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WG1 – Disciplinary mathematics and school mathematics

What's wrong with this title? "Disciplinary Mathematics and School Mathematics"

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Abstract
It turns out that I don't say much about disciplinary mathematics and school mathematics at all—beyond making the point that this relatively recent distinction might have survived its utility. It may be time to reconsider the complex relationship between mathematical activities in schools and mathematical activities in more formal research settings.

A Context:
Mathematics Education Research, Mathematics, and Mathematics-for-Teaching

At the moment on this writing, I am sitting in an airport, awaiting a homeward flight after a week in a mountain retreat with a handful of colleagues. Seven of us with diverse interests that collect around the learning of mathematics,[1] gathered around the question of whether we might collectively craft a text for use in courses on mathematics intended for prospective teachers.

To our knowledge, the project is unique in the extent to which mathematics education researchers and mathematicians are uniting in the effort. This type of synergy might be contrasted with the structured that define most North American pre-service teacher education programs for mathematics teachers, where there tends to be a rather sharp line drawn between courses in disciplinary mathematics (usually taught in Departments of Mathematics) and courses in school mathematics (i.e., curriculum and pedagogy, taught in Faculties of Education). Our experience in the retreat not only confirmed the suspicion that such a separation is troublesome, for us it underscored the desperate need for research mathematicians and mathematics education researchers to work together. Throughout our discussions it was clear that mathematics education researchers are often justly suspicious of the pedagogical emphases in many university mathematics courses, just many university-based mathematicians are right to wonder about the interpretations of disciplinary concepts within courses on curriculum and pedagogy. For example, on the latter, I learned that in spite a decade of post-secondary education organized in the main around mathematics, another decade of public school teaching experience, and 15 years as an educational researcher and teacher educator,

[1] The other participants were Malgorzata Dubiel (Mathematics, Simon Fraser University), Bernice Kastner (Mathematics, Simon Fraser University), John Mighton (Mathematics, University of Toronto), John Mason (Mathematics, Open University), Elaine Simmt (Mathematics Education, University of Alberta), and Anne Watson (Mathematics Education, University of Oxford). Keith Devlin (Mathematics, Stanford University) is also part of the group, but was unable to participate in the retreat.

some of my mathematical knowings are incommensurate with contemporary formal mathematics knowledge (e.g., my habit of conflating "rational number" with "fraction"). But I was comforted that mathematician colleagues with deep interests in education seem to be making analogous "errors" about learning, teaching, and the mathematical competencies that teachers have and need.

On this point, it turns out that most pre-service teacher education programs in North America require prospective elementary school teachers to take at least one course in mathematics. If the ones offered in British Columbian post-secondary institutions are any indication, most of these courses are organized around two intertwining emphases: (1) affecting attitudes toward mathematics and (2) developing some proficiency in problem solving. On the first point, in my experience a majority of students arrives with a distaste of the subject matter, sometimes articulated as a fear of further cognitive bullying, other times as a loathing of a perceived irrelevance, other times as an indifference borne of an imperative to perform without understanding. One informal poll after another has revealed that almost all enrolled in such courses arrive with a perception of mathematics as a collection of fixed procedures to be deployed appropriately and efficiently. An emphasis on problem solving is intended to interrupt such assumptions, inviting students to linger, experiment, invent, generalize, and so on. For most, these sorts of aspects of mathematical engagement would have been concealed behind the tidy, linearized texts that defined the bulk of their school experiences—in what Lakatos (1976) dubbed "pedagogical falsifications."

The mathematics educators in our group agreed with the mathematicians that these emphases are important and appropriate. However, informed by recent research into "mathematics-for-teaching" (cf. Davis & Simmt, 2006), we also wondered about the possible need to develop explicitly some of the conceptual elements that are met in grade school mathematics. On this matter, we were profoundly influenced by the now-commonsensical assertion that mathematics-for-teaching is not a matter of knowing *more* or *more deeply* than one's students, but of knowing *differently*.

