Empirical and theoretical reflections on researching mathematics for teaching in mathematics teacher education

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Introduction

Teacher education, both pre-service and ongoing professional development, is increasingly in the spotlight, indexing a recognition that the production of quality teaching is central to any education system. In South Africa an interesting and challenging feature of a new undergraduate Bachelor of Education degree and a variety of formalised inservice mathematics teacher education programmes is *what* and *how* mathematical knowledge and practices are distributed, and the consequent opportunities for the education of teachers. While programmes vary across institutions, there is a commonly stated desire to impact on teachers' mathematical knowledge and practices. This is a function of a context of rapid and intense educational change generated by national imperatives to impact generally on the quality of mathematics education and specifically on attempts to eradicate apartheid's legacy of the reproduction of deep educational (and more general) inequality.

Most current *mathematics-focused* teacher education programmes are located in higher education institutions (HEIs), all of which enjoy relative autonomy in curriculum design. As we have argued elsewhere (Parker & Adler, 2005), this opens up spaces for agents in HEIs to construct what they would see as worthwhile programmes[1]. However, there are always tensions or dilemmas of both selection and integration of knowledge(s) and practice(s) in teacher education (Graven, 2005), with related consequences for the quality of programmes. Teacher education in South Africa thus provides a rich context for a study of what and how mathematical knowledge and related practices come to be produced in and across a range of institutional offerings, and their possible effects. This is a central focus of the QUANTUM research project[2]. In this short paper, I reflect on methodological and empirical progress in the project. This work speaks directly to the overall concern of Working Group 2 with the traditional dichotomy between content knowledge and pedagogical knowledge. In QUANTUM, we have developed a theoretical orientation to studying teacher education practice that foregrounds the relationship between the practice of teaching and foundations of mathematics in teacher education, and engages the concept of mathematics for teaching.

From our perspective, how mathematical knowledge and practices come to be constituted across sites of practice provides a different and complementary take on the understanding of *mathematics for teaching* (MfT). MfT, the specialised mathematical knowledge and practices required in the work of teaching, is currently studied in different ways, in different contexts. This body of research has its roots in the seminal work of Lee Shulman (1986, 1987) and his notion of *pedagogic content knowledge* (PCK). Current studies related to PCK and SMK (*subject matter knowledge*) in mathematics education (e.g. Ball, Bass & Hill, 2004; Brodie, 2004; Even, 1990, 1993; Ma, 1999; Marks, 1992) can be broadly divided between those that refine and develop categories of knowledge for teaching mathematics (e.g. Ma and Even) and those where attention shifts towards the practice of teaching, and so to an identification of tasks of teaching and their mathematical entailments. This latter orientation is led by Ball et al., signalled by a discursive shift from PCK/SMK to MfT, and extends to the development of measures of MfT, and its relationship to teaching and learning in school. QUANTUM adds to this growing body of knowledge. We align ourselves with a practice-based notion of MfT, though our project is somewhat different. Working with a social orientation to knowing and to

[1] Of course programmes might well be approached expediently, and without due concern for quality and impact. This problematic is beyond the scope of the paper and the study that frames it.

[2] QUANTUM is an R & D project on quality mathematical education for teachers in South Africa. Principal investigators who have contributed to research progress include: Dr Zain Davis, University of Cape Town; Diane Parker, University of Kwazulu Natal; Dr Mercy Kazima, Chancellor College Malawi.

knowledge (re)production, we understand that what comes to be MfT in any practice is structured by pedagogic discourse, be this in teacher education or school practice. In other words, there is a structuring of mathematics by the activity of teaching[3]. As such our methodology is sensitive to context and conditions. We are specifically interested in the kinds of MfT and related pedagogic practices that are emerging across a range of mathematics teacher education programmes in South Africa.

Our contribution to this emerging field of study is to understand what happens inside mathematics teacher education. We start from the assumption that in mathematics teacher education, there are multiple goals and at least two objects of transmission and acquisition: teaching and mathematics. We also assume that these two objects are co-constitutive: each shapes and is shaped by the other as they come to live in pedagogic practice and so constitute MfT in mathematics teacher education. Following Boaler (1997), and Bernstein's (1996) more general theorisation of pedagogy, we work with the proposition that the forms of knowledge and practices produced are a function of the pedagogical practice in which they are elaborated. What are these emergent forms of MfT? How do they relate to pedagogic practice inside teacher education? How do we explain these and what do they tell us about possibilities for developing and improving mathematics teacher education? In this short paper, I provide a brief description of the methodology we are using to pursue these questions as well as some findings that open up questions for further investigation.

QUANTUM – methodology and theoretical framing

As MfT is embedded in pedagogic practice, it cannot be grasped directly. Methods need to be developed and put to work to describe and explain what MfT is and how it is constituted across varying sites of practice. As noted, the added complication in mathematics teacher education pedagogy is that there are two objects in play: mathematics and teaching. Our overarching theoretical orientation is elaborated in Adler & Davis (2006) and draws significantly from Davis (2005). We began with a theoretical orientation developed from Basil Bernstein's theory of pedagogic discourse and his general account of the structuring effects of the pedagogic device on pedagogic practice. We kept in view the frameworks that had been developed to describe PCK and SMK. We began by recruiting Bernstein's (1996) proposition stating that the whole of the pedagogic device is condensed in evaluation: we saw a productive way into this complex field of MfT through an examination of evaluation.

