From Dienes' Blocks to JavaBars: A Personal Odyssey in the use of artifacts, materials and tools for learning and teaching mathematics.

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As we celebrate the centennial of ICMI, I am moved to look back over the past 40 years of my own involvement in mathematical instruction, with a particular focus on the use of concrete materials, calculating machines and computer software. I completed my degree in mathematics at Manchester University in England in June of 1967 and began my career as an educator that same year by enrolling in the Post Graduate Certificate in Education at Leicester University, with an emphasis on the primary school. It was during my time in the P.G.C.E. program at Leicester that I was introduced to the structured materials for learning mathematics that were being developed by Zoltan P. Dienes (1971).

Dienes' Logic Blocks

I had the good fortune to be in the right place at the right time. I requested that I do my student teaching at my old village primary school in Burbage, Leicestershire. The headmaster at that time was Philip Sherwood, a progressive educator who was working closely with Dienes. Burbage Primary School was one of Dienes' lab schools and Philip lost no time in introducing me to the visionary mathematics educator who took me under his wing. I worked with Dienes over the next two years, trying out many of his new games and structured materials with my class of 9 and 10 year-olds. I was particularly interested in his logic games for young children, and in the use of his Multibase Arithmetic Blocks (known world-wide as Dienes' Multibase Blocks) that were designed to provide children with multiple embodiments of the structure of our place-value number system. My first research project was conducted during this period with 8 and 9-year old children at the school. I investigated the practicality of implementing the course on teaching logic to young children that Dienes was developing: "La Logique á L'Elementaire" produced by the International Study Group for Mathematics Learning (ISGML) at the University of Sherbrooke in 1967. Dienes created a structured set of attribute blocks that consisted of 48 plastic blocks: 4 different shapes (circle, triangle square and non-square rectangle), 3 different colors (red, blue and yellow), 2 sizes (large and small) and 2 thichnesses (thick and thin). The children were introduced to this structured material through cycles of "free play -- structured games -- practice games" that were designed to help young children form insights about the logical relations among blocks based on the properties of set union, intersection and complement. My research (Olive, 1968) indicated that these young children had most difficulty (at first) with games of logical disjunction (e.g. form the set of blocks that are either red or triangular – they chose only the red triangles) and that they chose to use the color attribute, even when the other attributes provided simpler solutions to the game situation. I modified my approach, based on these initial findings and began a sequence of games, starting with work on defining sets of blocks (e.g. all the red blocks), unions and intersections of sets (e.g. all the red blocks in one hoop, all the triangles in another hoop, what happens to the red triangles?). I then introduced the language of logical conjunction (e.g. find a block that is both red and triangular) and simple negation (e.g. find a block that is NOT red). I followed these games with logical disjunction (e.g. find the blocks that are either red or triangular), logical negation (e.g. find the blocks that are NOT both red and triangular) and logical implication (e.g. form the set of blocks for which the following implication is true: "if a block is red then it is a triangle"). The four children with whom I worked in this second round progressed rapidly through this sequence of games and one student even produced a contra-positive from a set of blocks (e.g. for the set of blocks for which the implication "if red then triangular" is true [the union of triangles with the blocks that are NOT red] the student also found that the contra-positive statement was also true: "If not triangular then not red"). They eventually were able to make simple logical deductions, such as the following: We have a set of blocks that are either red or triangular; if I pick a block that is NOT red, what can you tell me about it?

My conclusions, based on this small, investigative study were the following:

1. It was necessary for me to engage the children in an analysis of each game situation in order for them to understand the role of the logical connectives. When left to play the games with the blocks themselves, the children seldom generated logical sentences.
2. Children should be encouraged to invent their own symbol systems for recording the logical relations before introducing any kind of formal system. Any symbol system should first be used to represent physical situations that can be analyzed meaningfully by the children.
3. The children needed to go through cycles of physically building a logical structure (using the logic blocks), then analyzing the structure, before further building could take place.
4. After just 8 one-hour sessions over a period of 7 weeks, these children were ready to engage in logical deductive reasoning based on the sets of blocks. Two of the children were able to transfer this reasoning ability to other situations and contexts.

