethnoscience), Universidade Pedagógica, C.P. 915, Maputo, Mozambique (pgerdes@virconn.com)

Mathematics, culture and education in Africa

Otto Raum's book *Arithmetic in Africa* (1938) is one of the earlier reflections about the necessity to take the home culture of the pupils as a starting point in mathematics education in Africa. Internationally well known is the study by the Americans Gay and Cole (1967) about mathematics teaching and learning among the Kpelle in Liberia, who attend 'western-oriented' schools where they are "taught things that have no point or meaning within their culture." Gay and Cole proposed a creative mathematics education that uses 'indigenous' mathematical concepts as a starting point. Zaslavsky's classical study *Africa Counts* (1973) was the first book on mathematical ideas in the socio-cultural context of Africa South of the Sahara. In general, the famous historian Joseph Ki-Zerbo elaborated in his book *Educate or Perish: Africa's Impass and Prospects* (1990) a profound analysis of the dominant neglect of culture in the colonial and post-colonial transplanted school system, underscoring the necessity for Africa education to re-enroot itself.

At the 2nd Pan-African Congress of Mathematicians (Jos, Nigeria, 1986), the African Mathematical Union decided to create a commission on the history of mathematics in Africa (AMUCHMA), alongside its commission on mathematics education (AMUCME). The aim of AMUCHMA is to study and divulge the history of mathematics in Africa, and to give information for teachers and scientists to understand the roots of mathematics in African history and cultures. AMUCHMA produces a newsletter.² Twenty years of information gathering has been joined in an annotated bibliography on Mathematics in African History and Cultures (Gerdes & Djebbar, 2007), including hundreds of references to culture and mathematics education in African countries. The emergence of ethnomathematics as a research field in Africa is explained in (Gerdes, 1995).

Interest in the study of cultural factors is manifested in African doctoral theses in mathematics education since the middle of the 1970s (see e.g. the theses by Geofrey Mmari (1974) [Tanzania], George Eshiwani (1975) [Kenya], Peter Lassa (1975) [Nigeria], Bukunola Mabogunje Osibodu (1975) [Nigeria], Souad Redjeb (1977) [Tunisia], Fatoumata Camara Diallo (1979) [Mali], and Mohamed Moustafa (1979) [Egypt]) until the more recent dissertations by Kalifa Traoré (2006) [Burkina Faso] and Mamadou Kanouté (2007) [Mali]. In the case of southern Africa, the cultural basis of mathematics education, incl. gender, language, cultural games, technology, is increasingly explored in doctoral theses since the beginning of the 21st century; see e.g. the dissertations by Abdulcarimo Ismael (2001) [Mozambique], Elias Kaphesi (2001) [Malawi], Mogege Mosimege (2001), Mathume Bopape (2001) [South Africa], Catherine Chamdimba (2002) [Malawi], Marcos Cherinda (2002), Sarifa Fagilde (2002) [Mozambique], Kgomotso Garegae (2002) [Botswana], David Mogari (2002), Mamokgethi Setati (2002) [South Africa].³ Mosimege and Ismael explore possibilities to use 'cultural games' like board games of the 'mancala' type in mathematics education (cf. Mosimege & Ismael, 2007). Cherinda developed a weaving-board for the exploration of mat weaving in Mozambican classrooms. Daniel Soares is concluding a doctoral thesis on the exploration of ancestral house building techniques in teaching geometry to and by (future) mathematics teachers (see

² Accessible on the web: www.math.buffalo.edu/mad/AMU/amuchma_online.html.

³ See Gerdes, Paulus (2007), *African Doctorates in Mathematics: A Catalogue*, lulu.com, Morrisville NC, 382 pp., and Gerdes, Paulus (2006), African Doctorate Holders in Mathematics Education, *AMUCHMA Newsletter*, Maputo, No. 32.

also the exploration of sticks and ropes in house building in Gay & Cole, 1967; Gerdes, 1988b, Traoré, 2006). The role of technology and activity in the early development of geometrical thinking was analysed in Gerdes' doctoral thesis (1985; English version: 2003a).

Cultural practices, technologies and mathematics teacher education in Mozambique

The first generations of students who came, after Mozambique's independence in 1975, to the mathematics teacher education programmes believed that mathematics was something 'alien', not rooted in African cultures (Gerdes, 1981, 1986, 1998). Stimulated by the political context of 'cultural rebirth', research on mathematical ideas embedded in cultural activities started. Since then, exploration of mathematical ideas embedded in and derived from technologies in various African cultural practices has been a dominant feature of our educational praxis and research, will be illustrated by a few examples (cf. Gerdes, 1986, 1988b, 1999, 2007c).

