TEACHERS' ASSUMPTIONS ABOUT STUDENT "LEARNING PATHS"

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INTRODUCTION

Ball, Bass, Sleep and Thames (2005) propose a framework describing knowledge associated with mathematics for teaching. The framework consists of four "distinct domains" (p. 3): (1) *common content knowledge (CCK)* — the mathematical knowledge of the school curriculum, (2) *specialized content knowledge (SCK)* — the mathematical knowledge that teachers use in teaching that goes beyond the mathematics of the curriculum itself, (3) *knowledge of students and content (KSC)* - the intersection of knowledge about students and knowledge about mathematics, and (4) *knowledge of teaching and content (KTC)* - intersection of knowledge about teaching and knowledge about mathematics (p. 4). This research examines the relationship between the second (SCK) and third (KSC) domains, with the locus of inquiry resting in the third domain (KSC). The other domains, although described as distinct, intersect, if not overlap, most closely within KSC – making this domain an important point of contemplation.

KSC is closely linked to assertions made by Murata and Fuson (2006) about "learning paths" (p. 424). Murata and Fuson argue that in relation to understanding students' mathematical thinking (i.e., KSC) "there are *not* [my emphasis] 20 to 35 different *learning paths* [authors' emphasis] or strategies for teachers to understand and assist" (p. 424). Rather, student thinking and learning can be isolated to a few specific *and* predictable trajectories, or learning paths. Murata and Fuson make clear that these few predictable trajectories are *not* a closed set, and that other trajectories are possible; hence, teachers must be open to these other trajectories.

The research questions guiding this work are: (1) what are the assumptions teachers make about student "learning paths" (i.e., KSC)? And, (2) in analyzing such assumptions, what conceptual and pedagogical insights might be mined to support knowledge development in the other domains defined by Ball et al. (2005)? To explore these questions, pairs of students and pairs of teachers were given a common mathematical task. Teachers were asked to model potential student learning paths.

THEORETICAL FRAMEWORK

Shulman (1986; 1987) was among the first to begin making distinctions between the types of knowledge needed for teaching in his conceptualization of *Pedagogical Content Knowledge* (PCK). According to Shulman PCK "goes beyond knowledge of subject matter . . to the dimension of subject matter knowledge *for teaching* [author's emphasis]" (Shulman, 1986, p. 9). He says emphatically that teachers must have "ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). Ball et al. (2005) attribute their proposed domains of KSC and KTC to Shulman (1986; 1987). Ball et al. state that KSC and KTC "are closest to what is often meant by 'pedagogical content knowledge' — the unique blend of knowledge of mathematics and its pedagogy" (p. 4).

METHODOLOGY

Participants

Data for this research was collected over the 2005-2006 school years at multiple sites. In order to examine teachers' assumptions about student learning paths, a common task was completed by pairs of tenth-grade students (n = 23) and pairs of mathematics teachers (n = 13). The students were from two classes, each taught by one of the authors of this paper. The composition of students varied according to gender, ethnicity, and socio-economic status. The course, in which the students were enrolled, was an "advanced" mathematics course, geared toward students who were anticipating post-secondary education. Although the range of abilities within the classes varied, the majority of students achieved at least a Level 3- (72%) in this course.

The mathematics teachers who participated in this research were all mathematic department heads, in addition to classroom teachers, from various secondary schools , some urban and some rural, of one particular board of education. The data were gathered during a monthly organizational meeting. All of the teachers except for one had greater than 10 years of teaching experience. All of the teachers had advanced degrees in mathematics, education, and mathematics education. Consequently, we describe this group of teachers as having high SCK. This sample of teachers is a purposive effort to neutralize concerns over low SCK in relation to mathematics for teaching and to elaborate on research about teaching mathematics that largely focuses on teachers with low SCK.

The task: What is water pressure?

We developed the task, *What is water pressure*?, as a final assessment for a quadratics unit that spanned approximately four weeks (Kotsopoulos & Lavigne, 2007). Quadratics appears in each of the subsequent grades of mathematics instruction in our province. Therefore, the teachers in this research likely taught this material on multiple occasions, in multiple courses, during the current school year, as well as during their careers.

The task involved modeling the water flow from the drinking taps in a school, where the projection of the water from the spout forms a parabolic arch. Pairs of teachers and pairs of students were asked to determine various forms of the quadratic relations (e.g., standard form, factored form, and vertex form) that modeled the current flow of the particular tap they were investigating. Additionally, the pairs were asked to determine the quadratic relation representing an arbitrarily set 'ideal' water flow of 3 cm above the faucet guard at the fountain. The underlining impetus for determining the ideal water flow was based on notions of water conservation; that is, a reduced water flow is potentially is more cost-effective in terms of overall water consumption.

Teachers and students had 70 minutes (one class period) to complete the mathematical task. Teachers and students were randomly paired and assigned a fountain in either their school or the school in which the department heads' meeting was taking place. The group as a whole read through the mathematical task. The assessment rubric was provided and reviewed prior to beginning the inquiry. Teachers were given the additional instructions to complete the task as a "level 3"[1] response that might be

[1] Level 3 represents the acceptable Ministry of Education standards of achievement (i.e., approximately 75% average) in mathematics.

anticipated from a tenth-grade student. Students were not permitted to use graphing calculators on the task, but were permitted to use their own scientific calculators. Following the completion of the task with the teachers, we reconvened for a brief focus group session to discuss the task, and possible teaching and learning dilemmas.

Data collection and analysis

A content analysis, involving the coding and counting of features within the artifacts, was completed for all the solutions (Berg, 2004). A common coding structure was established initially by each independently completing a possible "answer key" for the task. From our individual solutions, we discussed features that might be important or noteworthy if missed or omitted and modified the coding structure accordingly. Each author independently coded all the solutions from all the pairs of teachers and students. We then compared our coding with each other and contemplated differences and omissions with a goal of reaching consensus. The collective set of student and teacher data was compared.

