High school pre-service teachers' beliefs about proof: some reflections for & from a training course

Francesca Morselli
Dipartimento di Matematica Università di Genova Italy

1. Introduction

This contribution is made up of two parts. In the first part, I discuss some basic findings concerning teachers' beliefs. Afterwards, I discuss some different directions of the research on teachers' beliefs, namely the analysis and theoretical description of teachers' beliefs, the study of their mutual relationship with practice and the more specific issue of changing beliefs. This will lead to a central theme in mathematics teacher education, that is to say the work on beliefs in pre-service teacher preparation. In the second part, I discuss the design and context of a training program (where I am the educator) whose aim is to lead pre-service teachers to reflect on their beliefs, with particular reference to beliefs about proof.

I underline that both parts are strictly "autobiographical". The first part relies on my readings (basically during my PhD studies) and on what I heard, discussed and discovered attending to the ICMI Study 15 "*The Professional Education and Development of Teachers of Mathematics*". This part mirrors my discovering and becoming aware of some crucial factors shaping teachers' behavior, that should be taken into consideration when designing a training course, see (Morselli, 2007). The second part is an account of my first years as a teacher educator and may be considered my personal way of (attempting to) facing the problem of the relationship between theory and practice. It is the reflection of a young researcher in mathematics education, who sees mathematics teacher education as a field where it is possible to *invest* her knowledge, and also the reflection of a debutant mathematics teacher education practitioner.

This contribution aims at providing some elements of discussion for the working group, with particular reference to the issue: *the demand for research based teacher education.*

2. Teachers' beliefs

Shulman (1986) describes the different components of teachers' knowledge. Among them, it is important to take into account two components: content knowledge and pedagogical content knowledge. Similarly, Borko et alii (1992) identify two components of the *knowledge for teaching*: subject matter knowledge and pedagogical matter knowledge. As Furinghetti (2007) points out, also teachers' beliefs[1] are part of the knowledge for teaching. Teachers' beliefs are organized into a dynamic system and are in a dialectic relationship with practice (Thompson, 1992).

Thompson (1992) underlines that it is impossible to distinguish beliefs from knowledge, because "teachers treat their beliefs as knowledge" (p. 127), and that it is important to take into account two kind of beliefs: beliefs about mathematics and beliefs about mathematical teaching and learning. These two kinds of beliefs are evidently linked to content knowledge and pedagogical content knowledge and, as Thompson explains, may have a crucial role in influencing the teachers' instructional practice. According to Ernest (1989), three key elements influence mathematical teaching: the system of beliefs about mathematics and its teaching and learning, the social context, the teachers' level of thought processes and reflection. The issue of reflection on instructional actions, on the subject matter and on one's beliefs seems to be very important, as evidenced by the study conducted by Thompson (1984).

Teachers' beliefs are studied by the researchers in mathematics education according to three different perspectives, as evidenced by the survey by Thompson (1992) and by the different

[1] It is well known that there is a problem of terminology. For example, Thompson (1992) deals with beliefs and conceptions, "viewed as a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences" (p. 130). In the following, I choose to use only the term "belief".

contributions to (Leder, Pehkonen and Törner, 2002): the study of teachers' beliefs, with particular reference to the construction of a suitable theoretical framework, the study of the dialectic relationship between teachers' beliefs and instructional practice, the study of situations and experiences that may change teachers' beliefs. This latter strand is directly linked to the emerging field of research in mathematics teacher education. As Adler et alii. (2005) underline, not every research on teachers' beliefs is a research in mathematics teacher education, but some researches in mathematics teacher education directly refer to the theme of beliefs, in the context of a pre-service or in-service training program. Hereunder, I focus on this third theme, with particular reference to the preparation of pre-service teachers.

3. Pre-service teachers' preparation: the role of beliefs

The issue of prospective teachers training may be framed within the following consideration: prospective teachers are former students and, of course, prospective teachers. As former students, they have experiences linked to mathematics that may have formed their beliefs about mathematics and its teaching and learning. As prospective teachers, these beliefs will influence not only the kind of teachers they will become, but also the way of attending to the training programs, to live their training experiences. It is important to keep in mind that pre-service teachers come to the training program with beliefs about mathematics and mathematics teaching and learning, as well as expectations as regards training. As Ball (1990) says, it is important to help prospective teachers to "learn to do something different from – and better than- what they experienced in mathematics classes" (p. 11). According to Ball, educators have two responsibilities: "to judge what prior learning can contribute to future growth and which may impede it" and "to construct the conditions for experiences which can foster future growth" (p.12). Ball refers "the tension between instilling new confidence and unsettling old assumptions" (p.14). This means that training is at the level of content knowledge and pedagogical content knowledge, with a crucial role of beliefs about mathematics and its teaching and learning.

