An Inquiry-Oriented Approach to Undergraduate Mathematics: Contributions to Instructional Design of Differential Equations

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If teaching and student learning are to improve, a university teacher should recognize the characteristics of effective teaching. The research literature contains many examples of successful standards and practices for effective teaching that are based on evidence of enhanced student learning. However, many university teachers were never introduced to this knowledge base during their graduate or postdoctoral years and have not acquired this perspective. These instructors may struggle through teaching assignments, often redeveloping techniques and approaches that others already have tested and disseminated. The aims of this paper are to share an innovative approach called “inquiry-oriented” instruction and to point out some constructive ideas on learning and teaching mathematics at the university level.

Over the past decades mathematics educators have been moving toward the creation of inquiry-oriented learning environments to support student’s meaningful mathematical learning. This paper describes one such effort, referred to as the Inquiry-Oriented Differential Equations (IO-DE) Project, as a case example of how undergraduate mathematics can draw on theoretical and instructional advances initiated at the K-12 level to create and sustain learning environments in which students learn mathematics with understanding. Adapting research efforts from K-12 to the tertiary level is significant in its own right, however, our work also offers insights for further K-12 research, as well as reveal issues of learning and teaching that may be distinctive to the tertiary level.

In mathematics classrooms that use a traditional, textbook-dominated approach, effective participation means that students listen to and watch the teacher demonstrate procedures, and then practice what was demonstrated by completing textbook exercises. This prevalent phenomenon of traditional approach to instruction is linked with some general existing ‘myths’ and practices in the teaching of the mathematics at the undergraduate level (Alsina, 2002). In the last two decades, there has been a growth in educational research on undergraduate mathematics education. Research began by investigating student’s learning and understanding of specific mathematical concepts. The results obtained offered convincing evidence of the limitations of traditional teaching practices. The evidence of the gap between what a teacher tells students and what is learned was noted by several researchers (for example, Alibert & Thomas, 1991; McDermott, Rosenquist, & van Zee, 1987; Svec, 1995; Selden & Selden, 1995)

Conceptualizing inquiry-oriented approach

Recent reports suggest that the aforementioned gap can be overcome by emphasizing an inquiry-oriented approach to instruction (e.g., Ebert-May, Brewer, & Allred, 1997; Kwon, Rasmussen, & Allen, 2005; Rasmussen, Kwon, Allen, Marrongelle, & Burtch, 2006). For example, in science education the National Research Council (1996) states that inquiry includes identification of assumptions, use of critical and logical thinking, and consideration of alternative explanations. In mathematics education, Richards (1991), characterizes inquiry as learning to speak and act mathematically by participating in mathematical discussions, posing conjectures, and solving new or unfamiliar problems.

Two salient characteristics of the IO-DE project are detailed in this paper. The first is the active verbal involvement of students in which learners routinely engage in explaining and justifying their thinking. Such an approach affords ample opportunities for students to build skills and concepts while moving forward with the mathematical agenda (Rasmussen & Marrongelle, 2006).

The second is the instructional materials in the IO-DE project were inspired by the instructional design theory of Realistic Mathematics Education (RME). From an RME perspective, mathematics is a human activity of organizing subject matter on one level, giving rise to higher level understanding (Freudenthal, 1991; Gravemeijer, 1999). The heart of this perspective lies with students creating meaningful mathematical ideas as they engage in challenging tasks. In this process, referred to as mathematizing, symbols, algorithms, and definitions are built from the bottom up through a process of suitably guided reinvention (for elaboration see Kwon 2003; Rasmussen & King, 2000; Rasmussen, Zandieh, King, & Teppo, 2005).

An example of inquiry-oriented instruction: The IO-DE project

The IO-DE project capitalizes on advances within the discipline of mathematics and on advances within the discipline of mathematics education, both at the K-12 and tertiary levels. Given the integrated leveraging of developments both within mathematics and mathematics education, the IO-DE project is paradigmatic of an approach to innovation in undergraduate mathematics, potentially serving as a model for other undergraduate course reforms.

From the Discipline of Mathematics. Taking a historical point of view reveals the centrality of finding closed form symbolic expressions for solutions to differential equations. This traditional approach, however, is restrictive in the types of differential equations that can be studied. In particular, most nonlinear differential equations (which are the most interesting for science and engineering applications) cannot be solved with the cadre of available analytic techniques. From a dynamical systems perspective, however, all differential equations, not just specially defined classes of equations amenable to analytic techniques, are candidates for analysis. Drawing on a dynamical systems point of view (Artigue & Gautheron, 1983; Blanchard, Devaney, & Hall, 1998; Kallaher, 1999; West, 1994), the IO-DE project treats differential equations as mechanisms that describe how functions evolve and change over time. Interpreting and characterizing the behavior and structure of these solution functions are important goals, with central ideas including the long-term behavior of solutions, the number and nature of equilibrium solutions, and the effect of varying parameters on the solution space.