The Dienes' Logic Blocks are just one of the many structured sets of materials that Dienes and his associates produced as artifacts for his mathematics program. (For a very concise overview of the program see Seaborne, 1975.) The logic blocks were used in Dienes' program for much more than the logical relations I have outlined in my own study. They were used for patterning activities, order relations, difference relations, and as an introduction to number operations and place value (along with many other structured materials, including the multi-base arithmetic blocks). One of the key aspects of the Dienes' mathematics program was the notion of "multi-embodiments" of the same mathematical structure (Dienes, 1960, 1971; Seaborne, 1975). Dienes encouraged teachers to have their students create their own sets of materials out of "found" products, such as cardboard disks, stick-on paper shapes, buttons, plastic animals, etc.

Dienes' Logic Tracks

Dienes also created a system of "dynamic" notation for sorting blocks and representing logical relations called "logic tracks" (Seaborne, 1975, pp. 128-147). These tracks were modifications of a traditional tree-sort diagram, with the upper track being the selected branch of the tree and all other branches being combined into a lower track (the rejects). Conjunctions of attributes were represented by a series of tracks along which blocks (or any other structured set of attribute material that students wanted to use, including themselves!) could move from right to left as in Figure 1 below. All members of the set start on the right and move along the upper track until they come to a junction. Only the elements of the set having the attribute defined by the label on the upper track (e.g. red) would continue along the upper track; all other elements would be shunted off to the lower track. At the next junction in Figure 1, only the red squares would continue along the upper track; all the non-square red pieces would be shunted off to the lower track and join the non-red pieces.

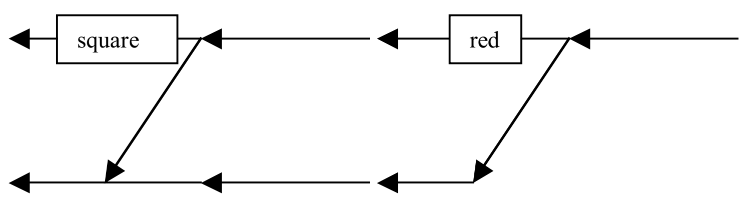

Figure 1: Representing the conjunction of red and square using Logic Tracks

The disjunction of two attributes was represented by a junction on the lower track shunting pieces to join the upper track according to a particular attribute (see Figure 2 below).

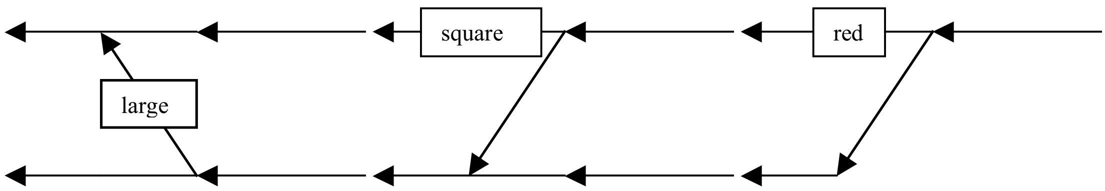

Figure 2: Disjunction of large pieces with the red squares

The final set emerging from the left of the upper track in Figure 2 would be the set of pieces that were either large or both red and square. Other track configurations were used for the negation of a set (a simple cross-over that exchanged upper and lower tracks) and equivalence (if and only if – then) that was achieved by a labeled cross over (only the pieces possessing the labeled attribute crossed over), followed by a negation (simple cross-over).