To that end, one of the strategies developed during the week to foreground aspects of teachers' knowledge of mathematics is represented in the following table:

<table>
<tr><th colspan="4">Elements of Teachers' Mathematics Knowledge</th></tr>
<tr><td colspan="2">The Mathematical
(i.e., qualities of mathematical engagement, typically associated with knowledge generation)</td><td colspan="2">The Mathematics
(i.e., established and often taken-to-be fixed concepts and conclusions)</td></tr>
<tr><td>Global Processes</td><td>Local Strategies</td><td>Explicit Content</td><td>Implicit Associations</td></tr>
<tr><td>e.g., abstraction, generalization, proving</td><td>e.g., asking a simpler question, generating an example space, etc.</td><td>e.g., single-digit multiplication facts; algorithms to multiply</td><td>e.g., images/metaphors that underpin/infuse multiplication concept</td></tr>
</table>

Importantly, the intention behind this rubric is not to parse up teacher knowledge. Nor is the table meant to suggest that it is in any way possible to capture the breadth and complexity of any category of mathematics knowing. (Obvious absences include the cultural and ecological implications of mathematics knowing.) Rather, it was developed as a device to highlight the varied areas of expertise and emphases of members of our group—and, at the same time, to identify possible sites that might call for some manner of elaboration in mathematics courses intended for teachers.

A Problematic:
Teachers and students as participants in the production of mathematics knowledge

One of the important contributions of the rubric to our discussions emerged around questions of school mathematics curriculum. In particular, it helped us to recognize that the question that most oriented our group was neither "What sort of mathematics *do* we learn at school?" nor "What sort of mathematics *should* we learn at school?"—nor even "What sort of mathematics *might* we learn at school?" Rather, the issue seemed to be one of more profound co-implication. In effect, I think we were asking, "How are teachers and their students participating in the production of mathematical knowledge?"

This is a question that is almost un-ask-able if the focus of school mathematics is on the third column of our rubric. If "school mathematics" is understood to refer only to the replication and perpetuation of existing truths—whatever that might mean—then it would seem quite obvious that teachers and students reside on the outside (or, at least, the periphery) of mathematics. Such would seem to be an assumption behind statements such as …

> The way fundamental mathematical ideas are presented to learners of mathematics will, of necessity, be shadow versions of the concept as held by mathematicians. … What are the consequences if a student stops learning mathematics before an idea is fully developed? (from p. 2 of Bill and Frédéric's WG description … and I hastily add that I don't mean to be critical; it's just that this particular statement is so ready at hand!)

To my reading, this manner of phrasing positions the grade school student's understanding as naïve, partial, and peripheral (i.e., shadowy and suspect) and the research mathematician's understanding as profound, fulsome, and central (i.e., illuminating and exemplary). By implication, the project of school mathematics becomes one of lighting the paths of those in darkness.

Such a sensibility is certainly not unique to mathematics. It would seem to be in keeping with prevailing beliefs about most subject matters and competencies. But it is appropriate? Drawing an analogy to the well-documented assertion that the most potent shapers of language are not adults, but children who plane off grammatical inconsistencies, invent more efficient modes of expression, and readily incorporate new vocabularies (see Deacon, 1997), it would seem reasonable to suspect that a similar phenomenon is at work in the culture of mathematics. Plainly stated, it may be the case that not only are children and their teachers *not* on the periphery, it could be that they play a definitive role in shaping mathematics as they co-select of images, metaphors, applications, gestures, examples, and exercises. These selections help to define the spaces of the imaginable for the next generation. That is, it may well be that teachers and their students are central participants in the production of mathematics, not merely consumers of or inductees into established knowledge.

