TEACHERS' BELIEFS AS TEACHERS' KNOWLEDGE

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Initial mathematics teacher education is primarily concerned with knowledge – the acquisition of knowledge required for the teaching of mathematics. Opinions as to what exactly comprise this knowledge and how it is best delivered and best learned varies widely across different contexts, but in general it is discussed as being comprised of two strands – *knowledge of mathematics* and *knowledge of teaching mathematics*[1]. Knowledge of mathematics pertains to mathematical concepts, use of mathematical techniques, mathematical reasoning, proof, etc. Knowledge of teaching mathematics is the knowledge regarding the conditions and ways of *mathematics* teaching and learning (Brousseau, 1997; Durand-Guerrier & Winsløw, 2006) and "captures both the link and the distinction between knowing something for oneself and being able to enable others to know it" (Rowland, Twaites, & Huckstep, 2006, p.1).

But discussions of teachers' knowledge cannot be strictly limited to these objective forms – teachers' subjective knowledge is also important. "It has become an accepted view that it is the [mathematics] teacher's subjective school related knowledge that determines for the most part what happens in the classroom" (Chapman, 2002, p. 177). One central aspect of subjective knowledge is beliefs (Op't Eynde, De Corte, & Verschaffel, 2002). In fact, Ernest (1989) suggests that beliefs are the primary regulators for mathematics teachers' professional behaviour in the classrooms. These beliefs do not develop within the practice of teaching, however. Prospective elementary teachers do not come to teacher education believing that they know nothing about teaching mathematics (Feiman-Nemser, Mcdiarmid, Malnick, & Parker, 1987). "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (Ball, 1988). These ideas are more than just feelings or fleeting notions about mathematics and mathematics teaching. During their time as students of mathematics they first formulated, and then concretized, deep seated beliefs about mathematics and what it means to learn and teach mathematics (Lortie, 1975). It is these beliefs that often form the foundation on which they will eventually build their own practice as teachers of mathematics (Fosnot, 1989; Millsaps, 2000; Skott, 2001; Uusimaki & Nason, 2004).

This distinction between knowledge and beliefs is a false dichotomy, however. In general, knowledge is seen as an "essentially a social construct" (Op 'T Eynde, De Corte, &

[1] In many contexts a further distinction is made between *knowledge of teaching* in general and *knowledge of teaching mathematics* in particular. This further segregation then leads to three strands - *content knowledge*, *pedagogical knowledge*, and *didactical knowledge* (Bromme, 1994; Comiti & Ball, 1996; Durand-Guerrier & Skott, 2005). Shulman (1987) refers to these same categories as *subject matter knowledge* (SMK), *pedagogical knowledge* (PK), and *pedagogical content knowledge* (PCK) respectively.

Verschaffel, 2002). That is, the division between knowledge and belief is the evaluations of these notions against some socially shared criteria. If the truth criterion is satisfied then the conception is deemed to be knowledge. But knowledge can also be seen as an 'individual construct'. Leatham (2006) articulates this argument nicely:

> *Of all the things we believe, there are some things that we "just believe" and other things we "more than believe – we know." Those things we "more than believe" we refer to as knowledge and those things we "just believe" we refer to as beliefs. Thus beliefs and knowledge can profitably be viewed as complementary subsets of the things we believe. (p. 92)*

Although viewing knowledge as a social construct is a convenient way to differentiate between knowledge and beliefs, individuals (for the most part) operate based on knowledge as an individual construct. That is, their actions are guided by what they believe to be true rather than what may actually be true. Mathematics teachers (preservice or inservice) are no different – their actions (i.e. teaching) is guided by what they believe to be true about mathematics and about the teaching and learning of mathematics. As such, any discussions about teacher knowledge, whether it be *knowledge of mathematics* or *knowledge of teaching mathematics*, needs to include discussions about teachers' beliefs.

Our Knowledge of Teachers' Knowledge

Both teacher knowledge and beliefs possess a duality within mathematics education – a duality that can be encapsulated as the tension between 'has' and 'should have'. That is, there is a constant tension in the literature between the knowledge and beliefs that a teacher 'has' and the knowledge and beliefs that a teacher 'should have'. In many ways this is a product of the constant confluence of theory, research, and practice within the field of mathematics education and cannot be, and should not be, resolved through the exclusion of one or the other. Our understanding of what knowledge and beliefs are needed for the teaching of mathematics is informed by the knowledge and beliefs possessed by teachers who are effectively (or not effectively) teaching that concept. This emerging understanding, in turn, informs our work in preservice and inservice teacher education as we work to develop the necessary knowledge and beliefs within teachers.

Beliefs about Mathematics

Dionne (1984) suggests that beliefs about mathematics are composed of three basic components called the traditional perspective, the formalist perspective and the constructivist perspective. Similarly, Ernest (1991) describes three philosophies of mathematics called instrumentalist, Platonist and problem solving, while Törner and Grigutsch (1994) name the three components as toolbox aspect, system aspect and process aspect. All these different notions correspond more or less with each other. In the *toolbox aspect*, mathematics is seen as a set of rules, formulae, skills and procedures,

while mathematical activity means calculating as well as using rules, procedures and formulae. In the *system aspect*, mathematics is characterized by logic, rigorous proofs, exact definitions and a precise mathematical language, and doing mathematics consists of accurate proofs as well as of the use of a precise and rigorous language. In the *process aspect*, mathematics is considered as a constructive process where relations between different notions and sentences play an important role. Here the mathematical activity involves creative steps, such as generating rules and formulae, thereby inventing or re-inventing the mathematics.

Mathematics Teachers Beliefs about Mathematics

How these beliefs, when held by mathematics teachers, link to pedagogy is obvious. A teacher with a view of mathematics as a *toolbox* will teach mathematics as such. This will mean an emphasis on rules, formula, and procedures with an abundance of practice to enforce memorization and mastery. A teacher with a *systems* view will make extensive use of definitions and proofs both as a pedagogical strategy and as content to be acquired. Finally, a teacher with a *process* view will incorporate progressive constructivist teaching methodologies into their teaching in order to have their students experience the 'doing' of mathematics.

Changing Mathematical Beliefs of Mathematics Teachers

Robust beliefs are difficult to change. However, an abundance of research purports to produce changes in preservice and inservice teachers of mathematics. Prominent in this research is an approach by which preservice teachers' beliefs are challenged (Feiman-Nemser et al., 1987). Another prominent method for producing change in preservice teachers is by involving them as learners of mathematics (and mathematics pedagogy), usually submersed in a constructivist environment (Ball, 1988; Feiman-Nemser & Featherstone, 1992). A third method for producing changes in belief structures is through preservice teachers' experiences with mathematical discovery, which has been shown to have a profound, and immediate, transformative effect on the beliefs regarding the nature of mathematics, as well as their beliefs regarding the teaching and learning of mathematics (Liljedahl, 2005). More recently, a combination of all three of these methods has been shown to be very effective in changing preservice teachers' beliefs (Liljedahl, Rolka, & Rösken, 2007; Rolka, & Rösken, & Liljedahl, 2006).

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