Thompson (1992) refers some studies aimed at changing pre-service teachers' beliefs, but she warns that "pre-service teachers' conceptions are not easily altered" (p. 139). Possible ways of changing prospective teachers' beliefs are involving them into innovative mathematical experiences or analyzing with them the students' ways of thinking. The different ways of changing beliefs are linked to the systemic nature of beliefs and to their dialectic relationship with practice.

I refer here to two examples of training programs whose aim was to affect prospective teachers' beliefs. I think they represent two ways of bringing the practitioners' competence in the research in mathematics education into the practice of mathematics teacher education, turning it into research in mathematics teacher education.

The first example is described in Furinghetti (2007) and deals with the use of history in a training program: by means of history, prospective teachers are led to question their beliefs about algebra and its teaching. In this way, Furinghetti challenges "their beliefs that they must reproduce the style of mathematics teaching seen in their school days".

The second example is the training program described by Liljedahl (2007) in his contribution to the 15th ICMI study. Liljedahl discusses "the effectiveness of a methods course designed around problem solving in challenging the beliefs of a group of pre-service teachers of mathematics". Through their experiences with problem solving, the prospective teachers changed their beliefs about the nature of mathematics, and what it means to teach and learn mathematics.

According to Thompson (1992), good results are obtained when the goal is to make the prospective teachers *to doubt* about their own beliefs. This is linked to the importance of creating a reflective teacher, who reflects on his own practice and beliefs. In my view, the crucial point is to make prospective teachers to reflect on their own beliefs, as a preliminary activity to enter the training program and as a first step towards the formation of a reflective teacher. Hereunder, I elaborate on this point.

4. Discussion of a training program

In the previous part I outlined some crucial points that a practitioner should take into account when planning and teaching a course for prospective teachers. In particular, a course should take into account the need of reconciling with mathematics and questioning the existent beliefs about mathematics and its teaching and learning. In this part I refer my own experience while designing and teaching a course to high school prospective teachers. Referring to Shulman's theory, the course should provide some elements of content knowledge and pedagogical content knowledge. The course (24 hours), administered during the first year of a two-years training program, is in my opinion the best place to lead pre-service teachers to reflect on their beliefs.

I point out here a peculiar feature of the prospective teachers I teach to. Since the course is addressed to prospective teachers that will teach both mathematics and physics in high school, the participants have different university backgrounds, namely they have a degree in mathematics, or in physics, or in engineering. A priori, this could be considered an additional difficulty to conceive the training program, since the different backgrounds entail different content knowledge and, probably, different beliefs concerning mathematics learning and teaching. As I will discuss, the choice of working on beliefs turns the presence of different backgrounds into a positive fact.

My choice was to focus on one crucial theme in Mathematics Education: the teaching and learning of proof. Proof is a central theme in mathematics and, as such, is worth having a crucial role in mathematics education. Consequently, it is important to take into consideration pre-service teachers' knowledge and beliefs about proof and its teaching and learning. The choice of the theme was, of course, influenced also by my previous experience as a PhD student and by my knowledge of the literature of research in Mathematics Education related to proof (for a survey of literature, I refer to the website http://www.lettredelapreuve.it).

The course started with a written questionnaire, to be filled individually, on proof and its teaching and learning. The questionnaire encompassed the following parts:

- Open-ended questions on proof and its teaching and learning
- Comparison and commentary, from the mathematical and educational point of view, of two different proofs of the Pythagorean Theorem.
- Individual written activity of conjecture and proof in Elementary Number Theory.

As regards the open questions, they were divided into several categories (for each section, I mention some significant questions):

- Questions about proof from a mathematical point of view (e.g.: *What is the role of proof in mathematics?*)
- Questions about the teaching of proof at secondary school level (e.g.: *What is the role of proof in school mathematics? How much time would you spend in teaching proof, also taking into account the kind of school where you teach?*)
- Questions about students' difficulties with proof (e.g.: *What are, in your opinion, the most widespread difficulties of students concerning proof?*)
- Questions about one's knowledge on proof (e.g.: *Write a statement whose proof you remember*)

Obviously there were not "right" or "wrong" answers: each open question was designed in order to elicit one's beliefs and in order to foster the subsequent discussion.

The written answers to the questionnaire were collected and preliminary analyzed by myself. I categorized the answers to each question and some representative excerpts from each category were discussed by the whole class in the subsequent sessions. Also the protocols of individual conjecture and proof were analyzed during the discussion.

5. Some excerpts from the questionnaire

Hereunder I present some relevant excerpts, just to give an idea of the discussion that took place.