From the Discipline of Mathematics Education. At the outset of the IO-DE project we conjectured that theoretical advances originating from K-12 classroom based research would be useful for informing and guiding the learning and teaching of undergraduate mathematics. There are two complementary lines of K-12 research from which we draw: the instructional design theory of RME (Freudenthal, 1991; Gravemeijer, 1999) and the social production of meaning (Cobb & Bauersfeld, 1995).

Central to RME is the design of instructional sequences that challenge learners to organize key subject matter at one level to produce new understanding at a higher level. In this process, referred to as mathematizing, graphs, algorithms, and definitions become useful tools when students build them from the bottom up through a process of suitably guided reinvention (for illustrative examples and further theoretical development, see Kwon, 2003; Rasmussen & King, 2000; Rasmussen, Zandieh, King, & Teppo, 2005).

The mathematization process is embodied in the core heuristics of guided reinvention and emergent models. Guided reinvention speaks to the need to locate instructional starting points that are experientially real to students and that take into account students' current mathematical ways of knowing. One aspect of the reinvention principle entails examination of students' informal solution strategies and interpretations that might suggest pathways by which more formal mathematical practices might be developed. Another aspect of the reinvention principle involves looking at the history of mathematics for insights into how particular mathematical concepts and practices evolved over time as well as potential barriers and breakthroughs. The IO-DE project further expanded the reinvention principle to include possible starting points due to technology that historical examinations would not reveal (Rasmussen & King, 2000).

Finally, the heuristic of emergent models highlights the need for instructional sequences to be a connected, long-term series of problems in which students create and elaborate symbolic models of their informal mathematical activity (Gravemeijer, 1999). The use of the term model is overarching idea that encompasses students' evolving ideas and activity with a chain of symbols, such as number tables, algorithms, graphs, and analytic expressions. From the perspective of RME, there is not just one model, but a series of models where students first develop *models of* their mathematical activity in an experientially real task setting, which later becomes *models for* reasoning about mathematical relationships.

As researchers working within an RME approach emphasize, a coherent sequence of learning tasks does not guarantee that students will learn mathematics with understanding (Graveimeijer, Cobb, Bowers, & Whitenack, 2000; Treffers, 1987). In addition to theoretically informed design with extensive classroom testing, the IO-DE project works from the premise that the way in which instructional tasks are constituted is as important as the material itself, and it is toward this aspect that we now turn.

An explicit intention of IO-DE project classrooms is to create a learning environment where students routinely offer explanations of and justifications for their reasoning. Because of the strong emphasis on argumentation in inquiry-oriented classrooms, we conjectured that the theoretical constructs arising from research in inquiry-oriented elementary school classrooms would be useful for learning advanced mathematics, such as differential equations. After all, mathematicians engage in similar forms of argumentation when creating new mathematics. .

Specifically, we found the constructs of social norms and sociomathematical norms (Yackel & Cobb, 1996) useful because they offer a way of thinking about the multiple and complementary roles of argumentation as a means to conceptualize processes by which teaching mathematics for understanding can occur (see also Rasmussen, Yackel, & King, 2003; Yackel & Rasmussen, 2002). Social norms pertaining to explanation and justification are discursive regularities that are constituted and reconstituted through ongoing interaction. Being intentional about fostering such norms is a pedagogical goal of IO-DE project classroom teachers. Examples of social norms include students' routinely explaining their thinking, listening to and questioning others'thinking and responding to others' questions and challenges. Sociomathematical norms refer to criteria for that what counts, for example, as an acceptable explanation, a different explanation, and as an elegant justification. Like social norms, sociomathematical norms are not rules set out in advance but rather emerge as joint accomplishments.

Research focusing on student conceptions in differential equations also pointed to a number of relevant issues for instructional design and teacher planning. For example, in one case study of a differential equations class that treated contemporary topics in dynamical systems, Rasmussen (2001) found that rather than building relational understandings (Skemp, 1987); students were learning analytical, graphical, and numerical methods in a compartmentalized manner. An important lesson from this research is that working with multiple modalities does not guarantee that students will build a coherent network of ideas. It is also important to have a long-term, coherent sequence of tasks, and our adaptation of RME was useful for this purpose.

