The Professional Knowledge of German Secondary Mathematics Teachers: Investigations in the Context of the COACTIV Project

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1 Introduction

The relevance of teachers' domain-specific knowledge for high-quality instruction has been emphasised repeatedly, particularly in the context of mathematics teaching (e.g., Ball, Lubienski, & Mewborn, 2001). However, despite the eminent role that is attributed to teachers' professional knowledge for creating powerful learning environments, today only very few instruments are available to tap teachers' knowledge directly (Baumert, Blum & Neubrand, 2004). As a consequence, many questions on mathematics teachers' knowledge, i.e., on its content, its structure, or on the way how it influences teaching and learning, are still empirically unanswered.

One of the aims of the COACTIV Project (**Co**gnitive **Activ**ation in the Classroom) was to conceptualise the professional knowledge of secondary mathematics teachers and to construct reliable tests accordingly. COACTIV, which was funded by the German research foundation (DFG) from 2002-2006 (Project directors: Jürgen Baumert, Berlin; Werner Blum, Kassel; Michael Neubrand, Oldenburg), investigated the mathematics teachers of the German PISA classes 2003 and 2004. Besides knowledge tests, in COACTIV a broad battery of instruments was newly developed (or adapted) that tapped, among other things, teachers' biographical background, motivational orientations, professional beliefs and self regulation (Fig. 1; for an overview on the instruments see, e.g., Krauss et al., 2004). As it becomes clear from Figure 1, the close relationship between COACTIV and PISA allows, for the first time in Germany, a combined analysis of large data sets on teachers and their lessons (COACTIV) as well as on their students (PISA; see Prenzel et al., 2004) in a common technical and conceptual framework (for an overview on results of COACTIV see, in particular, Kunter et al., in press, or Brunner et al., 2006a).

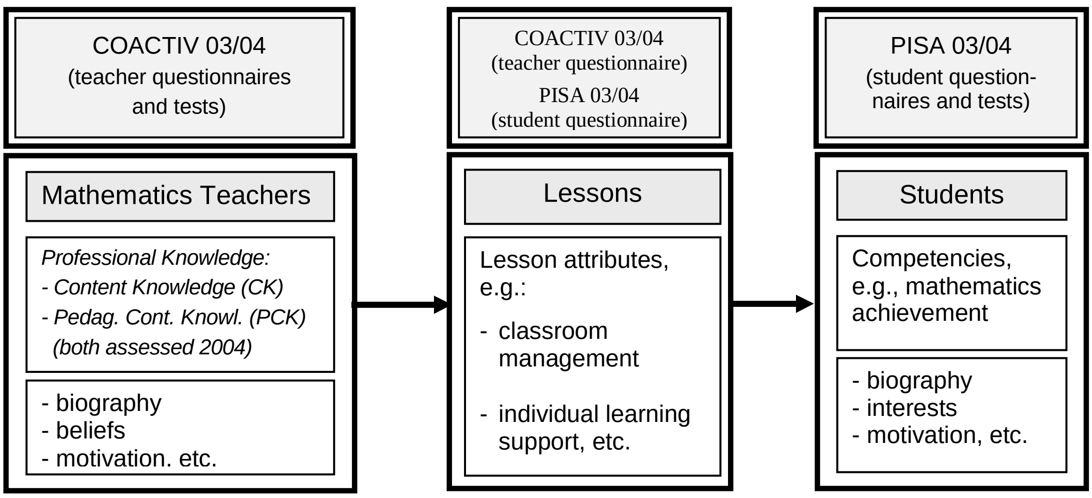

Figure 1: Conceptual connection of the COACTIV Study 03/04 and the PISA Study 03/04

In the present paper, we shall introduce the COACTIV instrument for testing mathematics teachers' *professional knowledge*, concentrating on the *subject-specific* knowledge (italic in Figure 1). Following Shulman (1986), a theoretical distinction is often drawn between domain-specific subject matter knowledge, *content knowledge* (CK), and the knowledge needed for teaching a specific subject, *pedagogical content knowledge* (PCK).[1] In the following, we will describe the conceptualisation of these two subject-specific knowledge categories in COACTIV, outline how tests on PCK and CK were constructed and implemented with German secondary mathematics teachers, and report some relevant results.

2 Conceptualisation of PCK and CK, test construction and test implementation

2.1 Pedagogical content knowledge (PCK). We have used three aspects that are specifically important to successful mathematics teaching in order to conceptualise pedagogical content knowledge and to guide test construction (see Krauss et al., in press).

(1) *Tasks* play a central role in teaching mathematics, accounting for much of the time allocated to mathematics lessons. Appropriately selected and implemented mathematical tasks lay the foundations for students' construction of knowledge and represent powerful learning opportunities (e.g., Jordan et al., 2006). Because the potential of tasks for students' learning can be exploited specifically by considering various solution paths (e.g., Silver et al., 2005), we assessed teacher' knowledge of tasks by testing their awareness of multiple solutions. In our PCK test, four items required teachers to list as many different ways of solving a given task as possible.