Squares of cardboard paper have been employed to imitate a basket weaver's technique to produce a pyramidal funnel used for salt production. The basis of the pyramid is an equilateral triangle. The woven pyramids have been explored in the study of polyhedra in space and their volumes. The transformation by the basket weavers of a square mat into a pyramidal funnel has been explored to construct regular polygons.

Traditional hat, basket and fish trap weaving techniques have been explored to weave hexagonal bands and patterns, and derive knowledge about triangles and several polygons. The hexagonal tiling is explored to obtain models, like, e.g. of a layer of graphite. 'Hexastrips' were introduced in the beginning of the 1980s to explore geometry in space by weaving polyhedra with hexagonal, pentagonal, square and triangular holes. Since the discovery of Buckministerfullerene in 1985, hexastrips have been explored to construct models of fullerenes and quasi-fullerenes.

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{i$$

Shape of a hexastrip of cardboard paper. The dotted lines are fold lines. Figure 1

The weaving of a nonahedral musical instrument (Figure 2 displays its shape) has been explored to weave it with cardboard strips (see a strip with fold lines in Figure 2b) and to discover other polyhedra in space (Gerdes, 2005, 2007c).

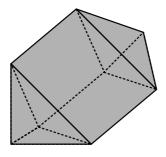


Figure 2

Illustrations in the sand that accompany stories told by Cokwe male educators (Angola) have been studied and explored in mathematics education (cf. Gerdes, 1988a, 1997, 1999, 2006, 2007b; see e.g. also Maffei & Favilli, 2007).

In the case of these examples and in many other instances, we have noted the interest, enthusiasm, engagement and growing socio-cultural self-confidence of the pupils, students and teachers.

References

- Adler, Jill & M. Kazima, W. Mwakapenda, T. Nyabanyaba, S. Xolo (Eds.) (2007), Mathematics Teacher Education: Trends across twelve African countries, ICMI & Marang Centre for Mathematics and Science Education, Johannesburg
- Alsina, Claudi & J. Alvarez, B. Hodgson, C. Laborde, A. Pérez (Eds.) (1996), δth International Congress on Mathematical Education Selected Lectures, S.A.E.M. Thales, Sevilla
- Bishop, Alan & P. Damerow, P. Gerdes, Ch. Keitel (1988), *Mathematics, Education and Society*, in: Hirst o.c., 311-325
- Bishop, Alan & K. Clements, C. Keitel, J. Kilpatrick, C. Laborde (1996), *International Handbook of Mathematics Education*, Kluwer Academic Publishers, Dordrecht
- Carss, Marjorie (Ed.) (1986), Proceedings of the Fifth International Congress of Mathematical Education, Birkhäuser, Boston
- Cherinda, Marcos (2002), *The use of a cultural activity in the teaching and learning of mathematics: The exploration of twill weaving in Mozambican classrooms*, University of the Witwatersrand, Johannesburg
- D'Ambrosio, Ubiratan (1985), Socio-cultural bases for mathematics education, UNICAMP, Campinas
- D'Ambrosio, Ubiratan (1996), Ethnomathematics: Where does it come from? And where does it go?, in: Alsina o.c., 119-128
- Favilli, Franco (Ed.) (2007), Ethnomathematics and Mathematics Education, Proceedings of the 10th International Congress of Mathematics Education, Discussion Group 15: Ethnomathematics, Tipografia Editrice Pisana, Pisa
- Gay, John & Michael Cole (1967), *The New Mathematics in an Old Culture: A Study* of Learning among the Kpelle of Liberia, Holt, Rinehart & Winston, New York
- Gerdes, Paulus (1981) Changing mathematics education in Mozambique, *Educational Studies in Mathematics*, Vol.12, 455-477
- ____ (1986) On culture, mathematics and curriculum development in Mozambique, in: Mellin-Olsen, S. & Johnsen Hoines, M. (Eds.), *Mathematics and Culture, a seminar report*, Caspar Forlag, Radal, 15-42
- (1988a), On possible uses of traditional Angolan sand drawings in the mathematics classroom, *Educational Studies in Mathematics*, Dordrecht / Boston, 1988, Vol.19, No.1, 3-22
- (1988b), On culture, geometrical thinking and mathematics education, *Educational Studies in Mathematics*, Vol.19, No.3, 137-162 (Reproduced in: A. Powell & M. Frankenstein (Eds.), *Ethnomathematics: challenging eurocentrism in mathematics education*, State University of New York Press, 1997, 223-247)
- (1995), *Ethnomathematics and Education in Africa*, University of Stockholm, 1995, 184 pp.
- (1996a), Culture and mathematics education in (southern) Africa, in: Alsina o.c., 221-231