I used versions of these tracks made out of thick cardboard rectangles (tri-wall) with silver tape for the tracks with my 8 and 9-year old students at the Paidea School in Atlanta, Georgia. The students enjoyed making complicated tracks and predicting which pieces (or students) would end up in the "castle" (where the upper track ended) and which would end up in the railway yard (where the lower track ended). My students made a pilot for a children's television program (Olive, 1977) that featured this tracks activity using thick masking tape on the floor for the tracks and the children themselves as the pieces. The children moved along the tracks according to the labeled attributes to the tune of the Beatle's "Magical Mystery Tour." The viewers were to figure out who ended up in the castle.

Computer Instantiations of Logic Games

Television was not the only technological medium in which Dienes' logic games emerged during the late 1970's and 1980's. The Learning Company published a set of computer programs during the 1980's based on these logical games. *Rocky's Boots* (Robinett & Grimm, 1982) won many awards and was a computer version of the Logic Tracks in which the user joined different *logic gates* together in order to determine the set of pieces that would travel along the conveyor belt (the rejected pieces were *booted off* the conveyor belt by a raccoon named *Rocky*). The "order" games and "difference" games were the basis for *Gertrude's Secrets* that was published by The Learning Company in 1986. The game featured rooms filled with puzzle shapes similar to the Logic Blocks. Users solved the puzzles by arranging the objects by shape and color according to some pre-determined rules. Gertrude was a goose that brought the different pieces into each "room" in the game.

More recently, TERC (formerly Technology Education Resource Center) created a very successful set of computer games for both home and school, called *Zoombinis Logical Journey* now published by Sunburst. To quote from TERC's (2007) web site:

> Winner of the prestigious 1997 Cody Excellence Award, the Bologna International New Media Prize, and numerous other accolades, this entertaining computer game takes children on a puzzle-filled adventure that builds math thinking skills along the way! Choosing from different Zoombini attributes of hair style, nose color, eyes, and mobility, kids create their band of Zoombinis. As they solve the puzzles to reach Zoombiniville, children can develop skills in data analysis and sorting, hypotheses formation, graphing, logical reasoning, statistical thinking, pattern recognition and set theory.

Dienes' Multibase Arithmetic Blocks

Dienes' Multibase Arithmetic Blocks (MAB) have been used in elementary classrooms around the world for more than 3 decades. The blocks consist of small wooden cubes (approximately .5 cm on an edge) called "singles", linear groupings of these cubes according to various base numbers (2, 3, 4, 5, 6 and ten), called "longs", groupings of these long blocks into square regions called "flats" (using the same base number) and groupings of these flats into larger cubes (using the same base number). The sets using the smaller base numbers (2, 3 and 4) also contained "super longs" and "super flats" that continued the grouping structure using the large cubes. Dienes' rationale for using different grouping bases (rather than just base ten) was based on his theory of multi-embodiment: By grouping pieces using different numbers of blocks, but following the same grouping rules and forming similar structures at each level of grouping, the children would be able to more readily abstract the mathematical structure of the groupings and relate this to our base ten number system. My own experiences using these materials with children in my elementary classrooms and with pre-service elementary teachers at the university, in both England and the USA, supports Dienes' rationale and his theory of multi-embodiments. Use of the MAB provided children in my classes with opportunities for developing their own algorithms for multi-digit arithmetic operations that were both more efficient and more sophisticated than the standard algorithms. The pre-service teachers in my university classes were challenged to think deeply about place value and multi-digit arithmetic when working in different bases with the MAB. They developed a respect and understanding for the difficulties young children have when working in base ten with multi-digit arithmetic for the first time.

