Insight into teachers' unconscious behaviour while dealing with mathematical modelling problems and implications for teacher education

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1 The projects COM² and DISUM

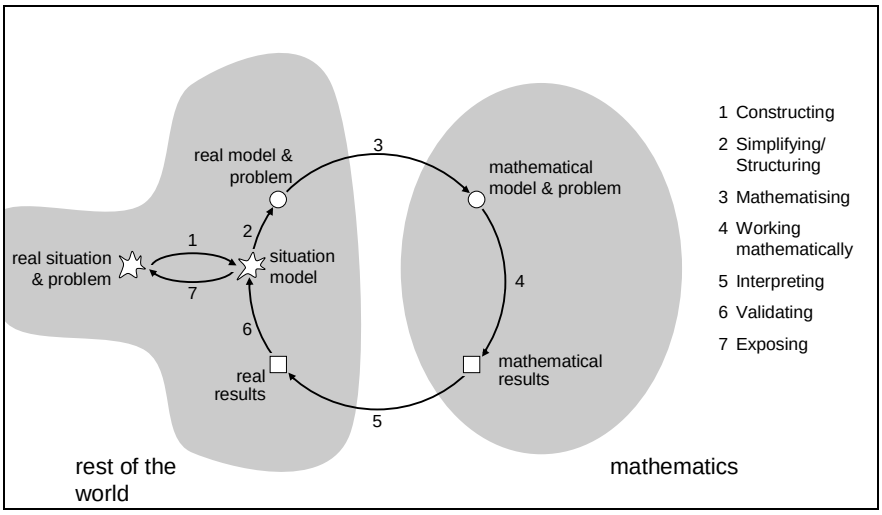
Within the discussion on mathematical modelling it is still an interesting question how teachers actually deal with modelling problems in the classroom.
A lot of studies make clear how important the role of the teacher during the modelling process is (see e.g. Lesh/Doerr 2003, Schorr/Lesh 2003, Blum/Leiß 2007a). Lesh/Doerr (2003, 126) pointed out that most studies focus on what "the teacher does in a particular situation, but not how the teacher thought about the context, what alternatives she considered, what purposes she had in mind or what elements of the situation she attended to and the meaning of those elements." The latter was the focus of their study and they conclude that the development of teachers' models during modelling-eliciting activities is of great importance. Doing practical work with teachers is a good and effective way of revealing teachers thinking. Schorr/Lesh (2003) developed so called "thought-revealing activities" for teachers who participated in a 3-year study. The results of this study also make clear that a big change happened with the teachers on different levels concerning their behaviour while pupils worked on modelling problems.
In this paper, I will report on two studies where experienced German teachers were observed while dealing with modelling tasks in grades 8-10 (14-16-year-olds). A common feature was that most of the teachers have not reflected on how they deal with modelling problems in the classroom concerning their preferred mathematical thinking style or their preferred types of interventions. Their actions and reactions were often on an unconscious level. One aim of the two studies was to make better visible what the teachers actually do. I will now introduce these two studies briefly:
The aim of the project COM² („**Co**gnitive-psychological analysis of **m**odelling processes in **m**athematics lessons"), my own project, is to analyse teachers' and students' actions, interactions and reactions while working on modelling problems in mathematics lessons from a cognitive perspective. For that aim, "theoretical glasses" are needed which make clear how the data were finally analysed and interpreted in this sense. These "glasses" are the theory of *mathematical thinking styles* (Borromeo Ferri 2004; for roots of this concept in cognitive psychology see Sternberg 1997). The term *mathematical thinking style* denotes "the way in which an individual prefers to present, to understand and to think through mathematical facts and connections using certain internal imaginations and/or externalized representations. Hence, mathematical style is based on two components: 1) internal imaginations and externalized representations, 2) the holistic respectively the dissecting way of proceeding." (cf. Borromeo Ferri 2004, 50). Empirically, three mathematical thinking styles of students attending grades 9/10 could be reconstructed: visual, analytic and integrated thinking styles.
The leading questions of the project DISUM („**D**idaktische **I**nterventionsformen für einen **s**elbständigkeitsorientierten aufgabengesteuerten **U**nterricht in **M**athematik"), directed by the second author together with R. Messner (both University of Kassel) and R. Pekrun (University of München), with research staff D. Leiß, S. Schukajlow and M. Müller, are (see Blum/Leiß 2007b):

- Which cognitive potential do given modelling tasks have, and how is that potential used by students and teachers, how do they actually deal with such tasks?
- Which effects do different teaching conceptions and different types of teachers' interventions have?

Students and teachers were observed both in laboratory situations (pairs of students working together on modelling tasks, partly with and partly without the support of a teacher) and in the classroom. The theoretical background for our observations of teachers' interventions in particular is the classification according to Leiß/Wiegand (2005): organisational, affective, content-related and strategic interventions. The focus of all observations is the crucial question how the subtle *balance between students' independence and teachers' guidance* is realised in the classroom (according to Maria Montessori's principle: "Help me to do it by myself!")

Both projects also aim at implementing their insights into mathematics teaching and mathematics teacher education.