There were seven codes used for the analysis. Evidence of an error, either in the solutions or the graphical representations (e.g., incorrect *x*-intercepts on the graphical representation), was coded as *conceptual error.* Solutions that did not include a diagram, make use of the available physical model, were highly abstract in reasoning (i.e., factoring or using the quadratic formula to find the zeros of the quadratic relation), and/or achieved via graphing calculator (i.e., quadratic regression) were coded as *theoretical reasoning*. Solutions that were incomplete in one or more of the requirements of the task were coded as *incomplete*, regardless of the extent that the solution was incomplete. Solutions that were completed using fractions as opposed to decimals were coded as *fractions.* Transformation of graphical representations of water fountains that appear to flow into the second quadrant of the Cartesian plane, to the first quadrant were coded as *transformed model*. We also coded instances in which solutions showed the intentional use of *friendly numbers*, indicating that students or teachers understood the numbers used were somewhat arbitrary and could be manipulated slightly to facilitate easier calculations. Finally, we coded, using the Achievement Chart from the current curriculum documents in mathematics (Ontario Ministry of Education/OME, 2005), the overall level of communication of mathematical findings of the solutions as either a *L1, L2, L3*, or *L4* – with L3 representing current acceptable ministry standards, L4 exceeding standards, and L1 significantly below ministry standards.

RESULTS AND DISCUSSION

Our results suggest that the teachers in this study were not able to easily anticipate the learning paths (i.e., KSC) of students (see Table 1). Surprisingly, teachers made significantly more conceptual errors than students. Teachers were more likely to over theorize as well. The conceptual errors and the theoretical reasoning of the teachers are intertwined and directly related to SCK. For example, many teachers neglected to transform the *x*-intercepts of their graphical representations. This error was seen as unrelated to the mathematics, but rather to the perception that the model was perhaps unnecessary in order for them to complete the task. Consequently, the physical model was not fully incorporated into their solution. The teachers showed more dependency on their theoretical understanding of the quadratic relation (or high SCK). Their results were less precise because of this, although we believe that the teachers' intentions were increased precision.

Students adapted to the task somewhat more concretely and did better overall because of their use of the physical model. Students' concrete thinking was further evidenced by the creative ways in which their graphical representations were transformed in order to make the calculations more straight forward (i.e., from the second to the first quadrant. This was surprisingly not anticipated by the teachers or us - the researchers. This leads us to question whether student learning paths might potentially be restricted or develop at all, if higher SCK informs the decisions teachers make in relation to KTC. Also compelling is the evidence that many teachers, with significant SCK, were unable to complete the task. Again, here we assert that over-theorization was the contributing factor. In spite of this assertion, we wonder how teachers come to zoom in on the particulars of CCK and KTC in order to develop an understanding of KSC, given the possible interference of SCK.

CONCLUSIONS

Our research demonstrates that more research is needed to examine, not only those with limited SCK, but also those with significant SCK. Indeed, a *different* type of mathematics for teaching education may be required for those with high SCK. The teachers in our research, in our opinion, would have been the most likely to anticipate student learning paths, given their personal histories in mathematics education. Yet, the learning paths of students were not well anticipated, which we contend was not intentional. Despite a teacher's experience and high levels of SCK, student thinking *can* still be surprising and reveal alternative ways of thinking about mathematics, which highlights the merit in engaging in an exercise of modeling student thinking as professional development for teachers. High SCK might permit teachers to look at alternative learning paths and see the validity and merit in those learning paths, whereas those teachers with limited SCK may not have the same sort of elasticity.

Our findings demonstrate that indeed the loci of mathematics for teaching, within Ball et al.'s (2005) domains, does rest in KSC. Teachers' knowledge of the other domains can be examined, by extension, through an analysis of KSC. Furthermore, we propose that our research suggests that perhaps a fruitful starting point for mathematics teacher education may be through an examination of student learning paths or KSC.

In our research, we did not return to the teachers and present the student results in relation to their own projection of student learning paths. This remains an important area of further inquiry and, as mentioned earlier, could be a productive way of educating both current and in-service teachers about the various domains of mathematics for teaching. Furthermore, we do not explore the factors that *do* contribute to student learning. Further analysis is needed to see what other factors contribute to student understanding, particularly when the learning paths of students and differ from those projected by teachers.

We recognize that our role as these students' regular classroom teachers may have influenced our *own* assumptions. However, aside from the common task, the individual choices each of us made within our classrooms, as teachers, were not common or shared. We each made independent decisions about our classroom practices. However, like our colleagues, who engaged in this research, our choices are restricted by the current policies in mathematics education (i.e., curriculum, assessment, and authorized texts) so there is a commonality in pedagogy to an extent. This commonality between our fellow teachers and us should have resulted in a more cohesive set of learning paths between the students and the teachers. However, this was not the case.

The assumptions teachers make about student thinking potentially correlates to the ways in which teachers teach. By examining teachers' assumptions about student thinking, we can then begin to unpack the assumptions teachers make. Furthermore, we can begin to understand the kinds of additional knowledge that teachers might need to be more effective at teaching mathematics.

Table 1: **Overall results from the coding, as percentages, for the teachers and students**

Code	Teachers	Students
Conceptual error	69%	26%
Theoretical reasoning	31%	13%
Incomplete	31%	44%
Fractions	54%	48%
Transformed model	38%	74%
Friendly numbers	92%	96%
Overall level of communication of mathematical findings	L4: 62% L3: 8% L2: 22% L1: 8%	L4: 74% L3: 26% L2: 0 L1: 0

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