

Disciplinary Mathematics and School Mathematics:

Working Group 1

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Note: In this discussion document papers are named by the author who will be present at Rome. Many papers have several authors, some of whom will not be attending.

As we look into the nature of Disciplinary Mathematics and School Mathematics, the diverse responses to the discussion paper indicate the complexity of this topic. As remarked by Siu in his paper, we are reminded of the double discontinuity as described by Felix Klein.

The young university student found himself, at the outset, confronted with problems that did not suggest, in any particular, the things with which he had been concerned at school. Naturally he forgot these things quickly and thoroughly. When, after finishing his course of study, he became a teacher, he suddenly found himself expected to teach the traditional elementary mathematics in the old pedantic way; and, since he was scarcely able, unaided, to discern any connection between this task and his university mathematics, he soon fell in with the time honoured way of teaching, and his university studies remained only a more or less pleasant memory which had no influence upon his teaching.

Felix Klein (1908, 1933), *Elementary Mathematics from an Advanced Standpoint* p.1.

Klein went on to note a movement to ameliorate this situation. A hundred years later, however, we find ourselves concerned that the discontinuity has yet to be patched.

In order that our Working Group is able to make a significant contribution to this question, we offer this summary of the contributions, and suggestions for our deliberations in Rome.

Central Questions: Parallelism

The most apparent question implicit in many of the papers offered to WG1 is *whether* school mathematics should reflect disciplinary mathematics, or, at least, to what extent should such a connection be evident. On the one hand, Watson argues that school mathematics is a very different sort of thing, and therefore needs to be different in type. Wood and Moro/Soares indicate their view that “school mathematics would not essentially be different in nature from mathematics as a science” (Moro/Soares). A recurring theme in this discussion is that the parallelism should not be with respect to content, but that school mathematics needs to mirror the processes and ways of working like a mathematician (see, for example, Zazkis). Hersant, Alekseev, and Hanna reflect on particular examples of such processes. Many of the other papers offered to WG1 discuss other issues (the nature of university mathematics or teacher preparation) but imply by default that university mathematics should be reflected in schools. Finally, Durand-Guerrier asks us to consider a third option, that disciplinary mathematics could benefit from reflecting aspects of school mathematics.

We propose, therefore, that the following questions be the focus of early discussion within our group.

Questions.1.

To what extent do we consider that school mathematics should reflect disciplinary mathematics? In what ways is this possible and in what ways should it be different? What are the factors that might affect the way that school mathematics reflects disciplinary mathematics?

In thinking about this we need to be careful not to get bogged down in discussions as to how many students will become mathematicians or whether we are serving all secondary students adequately. These are important questions, but can be regarded at the periphery of the central issue. Let us consider the preparation of mathematics postgraduate students as a separate issue if necessary.

So why might school mathematics not be like disciplinary mathematics? Watson argues that it has different purposes, different forms of reasoning, and different activities. She paints a picture of mathematics as a social activity and a mode of intellectual enquiry including empirical and deductive activity, reworking problems and conjecturing, proving and refuting. School mathematics has some more specific and psychological aims. More importantly, Watson describes a suite of developmental transitions that are particular to school mathematics: additive to multiplicative thinking, probabilistic argumentation; following procedures to making choices; constructing multi-stage arguments; problem-solving in Polya's sense; and mathematical modelling. School mathematics needs to facilitate these epistemological transitions, not merely reproduce the end product.

A further example from Watson of significant difference is the place of unifying concepts. These are vital sources of questions and answers in the discipline, but school mathematics is the place to acquire the multiple experiences which, later, become unified under a greater concept. The greater idea loses its power without the experiences. Finally Watson argues that constraints on teaching make it impossible for school mathematics to emulate disciplinary mathematics, with a particular emphasis on the nature of authority in each.

In apparent contradiction to this view, Wood and Moro/Soares believe that it is important that learning practices should reflect the disciplinary practices in mathematics. While acknowledging different purposes and acknowledging the distinction between learning practices (those that help understand existing mathematics) and disciplinary practices (those concerning developing mathematics), both papers argue that students learn through reasoning that resembles mathematical thought (i.e. using abstraction and generalisation). Wood goes further to claim that then students understand the subject in a more conceptual way. Furthermore, to the extent that classrooms are an induction to a set of social norms that make up a discipline, those environments must reflect that particular habitas. Boaler's research is cited as evidence for the view that how we learn directly impacts on how we come to see and do mathematics, hence the essence of disciplinary mathematics must be present in the culture of learning.

Moro/Soares go on to consider the development of topics, and note that, for example, psychological, cognitive and socio-cultural factors come into play as we develop concepts, not just mathematical ones. They thus dismiss the idea of any "best" development sequence.

We are reminded not to regard the theme of our Working Group as applying only to secondary mathematics. Hersant examines young children's responses to "problèmes pour chercher" and asks us to consider what aspects of their answers reflect mathematicians' thinking, or, rather, in what ways can we consider that the students' answers approach a mathematician's point of view, and what aspects of mathematician's thinking are they apparently unable to approach.

To what extent are the views in the contributions really at odds with each other. On the surface there appears to be disagreement about how different school mathematics can (or should) be from mathematical practices. However, when the detail is examined, the papers also contain many points in common. We need to tease out how we feel as a group on this issue.

Central Questions: The Nature of the Discipline

A second issue that emerges from the contributions is the difficulty of describing “disciplinary mathematics” sufficiently to know what it would look like if it was to be mirrored in schools. Engelbrecht asks this question directly, Henderson asks the same question by discussing what skills undergraduates need to have to deal with disciplinary mathematics. Lins, by imagining Euler in a modern university context, thinks rather of a mathematical environment in which we can manage education through mathematics. Davis and Holton have a much wider purview, suggesting that we should look at the whole question of mathematics knowledge generation from a wider perspective. Implied in this is a critique of trying to categorise disciplinary mathematics (or school mathematics) as something that can be encapsulated at all. Davis in particular, asks us to at least consider the usefulness of the distinction in our title.