- (1996b), Ethnomathematics and Mathematics Education, in: Bishop o.c., 909-943
- (1997), Recréations géométriques d'Afrique Lusona Geometrical recreations of Africa, L'Harmattan, Paris, 127 pp.
- (1998), Culture and mathematics teacher education, *Journal of Mathematics Teacher Education*, Dordrecht, Vol. 1, No.1, 33-53
- (1999), *Geometry from Africa: Mathematical and Educational Explorations*, The Mathematical Association of America, Washington DC
- ____ (2003a), Awakening of Geometrical Thought in Early Culture, MEP Press, Minneapolis MN, 184 pp.
- (2003b), *Sipatsi: Cestaria e Geometria na Cultura Tonga de Inhambane*, Moçambique Editora, Maputo, 176 pp.
- (2005), Nirrosula, an African musical instrument as a source of inspiration for mathematical exploration, in: Rosemond, Frances & Larry Copes (Eds.), Educational Transformations: Changing our lives through mathematics; A tribute to Stephen Ira Brown, AuthorHouse, Bloomington Indiana, 367-378
- ____ (2006), Sona Geometry from Angola: Mathematics of an African Tradition, Polimetrica International Science Publishers, Monza, 232 pp.
- ____ (2007a), Etnomatemática Reflexões sobre Matemática e Diversidade Cultural, Edições Húmus, Ribeirão, 285 pp.
- ____ (2007b), *Drawings from Angola: Living Mathematics*, Lulu.com, Morrisville NC, 2007, 72 pp.
- (2007c), Otthava: Fazer Cestos e Geometria na Cultura Makhuwa do Nordeste de Moçambique, Universidade Lúrio, Nampula & Lulu.com, Morrisville NC, 292 pp.
- Gerdes, Paulus & Ahmed Djebbar (2007), *Mathematics in African History and Cultures: An annotated Bibliography*, Lulu.com, Morrisville NC, 430 pp. (also published in French: *Les Mathématiques dans l'Histoire et les Cultures Africaines*, Union Mathématique Africaine & Université des Sciences et des Technologies, Lille, 2007, 332 pp.)
- Hirst, Ann & Keith Hirst (Eds.) (1988), Proceedings of the Sixth International Congress on Mathematical Education, ICMI Secretariat & János Bolyai Mathematical Society, Budapest, 397 pp.
- Ismael, Abdulcarimo (2001), An ethnomathematical study of Tchadji about a Mancala type board game played in Mozambique and possibilities for its use in Mathematics Education, University of the Witwatersrand, Johannesburg
- Kanouté, Mamadou (2007), L'influence de l'apprenstisage en Bamana sur l'enseignement et l'apprentisage des mathématiques au premier cycle de l'enseignement fondamental, Université de Bamako
- Keitel, Christine & P. Damerow, A. Bishop, P. Gerdes (Eds.) (1989), Mathematics, Education and Society (Reports and papers presented in the Fifth Day Special Programme on "Mathematics, Education, and Society" at the 6th International Congress on Mathematical Education Budapest, 27 July 3 August, 1988), Science and Technology Education Document Series No. 35, UNESCO, Paris
- Ki-Zerbo, Joseph (1990), Educate or Perish: Africa's Impass and Prospects, UNESCO, Dakar & UNICEF, Abidjan
- Maffei, Laura & Franco Favilli (2007), Piloting the software sonapolygonals_1.0: a didactic proposal for the gcd, in: Favilli o.c., 99-106
- Mosimege, Mogege & Abdulcarimo Ismael (2007), Ethnomathematical studies on indigenous games: examples from Southern Africa, in: Favilli, o.c. 119-137
- Raum, Otto (1938), Arithmetic in Africa, Evans Brothers, London

Traoré, Kalifa (2006), Étude des pratiques mathématiques développées en contexte par les Siamous au Burkina Faso, Université de Québec, Montréal

Zaslavsky, Claudia (1973), Africa Counts: Number and Pattern in African Culture, Prindle, Weber & Schmidt, Boston