Between 2003 and 2006 we surveyed higher education institutions offering formalised (i.e. accredited) mathematics teacher education programmes. In the first phase of the study we focused on formal assessment (actual tasks) used in in-service programmes in our archive of course information. We examined what and how mathematics and teaching competence were expected to be demonstrated and so what kind of MfT was privileged in these tasks. The details of this part of the study have been reported in Adler & Davis (2006a) and are not repeated here due to space limitations. Of course, that analysis could not provide insight into the pedagogical practice of which the assessment tasks were a part. In phase 2 we focused in on in-depth study of selected courses, which required an elaboration of the language we had developed to that point. The unit of analysis required rethinking, as pedagogic practice functions over time, unlike static assessment tasks. In pedagogic practice the purpose of ongoing evaluation is to transmit criteria for the production of legitimate texts. Indeed, any evaluative act, implicitly or explicitly, has to appeal to some or other authorising ground in order to justify the selection of criteria. Given the complexity of teaching and more so, teacher education, what come to be taken as the grounds for evaluation are likely to vary substantially within and across sites of pedagogic practice in teacher education.

The unit of analysis in this phase is what we call an *evaluative event*, that is, a teaching-learning sequence focused on mathematics and/or teaching. Events are marked by punctuations in pedagogic discourse, when meanings are set through pedagogic judgement. Each course, all its contact sessions and related materials, were analysed, and chunked into

[3] We are aware of the complementarity of work in France developed through studying didactic situations. Here there is particular resonance with Chevellard's notion of institutionalisation.

evaluative events. Figure 1 presents a network we have used and reflects our dual and simultaneous focus on mathematics and teaching as specialised activities, and how they emerge as objects of study over time in each of the courses.

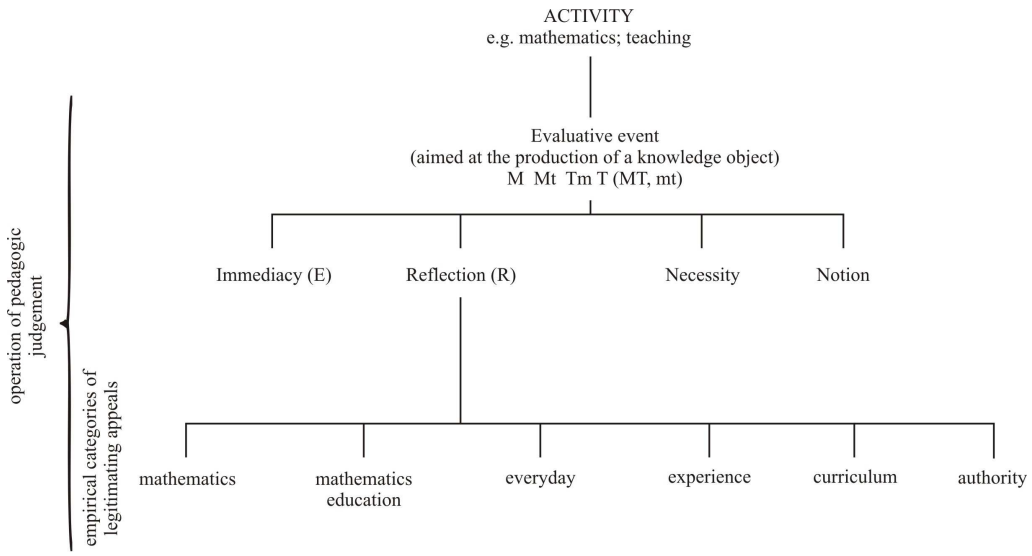

Figure 1:

After identifying starting and endpoints of each event, we first coded whether the object of attention was mathematical (M) and/or teaching (T), or both, and then whether elements of the object(s) were assumed known, rather than being a focus of study (and were then coded either m or t). The next row emerges through a recontextualisation of Hegel's theory of judgement (1969) and the proposition that judgement in general, and hence pedagogic judgement in particular, is itself constituted by a series of dialectically entailed judgements (of immediacy, reflection, necessity, and the Notion). We worked with the idea that in pedagogic practice, in order for something to be learned, known, it has to be represented. Initial orientation to the object, then, is one of immediacy—it exists in some initial (re)presented form; this is the E column. Pedagogic interaction (reflection, column R) then produces a field of possibilities for the object, and through related judgements made on what is and is not the object (Legitimating Appeals columns), so possibilities are generated (or not) for learners to grasp the object (N).[4] In other words, the legitimating appeals can be thought of as qualifying reflection. An examination of *what is appealed to* and *how appeals are made* delivers up insights into how MfT is being constituted in mathematics teacher education.