(2) Teachers need to work with students' existing conceptions and prior knowledge. Because mistakes can provide valuable insights into the implicit knowledge of the problem solver (Matz, 1982), it is important for teachers to be aware of typical student *misconceptions* and *difficulties*. In our PCK test, this aspect was assessed by confronting teachers with seven scenarios and asking them to detect, analyse (e.g., give cognitive reasons for a given problem), or predict a typical student error or a particular comprehension difficulty.

(3) Students' construction of knowledge is often only successful with instructional support and guidance, which may entail various forms of *explanations* or the explicit use of appropriate *representations*. Knowledge of subject-specific instructional strategies was assessed in our PCK test by eleven items requiring teachers to explain mathematical situations or to provide useful representations, analogies, illustrations, or examples to make mathematical content accessible to students (see Kirsch, 1977).

Consequently, our PCK test contained three *subscales*: knowledge of mathematical tasks ('*task*'), knowledge of student misconceptions and difficulties ('*student*'), and knowledge of mathematics-specific instructional strategies ('*instruction*'). Sample items for each of the three subscales are displayed in the Appendix.

2.2 Content knowledge (CK). Content knowledge describes a teacher's understanding of the structures of his or her domain. According to Shulman (1986), "the teacher need not only understand <u>that</u> something is so, the teacher must further understand <u>why</u> it is so". Clearly, teacher knowledge should go considerably beyond an awareness of the material to be mastered by students; rather, teachers should possess mathematical background knowledge of the content covered in the school curriculum at a much deeper level of understanding than their students. Thirteen items of this kind were constructed to cover relevant content areas (e.g., arithmetic, algebra, and geometry) and to tap conceptual or procedural skills (see Appendix for a sample item). For our CK test no subfacets were assumed (see Krauss et al., in press).

2.3 Procedure. 198 secondary mathematics teachers participated in the second COACTIV measurement point in 2004 where the tests of PCK and CK were implemented. Participants taught mathematics in the 10th grade classes sampled within the framework of PISA-2004 in Germany. Thus, our teacher sample can be considered fairly representative of 10th grade

[1] Another - subject-independent - knowledge category is *pedagogical knowledge* (PK), the knowledge on how to optimise learning situations in the classroom in general, which we will not address in this paper.

German mathematics teachers. Of the 198 teachers, 85 (55 % male) taught at the academic-track Gymnasium (“GY”), and 113 (43 % male) at other secondary school types (non-Gymnasium, “NGY”). The average age of participating teachers was 47.2 years (*SD* = 8.4); Teachers were paid 60 euro for participation. The assessment of PCK and CK was conducted individually in a separate room at the teacher’s school on a workday afternoon. It was administered by a trained test administrator, as a power test with no time constraints. The average time required to complete the 35 items was about 2 hours (approx. 65 min for the 22 PCK items, and 55 min for the 13 CK items).

All 35 items were open-ended. A scoring scheme was developed and 8 raters were extensively trained. Responses to each test item then were coded by two raters independently. The interrater reliability ρ was very satisfactory (on average across all items, ρ was .81).

3 Results

Both tests yielded satisfactory reliabilities (Cronbachs Alpha was .78 for PCK and .83 for CK). Thus, the test construction can be considered successful and the test results can be interpreted reliably. The largest source of variance in teachers’ performance was whether the teacher taught at Gymnasium or not. While the large differences in CK (see table 1) may be explained with the intensive education in mathematics of Gymnasium teachers at university, the advantage in PCK is somehow remarkable and may be interpreted as a first indication that CK substantially supports the development of PCK.

School type specific performance	M (SD) GY (N = 85)	M (SD) NGY (N = 113)	Effect Size d (GY vs. NGY)	Emp. Max GY	Emp. Max NGY
CK (13 Items)	8,5 (2,3)	4 (2,8)	1,73	13	12
PCK (22 Items)	22,6 (5,9)	18 (5,6)	0,80	37	29
Instruction (11 Items)	9,3 (3,4)	7,1 (3,2)	0,67	17	15
Student (7 Items)	5,8 (2,3)	4,3 (1,9)	0,71	11	9
Task (4 Items)	7,5 (1,8)	6,6 (2)	0,47	12	10

Table 1: School type differences: Means (standard deviations) and empirical maxima in two groups of teachers (GY = Gymnasium; NGY = non-Gymnasium). According to Cohen (1992), d = .20 is a small effect, d = .50 a medium effect, and d = .80 a large effect. *All differences are significant at the .01-level*

Further aspects of the test results of German mathematics teachers are noteworthy.
1) The knowledge development primarily seems to end after teacher education: In the COACTIV data, no positive correlation was found between either of the knowledge categories and the professional experience as a teacher (see Brunner et al, 2006b).
2) There is a close relationship between PCK and CK: The correlation between both knowledge categories was .60. PCK seems to „need“ a solid base of CK. This connection was much stronger in the group of Gymnasium teachers (GY) (see Krauss et al., in press).
3) PCK (and CK) are closely related to subjective constructivist learning theories. Teachers with high values in PCK and CK, for instance, tend to disagree with the view that mathematics is „just“ a toolbox consisting of facts and rules that „simply“ have to be recalled (see Kunter et al., in press). These teachers rather tend to think of mathematics as a process and that doing mathematics primarily means independent activity including insightful discoveries.
4) PCK can explain students’ achievement gains in a non-trivial way. Because COACTIV was “docked” onto the PISA study (see Figure 1), it is possible to relate teachers’ PCK to their students’ mathematics achievement gains over the year under investigation. Essentially, when mathematics achievement in grade 9 was kept constant, students taught by teachers with higher PCK scores performed significantly better in mathematics in grade 10 (see Baumert et

al., 2006, for details). This finding demonstrates that PCK is indeed a core candidate for creating powerful learning environments that support students' knowledge construction.

