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Knowledge of Teachers – Knowledge of Students:
Conceptualizations and outcomes of a Mathematics Teacher Education Study in Germany

In Germany, a unique chance to investigate systematically mathematics teachers' professional knowledge occurred in connection with the PISA-Study 2003 (OECD, 2004). Since a large scale study like PISA mainly provides descriptive data, there always remains an explanatory gap between student performance and system variables (Baumert, Blum & Neubrand, 2004). One salient variable, of course, is the professional knowledge of teachers. Thus, as a national option to PISA-2003, a study on the professional knowledge of teachers of mathematics was initiated, and linked to the student achievement data (on class level). In the sample are all those teachers who taught mathematics in the classes drawn for the international PISA-study (Krauss & al., 2004). The study's name is COACTIV[1] (Cognitive Activation in the Classroom: Learning Opportunities for the Enhancement of Mindful Mathematics Learning).

Four questions are immediately associated to such a study, addressed in the following paragraphs: What does it mean to say a mathematics teacher has professional knowledge? From where do teachers get this knowledge? What are the ways professional knowledge of mathematics teachers influences classroom instruction? Does professional knowledge of the teacher have an effect on students' achievement?

1 Conceptualization of teachers' professional knowledge

To conceptualize teachers' knowledge from a mathematics education point of view, is challenging, especially for teachers in secondary schools; primary teachers' professional knowledge already became studied by e.g. the Mathematics Teaching and Learning to Teach group at the University of Michigan (Ball, Hill & Bass, 2005). Studying mathematics teachers' knowledge requires to map specific issues of mathematics into concepts like content knowledge and pedagogical content knowledge. Doing so, COACTIV was directed by the following basic aspects:

(a) A mathematics lesson should students cognitively activate, teaching and learning should be centered around understanding. Thus as a *general orientation*, professional knowledge has the aim to foster students' thinking about mathematical concepts and procedures, and therefore any activity of a teacher has to be grounded in an authentic view of doing mathematics. Just to have methods of simplification at hand is not enough (cf. the ICME-1976 plenary lecture Kirsch, 1978).

(b) Any study on the professional knowledge of mathematics teachers needs to be *broad enough*: The concept of pedagogical content knowledge has to be mirrored in the

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potential of teaching and learning mathematics. Already Shulman pointed, besides the curricular knowledge, to the need of knowledge about "the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p 9). We easily see that knowledge on each of the edges of the triangle "content – students' learning and understanding – teachers' construction of lessons" is necessary.

(c) The instruments used in the study should reflect what one knows is effective in mathematics education research. Since the activities of the teacher in a lesson are widely organized by the tasks the teacher uses, *tasks are appropriate* to analyze professional knowledge. In mathematics education this way proved fruitful. E.g. tasks made visible different structures of lessons in the TIMSS-Video-Study (J. Neubrand, 2006); hence some instruments and analyzing categories were taken from there (Jordan & al., 2006).

As an overall structure of the COACTIV-Study, Shulman's distinction of Content Knowledge and Pedagogical Content Knowledge remains visible. But even under *content knowledge (CK)* one has several choices: Is it the every-day knowledge of mathematics, is it the knowledge taught to students at school, is it the academic knowledge taught at universities, or is it – what we finally took – a deeper understanding of the mathematics taught at school (Ma, 1999)? This is a task under the CK-perspective:

> Is it true that 0.999999.... = 1 ? – Please give detailed reasons for your answer.

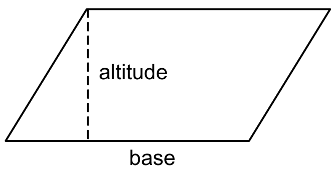
Considering *pedagogical content knowledge (PCK)*, three facets were contained in the COACTIC-Study, forming something like the edges of the aforementioned triangle:

– A teacher should know about the cognitive potential of mathematical tasks, since content is often brought by mathematical tasks into the classroom. This means knowing about students' strategies, to be able to judge the cognitive relevance of a task, and having multiple solution paths and representations at hand. A sample item is:

> Show in as many ways as you can give reasons for: "The square of an integer is always 1 bigger than the product of the two adjacent numbers."

– Knowledge about students' mathematical cognitions is necessary for adaptive teaching. Errors, difficulties, failures should be productive sources for concept development. A teacher must be able to recognize typical difficulties. A sample item is:

> The area of a parallelogram can be calculated by multiplying the length of its base by its altitude.