Again the issue of process versus content emerges as important, although nearly all the contributions leaned towards process and mathematical habits of working rather than a focus on content. It is interesting therefore, on the 100th anniversary of Klein’s book, that his approach was content-based, but in a somewhat removed sense. Klein sought to elucidate the links between school mathematics topics and broad areas of mathematics considered in both a historical and contemporary light. Rather than suggesting specific syllabus features, he presented his analysis as a challenge for teachers to think about. With the same intent, a second set of questions we could address is as follows.

Questions.2.

What aspects of disciplinary mathematics are important to mathematicians? What descriptions and what models of these aspects are available? What historical features of mathematics have been enduring, and what future features may emerge (or would we like to see emerge)? How can mathematics be seen as a coherent whole, how does it link together, and how might this be represented at school level?

Hanna addresses these questions through the issue of proofs, noting first that proofs occupy a different role in school mathematics, namely a pedagogical role, as well as any modelling of mathematical practice. Hanna argues strongly that the potential of this mathematical practice is underutilised and that some attention should be given to the ability of proofs to present new techniques, demonstrate their value, and provide explanations as well as act as warrants for mathematical statements. Hanna provides evidence that proofs contain new concepts and insights. While arguing, therefore, for their use in schools, Hanna is also challenging us to reconsider the mathematical role of proving.

Engelbrecht also addresses the nature of the discipline, but in a slightly different fashion, and raises some characteristics not present elsewhere: for example, the idea of repeatedly returning to a concept to converge on full understanding, and the idea of a balance between mathematical discipline and mathematical creativity. Similarly Holton draws our attention to characteristics such as linking skills together, and creating new skills. Zazkis gives us examples from teacher education courses of what “working like a mathematician” might look like when it is not done by mathematicians.

Other Discussions: Teacher Education

Both of these sets of questions can be approached through thinking about teacher preparation.

Galbraith argues strongly for providing teachers with authentic mathematical experiences, to be given problems that “force confrontation with understanding of mathematical specifics, using approaches that provide practice in the very attributes of argument and communication

that are the lifeblood of vigorous didactics.” Siu concurs, phrasing the need for teachers to engage in research—while noting that the research so-intended may look different from that of a research mathematician. Is this a contradiction?

The mathematical needs of teachers are also addressed in Novotna’s research, who asks whether mathematical structures unlikely to be used in any teaching could be part of a teacher’s preparation. Her answer is unequivocally “yes” for reasons that mirror those of the two authors above.

While the papers that discuss this issue are useful in considering the two main questions, there appears to be enough interest in teacher knowledge to warrant a specific discussion in our Working Group.

Other Discussions: History

The attempts made by the mathematics education community to make mathematics education in schools different from the simple acquisition of elementary mathematical knowledge are not new. It is therefore not surprising that another grouping of papers that contribute to the main questions but have a common theme are those of Brito, Zuccheri, and Volkert, who all reflect on historical exemplars of attempts to bridge the school and disciplinary divide.

Zuccheri’s paper describes a specific teaching method, based on cognitive theories attributed to Ernst Mach, the “Jacob Method” (see Jacob, 1913). Two central points of this method are that the kernel of mathematical concepts are real situations and concepts are the instruments made by humans to cope with experience. Thus questions derived from real life are central: for example, in calculus, this led the curriculum to be designed with the teaching of physics in mind. The Jacob method is claimed to have affinity with Klein’s principles of developing intuitive, deductive, and creative faculties in students.

Bruto’s paper reminds us that mathematical issues are not the only ones that affect the relationship between schools and the discipline. The American influence on Brazilian mathematics education through USAID, at university level through Bourbaki materials and in schools through texts from the Modern Mathematics Movement, are described in detail. What other barriers might there be to any hypothesised relationship between school mathematics and its parent discipline? There are other historical, and many recent, examples of government inspired school initiatives that might undermine high ideals with long-term motivations.

What history teaches us about mathematics teaching can be problematic. Volkert’s paper reviews the history of solid geometry, and uses it to show that our pedagogical intuition does not always follow disciplinary mathematics. For example, he describes an approach from the early 20th century based on the idea that the Euclidean simple to complex build-up does not seem a sensible approach when students come to class with wide experiences of 3-dimensional objects and their relations. However he also shows how a systematic approach to solid geometry needs more than intuition. This contradiction is a specific example of the tensions at work if school mathematics is to reflect the discipline, and answers we generate need to supply a resolution to such pedagogical conundrums.

It could be productive to bring together our collective knowledge of these and other historical issues in a focused session.

Other Discussions: Exemplars

A final set of papers describe, or argue for, particular positions. Novotna, Kwon, and Martignon each challenge us, through a particular example, to actualise our answers to the bigger questions by considering their particular cases. Is the early introduction of probabilistic

ideas using a heuristic approach a good example of the link between school statistics and the discipline of stochastics? Is an Inquiry-Oriented approach possible in undergraduate mathematics, and is this an example of school mathematics influencing the discipline at university level? Is exploring a non-standard algebraic structure the kind of experience that enables teachers to reflect the creative and surprising nature of mathematics in their teaching? We suggest that we use these papers, and our other collective experiences, not as a focus for discussion, but as a way of testing our answers to the big questions—a practical touchstone of whether our grand ideas will be possible in the real world of students confronting mathematical ideas in class.