An important instrument for analyses and observations in both projects is our version of the modelling cycle (see Blum/Leiß 2005, Blum/Leiß 2007a,b, Borromeo Ferri 2006).



2 Teachers' mathematical thinking styles

The central question of the COM² study was:

- How do mathematical thinking styles of teachers influence their way of dealing with mathematical modelling problems in the classroom? Are there differences with respect to the various phases of the modelling cycle (real situation, situation model, real model, mathematical model, mathematical results, real results)?

The design of this qualitative study is highly complex, because both teachers and pupils are in the focus. Quantitative research seems to be inappropriate given the focus of the study, the internal cognitive processes of learners and teachers.
Three grade 10 classes from different *Gymnasien* (German Grammar Schools) were chosen. The sample is comprised of 65 pupils and 3 teachers (one male, two female). Each individual of a class had to do a questionnaire on mathematical thinking styles which has been developed on the basis of the Ph.D. thesis Borromeo Ferri (2004).
Focused interviews were conducted with the teachers to reconstruct in each case his or her mathematical thinking style. In each lesson pupils worked in groups of five on different modelling problems. The video-camera was directed on one group desk and had a view of the whole class during plenary discussions to record all the interactions of the learners.

Additionally, the teachers were equipped with a minidisc-recorder strapped to their body in order to document the teacher's help or suggestions during modelling as this could possibly influence the student's modelling processes. After video-taping the lessons there was a stimulated recall with each of the teachers where they were shown sequences of their behavior in the classroom.
On the basis of this data analyses the following theses can be formulated concerning the research questions mentioned before:

- A teacher's mathematical thinking style can be reconstructed and manifests itself during individual pupil-teacher conversations as well as during discussions of solutions and while imparting knowledge of mathematical facts.
- Teachers who differ in their mathematical thinking styles have preferences of focusing on different parts of the modelling cycle while discussing the solutions of the problems and while helping students during their modelling processes.
- Teachers were mostly not aware of their behaviour during modelling activities in the classroom and were astonished about their preferences for certain parts of the modelling process, connected to their mathematical thinking style.

3 Teachers' interventions

The concentration in this paper lies on the question:

- How do teachers intervene, and how do they succeed in realising the balance between students' independence and teachers' guidance?

I report on a "Best Practice Study" with experienced teachers (all participating in a long-term reform project in Germany) in the context of the DISUM project. They observed teachers in 16 grade 9 or 10 classes teaching demanding modelling tasks in their own style.
Here are some general observation from this study:

- Teachers' interventions were mostly intuitive and not independence-preserving, they were mostly content-related or organisational and rarely strategic. The spectrum of interventions that a specific teacher used was mostly rather narrow, many teachers had their own "intervention style", independent of the individual students' needs.
- Often the teachers' own favourite solutions of the modelling tasks were unconsciously imposed on the students through their interventions, also because of an insufficient knowledge of the "task space" on the teachers' side. Most teachers were very surprised after this was revealed by the study.

There is no space here to report in detail on a recent case study within the DISUM project where two optimised teaching styles were compared, one more teacher-guided ("directive") style and more independence-oriented ("operative-strategic") style. All lessons (ten per class) were videotaped and analysed. There was a pre-test immediately before the teaching unit, a post-test immediately afterwards and a follow-up-test three months later. One of the most important results was that the progress in modelling competency of the students in the "operative-strategic" classes were substantially higher and, in particular, more enduring compared to the "directive" classes (and even more so compared to students working totally alone). The differences in progress resulted mostly from the stronger students whereas the progress of the weaker students was similar. The best results concerning progress in modelling competency were achieved in the class were the balance between students' independence and teachers' guidance was, according to our observations and ratings, realised best.

4 Some implications for teaching mathematical modelling

I have emphasised that these two studies also aim at using our insights for improving mathematics teaching and teacher education. Here are some obvious implications resulting from our studies:

- The version of the modelling cycle used in the two studies is helpful and even indispensable both for teachers (as a basis for their diagnoses and interventions) and for researchers (as a tool for describing actions and cognitive processes in learning environments with modelling tasks). For students, a simpler version seems to be appropriate (we have developed a four-step "Solution Plan" for students, see Blum/Leiß 2007b).
- It is necessary to make mathematics teachers aware of their own thinking styles and thus to support them in consciously finding an appropriate balance between thinking and acting within mathematics and thinking and acting within the real world. Reflecting on their own thinking styles will also help teachers to better communicate with their students, in particular with those whose thinking styles do not match with theirs.
- It is necessary to make mathematics teachers aware of their own intervention styles and to supply them with a broad spectrum of intervention modes in various teaching situations with modelling tasks, and thus to support them in better realising the afore-mentioned balance between students' independence and teachers' guidance. For that, a lot of reflected experience on the teachers' side is necessary.

We (also the members of the DISUM-Project) try to implement these aspects into pre-service and in-service teacher education. In particular, we try to link our approaches even closer together, that is to search for possible connections between teachers' mathematical thinking styles and teachers' intervention styles, both on a descriptive level and for a well-aimed broadening of teachers' repertoire of actions, to support students' successful solving of modelling tasks and students' developing of modelling competency.

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