4 Discussion

In previous studies, most conclusions about the nature of teachers' knowledge have been drawn using indicators that are rather distal to the concept, such as university grades, number of subject matter courses taken at university, or questionnaire data on beliefs or subjective theories. Consequently, numerous calls have been made in the literature for more valid and reliable assessments of teacher knowledge (e.g. Lanahan, Scotchmer & McLaughlin, 2004). In the present paper, we reported on the construction and implementation of tests to assess the pedagogical content knowledge and the content knowledge of secondary mathematics teachers *directly*. Both knowledge categories were measured reliably and the mean differences between teachers with different educational backgrounds provided evidence for the empirical validity of the tests.

Practically, our results have at least two implications. First, our instrument might find more widespread application as a psychometric *assessment tool* that measures teachers' competence directly. In the light of recent developments in the area of teacher education, selection, and accountability, this aspect is of increasing importance. Our research identifies a way of gauging teacher qualifications in terms of the assets that seem most important for their primary task of teaching. We certainly do not yet know enough about issues such as retest reliability or suitability for other samples, but addressing these questions is an important objective of our ongoing research agenda.

Second, our study provides some valuable insights into the "long arm" of university *teacher training*. Because no positive correlation was found between years of teaching practice and the two knowledge categories, teacher training can be assumed to be at the core of the development of the two knowledge categories. Thus, current efforts to improve teacher education by emphasising strongly subject-based pedagogical content knowledge more than at present are encouraged by our results. Future research may provide deeper insights into the acquisition of PCK and CK during teacher training. For instance, longitudinal implementation of our tests at several critical stages in teacher education might provide more accurate information on the timing (e.g., in which phases of teacher education are PCK and CK acquired?) and mechanisms (e.g., which is needed to acquire the other?) of professional expertise development. Such studies may have consequences for instructional programs (at university and in the classroom) to foster the CK and PCK of student teachers.

Last but not least, it is our hope that the COACTIV results might not only activate discussion on the professional knowledge of mathematics teachers, but also initiate similar endeavours for other school subjects.

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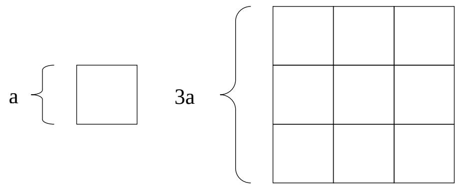
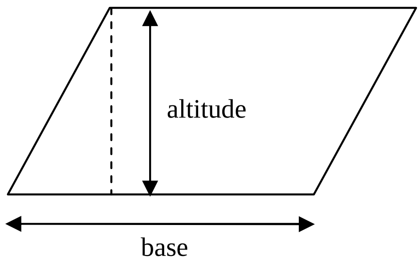
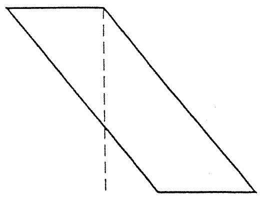
Appendix: Sample items and responses scoring 1 for the COACTIV tests on PCK and CK

Knowledge Category (Subscale)	Sample Item	Sample response (scoring 1)
PCK Task	How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem as possible.	<u>Algebraic response</u> Area of original square: a^2 Area of new square is then $(3a)^2 = 9a^2$; i.e., 9 times the area of the original square. <u>Geometric response</u> Nine times the area of the original square 
PCK Student	The area of a parallelogram can be calculated by multiplying the length of its base by its altitude.  Please sketch an example of a parallelogram to which students might fail to apply this formula.	 Note: The crucial aspect to be covered in this teacher response is that students might run into problems if the foot of the altitude is outside a given parallelogram.
PCK Instruction	A student says: I don't understand why $(-1)\cdot(-1)=1$ Please outline as many different ways as possible of explaining this mathematical fact to your student.	The "permanence principle," although it does not prove the statement, is one way to illustrate the logic behind the multiplication of two negative numbers: $3 \cdot (-1) = -3$ $2 \cdot (-1) = -2$ **-1** $1 \cdot (-1) = -1$ **+1** $0 \cdot (-1) = 0$ $(-1) \cdot (-1) = 1$ $(-2) \cdot (-1) = 2$
CK	Is it true that 0.999999.... = 1 ? Please give detailed reasons for your answer.	One possibility: Let 0.999… = a Then 10a = 9.99…, hence, $\underbrace{10a - a}_{9a} = \underbrace{9.99\ldots - 0.999\ldots}_{9}$ Therefore a = 1; hence, the statement is true