As regards the question "*What is the role of proof in school mathematics?*", four different roles were outlined by prospective teachers: proof convinces of the validity of mathematical statements, proof helps to memorize mathematical facts, proof is the best example of mathematical reasoning (and learning proof means learning to reason). One prospective teacher also wrote about the value of proof in the development of the individual.

> Excerpt 1: Proof convinces the student that the theorem is true.
>
> Excerpt 2: What is proved is better remembered by the students.
>
> Excerpt 3: Proof makes the students used to mathematical language and to logic rigour.
>
> Excerpt 4: Proof in school is useful not only in reference to the learning of mathematics, but more broadly to the development of cognitive faculties that are essential for every man. Learning to prove means becoming able to argument one's choices in an efficient way, that can be understood by all those who share the language and logic.

When answering the question "*How much time would you spend in teaching proof, also taking into account the kind of school where you teach?*", some prospective teachers affirmed that the teaching of proof should be limited to high school with high scientific orientation, while others defended the formative value of proof for any kind of school. Some prospective teachers mentioned some constraints (time constraints, students'motivation, …) to the teaching of proof.

> Excerpt 5: Proof is a useful tool in order to foster students' reasoning and lead them to argument in a correct way. Thus it should have a great role in a school that is intended to train the students to reason, rather than in technical and vocational schools where it is more important to focus on results and on the future work.
>
> Excerpt 6: I think that, where the students are enough motivated and interested, it is absolutely necessary to give wide place to proof and proving techniques. Indeed, I think that learning to prove is fundamental for life and only for mathematics, since it encompasses the learning of logic, of how to criticize, of how to distinguish. Such considerations stand independently form the kind of school. Students must be educated to proof. what can be different is the level and difficulty of the proofs that are proposed.

Some crucial points were discussed: for example, the coherency between considering proof important because it supports students' rationality and the choice of presenting proof only in some kind of schools.

The theme of proof allowed to address many other issues in mathematics education, such as students' motivation, institutional constraints in teaching, use of dynamic geometry software and so on. Proof revealed itself as an interesting theme per se, but also as a catalyser of issues concerning mathematics education.

The presence of prospective teachers with different backgrounds turned out to be useful: the comparison of different kind of answers made the prospective teachers to think about the influence of their school and university experience and to take into account different points of view. In

general, the discussion of the answers to the questionnaire led the prospective teachers to reflect on their own beliefs and to critically analyze them. Their initial assumptions, basically coming from previous experiences as students, were questioned and analyzed.

The analysis of the questionnaire was important in order to set a common ground as a starting point for the remainder of the course. In this sense, the questionnaire also helped me to know my students and to design the other activities in a more suitable way[2].

The analysis of the individual protocols of conjecture and proof allowed to work out some content knowledge concerning proof, but also to develop skills in analyzing students' ways of thinking and to reflect on the influence of personal factors in the activity of conjecture and proof.

The work on the whole questionnaire can be considered my personal attempt to bring my experience of research in mathematics education into the practice of teacher educator.

References

Adler, J., Ball, D.L., Krainer, K., Lin, F.L. & Novotna, J. (2005). Reflections on an emerging field: researching mathematics teacher education. *Educational Studies in Mathematics, 60(3)*, 359-381.

Ball, D.L. (1990). Breaking with experience in learning to teach mathematics: the role of a preservice methods course. *For the Learning of Mathematics, 10(2)*, 10-16.

Borko, H., Eisenhart, M., Brown, C.A., Underhill, R.G., Jones, D. & Agard, P.C. (1992). Learning to teach hard mathematics: do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education, 23*, 194-222.

Ernest, P. (1989). The impact of beliefs on teaching. In Keitel, C., Damerow, P., Bishop, A., Gerdes, P. (Eds.): *Mathematics Education and Society*, Paris, UNESCO, 99-101.

Furinghetti, F. (2007). Teacher education through the history of mathematics. *Educational Studies in Mathematics*.

Leder, G., Pehkonen, E. & Törner, G. (Eds.) (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer.

Morselli, F. (2007). Reflections on pre-service primary teachers' needs and difficulties: their "relation to mathematics". *Proceedings of the Fifteenth ICMI Study "The Professional Education and Development of Teachers of Mathematics"*, Águas de Lindóia, Brazil, 15-21 may 2005.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15* (5), 4-14.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15(2)*, 105-127.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: a synthesis of the research. In D.A. Grows (ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 127-146). New York: Macmillan.

[2] After this series of activities, we read together and discussed some research papers in mathematics education concerning proof. Other research papers were read in small groups and presented and discussed to the whole class. We also analyzed some students' protocols of conjecture and proof.