> Please sketch an example of a parallelogram to which students might fail to apply this formula.

– Knowledge of the variety of mathematics-specific methods of explaining is necessary. "The teacher must have at hand a veritable armamentarium of alternative forms of representation, …" (Shulman, 1986, p 9). This is one of the instances, content knowledge and pedagogical content knowledge influence one another. A sample item is:

> A student says: "I don't understand why (–1)(–1)=1" – Please (teacher) outline as many different ways you see of explaining this mathematical fact to that student.

2 The structure of German Teachers‘ professional knowledge

As an overall picture, content knowledge and pedagogical content knowledge were highly correlated (.60 is a typical overall correlation coefficient). However, the correlation was much lower when we only consider CK and PCK of the Non-Gymnasium teachers, and much higher for the Gymnasium teachers (Brunner & al., 2006). To understand this, it is necessary to know that the education of mathematics teachers in Germany is strictly separated along the academic (Gymnasium) and non-academic tracks of the schools. Gymnasium teachers receive an in-depth mathematics training at universities, comparable to a Master’s degree in mathematics, while the non-academic teachers‘ education gives much more emphasis to pedagogy and offers only limited mathematics courses. Our findings, that Gymnasium teacher show substantially higher PCK scores are remarkable, and may – although very tentatively – be interpreted as an indication that CK supports the development of PCK (Krauss & al., in press). Subject-specific knowledge seems to make it easier to integrate CK and PCK, a result well compatible with some findings of the psychological research on expertise in general.

The findings of COACTIV provide further theoretical and empirical evidence for the applicability of Shulman’s taxonomy of teacher knowledge, in a subject specific setting (Krauss & al., in press). They offer a differentiated view on how far CK and PCK can be separated (Hill & al., 2004).

3 Professional knowledge and classroom instruction

The design of COACTIV allowed to correlate aspects of the mathematics teachers‘ professional knowledge to what students reported from their classroom instruction. From the mathematics education perspective, a very satisfying result came out. PCK, to recall: measured by strictly mathematics bounded items, influences the potential of the teacher to foster cognitive activation in the classroom (Kunter & al., 2007). The more teachers know about how to make mathematical content accessible to students, the more their students feel cognitively challenged in the classroom.

4 Professional knowledge and students’ achievement

Of course, the salient question is in how far professional knowledge of a teacher is indeed a (positive) predictor for the achievement gains of the students in the class taught by that teacher. Never such results could be obtained on a representative level in Germany before. COACTIV correlated the achievement gains of the students (on class level), measured by the gain in PISA scores from 9^{th} to 10^{th} grade (in 2003 and 2004), to the degree of professional knowledge of the class teacher. Only those teachers came into consideration who permanently taught the class in that period.

Well-known from numerous studies is that the previous knowledge from Grade 9 is the strongest predictor. However, CK and PCK predict students’ learning considerably. The most promising model seems to be to assume that CK and PCK influence students’ achievement not directly but mediated by several factors. It turned out that in that model the most influential factor is the mathematical activation in the classroom. This could be measured in our study by a objective classification of the examination tasks the teacher gave in that class. The classification system used (Jordan & al., 2006) was intended to detect several instances for cognitive challenging tasks.

5 Conclusion

From the results of COACTIV already at this stage of the evaluation some conclusions can be drawn. It is not simply PCK which makes an effect on students' achievement. PCK seems to be positively influenced by a sound CK. However, it is definitely not in the reverse way: CK alone does not have an effect on students' achievement, nor does it influence classroom behavior directly into a more challenging setting. Cognitive activation in the classroom is a function of the PCK expertise of the teacher.

These results claim for fostering a strongly subject bounded, but nevertheless pedagogically oriented education of teachers, in the universities as well as in further education. This claim relates to different levels in the education of mathematics teachers (M. Neubrand, 2006): (i) Knowledge in psychology or pedagogy remains empty without being bounded to the subject. (ii) The mathematics education part of teacher education cannot be realized as just a methods course; rather it needs reflections on characteristic features of mathematics, be it in the sense of epistemology or by referring to students' ways of thinking. (iii) Mathematics content courses itself should for teachers also convey how mathematics proceeds, what mathematical thinking is, and how it is connected to other human activities, i.e. that it is a cognitive challenge. In all, there is no substitution of CK by PCK or pedagogical studies, nor vice versa.

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