Pat Thompson created a microworld called *BLOCKS* that was a computerized version of Dienes' MAB. In the *BLOCKS* microworld it was possible to designate any type of block as the unit block, thus decimal notation could be modeled using these virtual manipulatives. Together with his wife, Alba (Thompson & Thompson, 1990) he conducted an investigation that compared students' use of the physical MAB with use of the virtual BLOCKS in classroom lessons on decimal notation and decimal arithmetic. Results of this study were presented at the annual meeting of the ICMI's affiliate group, PME, in Mexico, 1990. Thompson & Thompson found that the medium and high-level students in their randomized sample of students using the BLOCKS microworld made better progress with respect to decimal notation and operations on decimals than their counterparts in the wooden blocks classroom. The Thompsons argued that the constraints of the computer microworld encouraged students to make connections between the arrangement and manipulation of the virtual blocks with the notational system for decimals (these were hot-linked within the microworld), whereas the students in the wooden blocks classroom (even those that made substantial progress) did not make such connections between their physical activities with the wooden blocks and the decimal notation. The virtual blocks provided students with the advantage of multiple, linked representational systems (Kaput, 1986).

There have been many computer versions of the MAB produced since Pat Thompson's BLOCKS microworld. My favorites are the Java applets available through the *National Library of Virtual Manipulatives* – NLVM (2007) at Utah State University. These on-screen manipulatives enable the user to create collections of blocks by clicking on the screen icon and generating a multi-digit number in the respective base. If as many or more blocks than the base number are created in a particular place-value region on the chart, the numerical link disappears. The user can assemble the blocks into the next higher type of structure and glue them together simply by surrounding them with a dragged selection rectangle. This higher-order block can then be moved one place to the left and the correct multi-digit number re-appears on the screen. There are a series of applets providing problems with multi-digit addition, subtraction, and work with decimal numbers. The NLVM also has Java applets for playing attribute games similar to the logic games developed by Dienes, as well as virtual manipulatives for practically every topic in the standard mathematics K-12 curriculum.

Computer-Based Interactive Learning Environments

The NVLM are but one example of *Computer-Based Interactive Learning Environments (CBILEs).* Topic Group 19 at ICME 8 in Seville, Spain focused on computer-based interactive learning. As reported in the proceedings from the Topic Group (Balacheff & Sánchez, 1996), "The main challenge of CBILEs design is to offer students a space in which they can explore freely a virtual world designed to support the construction of some mathematical knowledge." (p. 355). As part of this Topic Group, I presented results from a research project on children's construction of fractions (Steffe & Olive, 1990) through which we developed a set of virtual manipulatives called TIMA (Tools for Interactive Mathematical Activity) (Olive, 2000; 2002a). The children in our research project were able to construct complex operations involving multiplication and division of fractions using the TIMA that were different from the symbolic operations they had learned through their classroom instruction (Steffe & Olive, 1996). Activities using the TIMA: Sticks and TIMA: Bars microworlds (Olive & Steffe, 1994; Steffe & Olive, 2002) promoted the construction of an iterative fraction scheme that was more powerful for students' understanding of these operations on fractions than the traditional "parts-out-of-whole" concept of a fraction (Olive, 1999; Olive & Steffe, 2002; Steffe, 2002). Results of this research project have been reported at annual meetings of PME, an affiliate group of ICMI (Olive, 2001, 2002b, 2003). A free, downloadable version of TIMA: Bars, called JavaBars is now available from my webpage, along with a user manual and example activities (Olive, 2007). The JavaBars software is being used in classrooms around the world and I have had requests for creating Chinese and Turkish versions of the software. It is used extensively in our pre-service teacher education programs at The University of Georgia and at other higher education institutions both in the USA and abroad. A discussion of the effects of CBILEs in general and the TIMA in particular on knowledge and learning of mathematics will be published in the ICMI study volume currently being produced by the ICMI Study 17 working group on *Technology Revisited,* and will be presented at ICME 11 in Monterey, Mexico, July 2008.

Through this personal odyssey, I hope to have demonstrated how the design and use of structured materials for mathematics learning, pioneered by Dienes in the 1960's and 70's, have been both assimilated and transformed by the new technologies developed over the past 40 years. Dienes' (1960) theory of multiple embodiments together with Kaput's (1986) notion of multiple-linked representations that the new technologies provide, are key aspects of today's dynamic tools for teaching and learning mathematics.

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