

## Algorithms and proofs in school

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Are proofs necessary in school? To answer this question we need first to understand what people study and develop mathematics for. Mathematics always was applied science in the sense that it always contributed to mankind progress. A human being receives by his feels information about real world then produces reaction and affects to real world. Due to evolution a special system was created in the mind that processes this information. We do not know exactly now how this system works. But besides the ready reactions human being can create new reactions by analyzing the real world and extracting stable (repeated) events. Also he has the possibility of predictions or internal (logical) deduction of some regularities from other ones. Those regularities that concern quantity and geometrical forms are the subject of mathematics.

Important role of mathematics is stipulated by the fact that in real world there are many stable quantity and geometrical regularities that can be exploited for improvement of people life (for progress).

In my mind, mathematics is not divided automatically into school mathematics and high-level mathematics but has form like a pyramid. The top of it is high-level mathematics where some people analyze mathematical models for to extract new connections between their elements. The set of such models much richer than our intuitive reflection of real world. And which of this models better suit to real world is not clear. For example, according to the theory of relativity, we live not in 3-dimensional Euclidian space but in 4-dimensional Minkowski space.

The next level of mathematical pyramid is applied mathematics that searches for the ways of using extracted regularities for design of new algorithms for information processing.

The next level is mathematics in other sciences. Here scientists construct the mathematical models of phenomena of special type (physical, economical and so on) and use the designed algorithms for processing these models.

The last level is the use of designed mathematical models and algorithms for solution of practical problems. These problems can be hard (as for engineer) or very simple (as for salesman calculating change).

The school is the bases for this pyramid. To decide what part of this pyramid must be included into school is rather hard because school must teach all students who will be later at different levels of this pyramid. And we can not (now) determine this level in advance. So school has to give knowledge that can be useful for students but also to give a basis for those who will go higher in mathematics. Besides that, mathematics is part of the culture. School mathematics has to show the possibilities of mathematics and its role and place in people life. School has not only to give some methods but to form mathematical thinking.

Most people in their life are in the last level. That is why the most important for school mathematics is idea of algorithm i.e. of process that can be designed once and can be used many times. Because of wide expansion of computers this idea becomes central in mathematics. School students have to realize that design and using of algorithms lighten people life. So some sort of “algorithmic psychology” has to be worked out.

The idea of algorithm has to be formed not at once but by stages. The first examples of algorithms in school are methods for fast calculations. It is very nice example because it shows that acceleration can be achieved by extension of constant memory (table of multiplication) or by special organization of computations. Algebra plays the very important role for understanding idea of algorithm because it shows that the set of similar problems can be solved by single way (algorithm). Here students also realize the hardness of solution of inverse problems (solution of equations). Formulas for roots of quadratic equation give a simple example of algorithm.

Of course, wide expansion of computers allows us to entrust solution of many our problems to computer. In this case we can use some algorithms and know nothing about them. But the work with computer is itself some sequence of actions that is algorithm.

We need to develop in school idea that good and simple algorithms exist in those parts where good regularities exist and to show such parts in mathematics. School students have to feel that real world is arranged in a good way. But sometimes students have wrong feelings that mathematics can solve any problem. It is worth to explain that not all problems are reduced to quadratic equations and those problems from trigonometry or equations with logarithms that are studied in school are only small part of the problems but for more problems there are no methods for exact solution. But it is not a tragedy because in practice it is usually enough to obtain approximate solution and for it there are some algorithms but they are more complicated and are not studied in school.

For the higher levels of mathematical pyramid (mathematics in the sciences), it is important not only to use designed algorithms but also to construct a mathematical model. That is why it is worth to include into school mathematics more problems with life content. The solution of large amount of such problems always was a tradition of Russian school.

When constructing mathematical model, a man have to justify that this model is “correct”. So we need to bring to this people idea of “proof”. The proof is the necessary tool for algorithm designers. They have to realize and to be able to prove that their algorithms work correctly.

The problems of truth in mathematics its adequacy to real world and necessity of proofs in school are very important and complicated.

The nature put in our mind idea of induction. A child notices some repeated regularities and makes conclusion that they will repeat at the following (always). We do the same in elementary school. We detect some simple regularities and just generalize them.

But the nature also put in the brain of human being the grate possibility to work out new regularities from other ones and this possibility must be represented in school mathematics. We have to show that there are many regularities that are not recognized intuitively but can be worked out by logical rules (for example, Pythagorean theorem). Of course, the question arises if these rules are correct. For example, proof by contradiction is completely acceptable by classical mathematics but not fully acceptable by constructive mathematics. But this problem does not concern (now) the school mathematics because in small fragment of mathematics that is studied in school both sides have no contradictions.

The proofs are necessary in the schools for to raise a large amount of people for whom it is important not only “how” but also “why”. This is important not only for development of science but also for social life. The proofs must be the same as in disciplinary mathematics. Of course, most people do not need proofs of special theorems but proofs develop logic and explain internal structure of mathematics. I am sure that for more students it is important to understand but not only to believe and we must to support it. Maybe we do not need not to ask all students about proofs during exams but we have to give them proofs.

The Euclidian geometry plays very important role for development of logical thinking. It is worth to note that in Russia Euclidian geometry and algebra in secondary and high schools are given with proofs.

It is worth to show in school that not all “obvious” facts are true. Namely, it is worth to give some sophisms, for example, to give famous “obvious proof” that all triangles are equilateral. Students better understand by such examples necessity of accurate reasoning in mathematics and in life.

Of course, proofs in school must be not complicated. For example, proofs in the calculus are rather hard (they use difficult ideas of infinity and limit). That is why the basis of calculus in Russian schools is given without proofs and the calculus at universities is studied again from the beginning. I think that it is not a good situation.

I am sure that the main task of the school is to teach mathematical thinking but not only special topics and methods because the life is changing very fast and it is not known what sort of mathematics will be required in the near future. And school mathematics has to reflect the situation in disciplinary mathematics. For example, discrete structures and algorithms become more important due to wide expansion of computers. So it is possible that discrete mathematics

will displace calculus from school. Such topics as binary arithmetic, Boolean functions, graphs are more simple and have already penetrate to Russian school in main course "Informatics".

Progress of mankind requires support of the whole mathematical pyramid. Although high-level mathematicians looks like children playing with the sand, sometimes they find gold in this sand that gives new impulse to mankind progress. Methods for fast calculation contributed into commerce development, discovery of geometrical regularities simplified construction, the development of trigonometry is connected with accesses in astronomy, calculus gave possibility to develop mechanics and to solve optimization problems. I believe that many new regularities will be discovered in the future that will serve to mankind progress.

It is known that most productive age for people is 20-30 years and educational system has to raise gifted students to the top of mathematical pyramid to this age. It is necessary not only to teach gifted students logics and some proofs but to show them rather complicated regularities. It is not so important which part of mathematics will be presented but it must be surprisingly interesting and not much complicated. Naturally, this can not be done for all students. That is why it is worth and necessary to organize special mathematical classes and schools.

Thus, if mankind would like to have more profit (in all senses) from mathematics in the future then they have to teach in schools not only algorithms but also proofs.