If this is the case, the topics such as school mathematics curriculum and mathematics-for-teaching should not be discussed "in relation to mathematics" but as vital aspects of disciplinary mathematics. This sort of legitimation presents the real and timely possibility that mathematics (writ large) might be recast in more participatory terms. As Jenkins et al. explain,

> A participatory culture is a culture with relatively low barriers to artistic expression and civic engagement, strong support for creating and sharing one's creations, and some type of informal mentorship whereby what is known by the most experienced is passed along to novices. A

> participatory culture is also one in which members believe their contributions matter, and feel some degree of social connection with one another (at the least they care what other people think about what they have created). (2007, p. 3)

This suggestion is about more than what can or might happen in classrooms. On the level of personal responsibility, for example, it implicates students, teachers, curriculum developers, mathematicians, and mathematics education researchers. We are participants.

A Complication: The creation of genius

I’ve actually made the above argument (on the participatory nature of mathematics) in a variety of contexts, including among groups of research mathematicians. Often (in fact, *always*, in my experience) someone in the crowd raises the objection that while mathematical insight may well depend on cultural contexts, it is spurred along by exceptional minds. That is, as I interpret the point, most of us do actually reside in the shadowy periphery of mathematics knowledge, held in place by virtue of our rather ordinary individual intelligences.

This line of thought is rooted in the commonplace assumption that extraordinary talents are inborn—a belief that is actually suggested by the term *gifted*, understood as something handed over, unearned, fully formed upon bestowal.

The problem with this belief is that there are no validated accounts whatsoever of exceptional giftedness erupting full-blown without years of concentrated study and focused practice. Simply put, there is a strange lack of hard evidence to substantiate the belief in the importance of innate talent. By contrast, the preponderance of evidence suggests that exceptional abilities are not gifts at all; they are not given, they are earned. Experts aren’t born, they’re made. Across domains—writing, chess, music, athletics, mathematics, and so on—the top performers tend to share three qualities:

- they began early in life;
- they engage in intense (usually) solitary practice consisting mostly of repetitive drill work, direct copying, and/or focused examination of others’ work for more than 20 hours each week extending over at least a decade; and
- they engage in effortful study, which involves taking on challenges that are just beyond their competence. (cf., Ross, 2006)

The last point is of particular importance. It helps to explain why, even though many people might continue to practice, only a few tend to excel—explaining, for example, why in a nation of car drivers (and mathematics knowers), so few are exceptional. A critical quality is that practice must test the limits of one’s current abilities. “Effortful study” points to a sort of work/play that is undertaken not because it is easy, but because it is hard.

Some cogent evidence of this point is a contemporary proliferation of chess prodigies, which is believed to reflect the availability of chess programs and social software that make it possible for children to study many already-played games and engage in more practice with competent opponents (i.e., effortful study). Along similar lines, studies of professional athletes suggest that their success has to do more with training than with genetics—which is not to say that genetics do not matter. Of course they do, as do childhood experiences, dates of birth, social opportunities, precocity, and so on.

The point? The strategy of invoking the notion of 'genius' as a device to separate school mathematics (i.e., something intended for the masses) from disciplinary mathematics (i.e., something reserved for an elite) is simply untenable, especially given the role of contemporary schooling in enabling and/or constraining genius. In fact, I'd argue that the construct of mathematical genius only serves to further trouble the line drawn between school mathematics and disciplinary mathematics.

And so?

Where are we then?

I'm tempted to suggest that we are at a crossroads of sorts. Evolving circumstances (which include emergent issues of digitalization, globalization, multiculturalism, and politicization) are contributing to dramatic reformulations of what it means to participate in the production of knowledge, what it means to be educated, and what a school is and does. It may be time to abandon the distinction between school mathematics and disciplinary mathematics—which, it is important to emphasize, is a relatively recent invention. (It might predate the modern public school, but it certainly doesn't predate the modern university.) And it might be a distinction that is on the verge of a natural extinction, as evolving circumstances threaten the institution of the public school.

Well enough, I suppose. But I'm not sure what sorts of distinctions or practices might be useful as replacements. At the very least, I am comforted in the fact that the topic of "disciplinary mathematics and school mathematics" is not only included among the working group topics in the ICMI Centennial, it is (WG) *number one*. Perhaps there's a collective intuition that the matter is of central importance to what mathematics is becoming.

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