QUANTUM – progress – the empirical project and initial findings

I focus here on the in-depth case studies, specifically two contrasting mathematics education courses[5]. Through a saturation of data across the courses we were able to code and then see grounds for appeals across all evaluative events. Details are elaborated in Adler & Davis (2006b) and Parker, Davis and Adler (2005). Briefly, in a course entitled Algebra: Concepts and Methods (aimed at teachers working in Grades 7 – 9), content and method are integrated Appeals were predominantly to what we describe as empirical mathematics (mathematical explanations and representations grounded in concrete examples) and everyday knowledge (explanations that draw on everyday practices to locate some meaning for concepts or processes examined).

The course entitled Teaching and Learning Mathematical Reasoning (aimed at teachers across Grades 7 – 12) was not explicitly integrated, rather, it was an education course with a strong focus on mathematical practices. Meanings were frequently legitimated by appeals to

[4] All judgement, hence all evaluation, necessarily appeals to some or other locus of legitimation to ground itself, even if only implicitly.

[5] The third case is equally interesting, but space prohibits discussion of all three.

mathematics education (reported research) and to both empirical and principled mathematics (generalisation drove mathematical argument and justifications). Interestingly, there was a far greater density of appeals in the latter course: relative to the number of events, there were far more deliberate moves by the lecturer in the Reasoning course to ground and legitimate ideas with reference to established knowledge(s) of mathematics and mathematics education.

In addition to 'seeing' differing constitutions of MfT, through in-depth study of each course we became aware that both modelled teaching, though in different ways. Modelling the practice, we argue, is a necessary feature of all teacher education: there needs to be some demonstration/experience (real or virtual) of the valued practice—that is, of some image of what mathematics teaching performances should look like. In the Algebra course, the model was located in the performance of the lecturer whose concern (stated repeatedly through the course) was that the teachers themselves experience particular ways of learning mathematics. This experiential base was believed to be necessary if they were to enable others to learn in the same way. The mathematical examples and activities in the course thus mirrored those the teachers were to use in their Grades 7 – 9 algebra class. In the Reasoning course the model of teaching was externalised from both the lecturer and the teacher-students themselves, and located in images and records of the practice of teaching: particularly in videotapes of local teachers teaching mathematical reasoning, and related transcripts and copies of learner work. The externalising was supported by what we have called discursive resources (texts explaining, arguing, describing practice in systematic ways).

In Table 1 I summarise the observations of pedagogic performance alongside the description of MfT across the courses. We are still working on how best to systematise and theorise our observations and their complex inter-relatedness. As implied in the discussion above, we have drawn inspiration from Bernstein (how learners—in this case teachers—recognise themselves in pedagogic practice), Lacan (and his distinction between imaginary and symbolic modes of identification) and the ever-present tension in teacher education between theory (distancing through discursive means) and practice (embedding in experience). What we see in these two courses are different selections of and orientations to mathematical knowledge and teaching practice, and also their inter-relation as they project and constitute mathematics teaching and mathematics for teaching.

Course	MfT: nature of Math. appeals		Density of appeals		Modelling teaching			
					Image		Resources	
	Empirical	Principled	Low	High	Internal	External	Experience	Symbolic
Algebra	X		X		X		X	
Math. reasoning	X	X		X	X	X	X	X

Table 1:

We have identified how particular modes of identification with mathematics and teaching produce possibilities for learning different kinds of MfT. In the Reasoning course teachers are offered images of teaching together with symbolic resources that suggest opportunities for developing principled and practical orientations to mathematics and teaching. In the Algebra course the images of teaching are accompanied by empirical and concrete orientations to mathematics in particular. Such insights are critical in that they identify and describe different models at work. They are suggestive of potentialities and also the possibilities for the production of disadvantage (in relation to opportunities for teachers to learn appropriate mathematics). Of course, two examples, while contrasting, are only suggestive. Further study of additional and more contrasting sites is needed.

In conclusion

What we have found through our in-depth study is a function of the methodology we have used. Our findings thus need to be understood as a result of a particular lens, a lens that we believe has enabled a systematic description of what is going on 'inside' teacher education practice at two inter-related levels. The first level is 'what' comes to be the content of mathematics for teaching, i.e. the mathematical content and practices offered in these courses.

We are calling this MfT. It is not an idealised or advocated set of contents or practices. At the second level is the 'how'. This content is structured by a particular pedagogic discourse; and a key component in the 'how' that has emerged in the study, is the projection and modelling of the activity of teaching itself. In Bernstein's terms we have seen, through an examination of the evaluation at work and of how images of teaching are projected, that different MfT is offered to teachers in these programmes. The research we have done suggests that developing descriptions of what does or should constitute maths for teaching outside of a conception of how teaching is modelled, is only half the story.

Returning to the introduction to this paper and the South African context of where concerns with quality are accompanied by concerns to address inequality, important questions arise for further research. Do particular models of teaching necessarily give rise to a particular kind of MfT? What other models pertain in mathematics teacher education? How do the ranging models and forms of MfT relate to teachers' learning from and experiences of mathematics for teaching and, ultimately, the quality of their teaching? What possible consequences follow for social justice in and through teacher education itself?

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