Other informative research on student cognition in differential equations highlights students' concept images of Euler's method and students' informal or intuitive notions underlying equilibrium solutions, asymptotical behavior, and stability (Artigue, 1992; Rasmussen, 2001; Zandieh & McDonald, 1999). Knowledge of such informal or intuitive images was useful for the IO-DE project because it suggested task situations and instructional interventions that could engage and help reorganize students' informal and intuitive conceptions.

Quantitative Assessment of student learning. Rasmussen, Kwon, Allen, Marrongelle, & Burtch (2006) conducted an evaluation study to compare students' routine skills and conceptual understandings of central ideas and analytic methods for solving differential equations between students in inquiry-oriented and traditionally taught classes at four undergraduate institutions in Korea and US. Whereas IO-DE project classes at all sites typically followed an inquiry-oriented format, comparison classes at all sites typically followed a lecture-style format.

The assessment consisted of routine skill problems and conceptual understanding problems. Routine skill problems focused on students' instrumental understanding such as an analytic and numerical nature of differential equations. On the other hand, conceptual understanding problems were aimed at evaluating students' relational understandings of important ideas and concepts. There was no significant difference between the two groups on routine problems. However, the IO-DE group did score significantly higher than the comparison group on conceptual problems.

Further, Kwon, Rasmussen, & Allen (2005) investigated the follow-up study on the retention effect of conceptual and procedural knowledge one year after instruction for a subset of the students from the comparison study. Students' retention of knowledge was compared across a traditional and an IO-DE instructional approach. For the purpose of this analysis, procedurally oriented items were defined as those questions that were readily solved via analytic/symbolic techniques. Conceptually oriented items were defined into two categories, modeling tasks and qualitative/graphical tasks, each of which represent important and conceptually demanding thinking in mathematics, in general, and in differential equations, in particular. The two modeling tasks posed involved determining an appropriate differential equation to fit a given real-world situation. The qualitative/graphical tasks involved predicting and structuring the space of solutions. The analysis showed that there was no significant difference in retention between the two groups on the procedural oriented items. However, long-term retention of conceptual knowledge after students' participation in the IO-DE project as seen in student responses to modeling and qualitative/graphical problems was positive compared to retention by students in its traditional counterpart.

Conclusion

The implications of the IO-DE project are twofold. First, based on the results of the post-test and the delayed post-test (Kwon et al., 2005; Rasmussen et al. 2006), all IO-DE students from each of the four institutions, regardless of academic backgrounds and gender differences, outperformed traditionally taught comparison students on the post-test. Therefore, if the delayed post-test were given to all IO-DE students at the different sites, students may again similarly outperform comparison students. This result demonstrates that this instructional approach can be applicable to university mathematics regardless of academic preparations and gender differences. Secondly and more importantly, the instructional methods and curriculum design approach guided by RME are applicable to promoting student learning in all mathematics classrooms. Indeed, RME has its origins and broadest use in K-12 instructional settings. Yet, regardless of grade level or student differences, this inquiry-oriented instructional design could enhance long-term mathematics retention for all students. These findings support our conjecture that, when coupled with careful attention to developments within mathematics itself, theoretical advances that initially started at the elementary school classrooms (and which are beginning to spread to the rest of K-12) can be profitably leveraged and adapted to the university setting. As such, our work in differential equations may serve as a model for others interested in exploring the prospects and possibilities of improving undergraduate mathematics education in ways that connect with innovations at the K-12 level. Too often we hear teachers say that such and such an idea and approach to teaching and learning is fine at this or that level, but would not be appropriate for their level. We hope that this study calls into question such statements.

In our work we found that the heuristics from the instructional design theory of Realistic Mathematics Education and the constructs of social norms and sociomathematical norms constitute a useful collection of theoretical ideas that can inform and guide undergraduate mathematics education. Certainly these are not the only possibly useful theoretical ideas. Our position is, however, that innovations are more likely to result in positive outcomes if they are theoretically-driven rather than driven by the use of technology or collaborative learning, for example.

Additional significance of this work is evident in the extent to which the IO-DE research program is contributing to advancing the work of teachers and the professional development of mathematicians. For example, Rasmussen and Marrongelle (2006) develop teaching strategies or “tools” that are teaching counterparts to the instructional design theory of RME. Using the IO-DE project as a case example, Wagner, Speer, and Rossa (2006) detail the role of teachers’ knowledge in the implementation of curricular reforms.

Thus, the IO-DE may be an example of an inquiry-oriented instruction at the undergraduate level which can help the type of conceptual understanding that can make mathematics meaningful to students and develop students’ mathematical reasoning ability. The innovative approach to teaching differential equations suggested in this paper may give a possible arena for exploring the prospects and possibilities of improving undergraduate mathematics education.

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