SOME REFLECTIONS ABOUT OLD AND NEW MEDIA IN MATHEMATICS EDUCATION

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These days, when we talk about media, new information and communication technologies immediately come to mind: Internet, avatar, artificial intelligence, applets, multimodal language. Three decades ago, when we heard about information technology, calculators, scientific calculators, or maybe even graphing calculators might have come to mind. For instance, the Proceedings of the Second International Congress on Mathematical Education (Howson, 1973), held in Exeter in 1972, include references to the use of calculators and computers. Programmable calculators in school were *"seen both as an aid for the teacher when introducing mathematical concepts and also as a calculating aid for the pupils"* (p. 57). Computers were seen as motivating for students and promising for mathematics: *"Numerical and graphical methods will undoubtedly receive impetus from the advent of computers"* (p. 26). Clearly, the media was considered mainly as a sort of scaffolding at that time to help in the construction of mathematics but not as a fundamental part of the mathematical knowledge production in school. It is worth noting an interesting excerpt from the Proceedings of ICME 2 where the contributions to mathematics education of the first ICMI president, Felix Klein, were referred to: *"His books on* Elementary Mathematics from an Advanced Standpoint *are still read throughout the world – indeed, some of his suggestions, such as the use of calculating machines for teaching arithmetic to children, still have a modern ring about them!* (p.11). Klein died in 1925 and became an icon in mathematics education; it is also possible, in light of this quote, to credit him with being a pioneer in proposing the use of technology in schools.

Interest in the use of computers was increasing in the mathematics education community, and the first ICMI study was related with this theme: *The Influence of Computers and Informatics on Mathematics and its Teaching*. The Study Conference was held in Strasbourg, France, in March 1985. The work in this particular study focussed on three issues: the effects of computers and informatics on mathematics and on the design of new curricula, and the uses of computers as aides in the teaching of mathematics. In the discussion document that was prepared prior to the conference, when referring to the effects of computers on mathematics, it was recognised that *"Only on rare occasions has it been allowed to influence and infiltrate our teaching"* (Churchhouse et al, 1984, p. 164). In spite of being an assertion made more than twenty years ago, this is still the case in many schools in Argentina. For example, many educators consider that the use of calculators by students will prevent them from using mathematical reasoning. The following story illustrates a position of resistance towards the use of calculators. In a secondary school during a mathematics class for first graders (12-13 years old), the teacher posed the following task: *If the 9 key on your calculator doesn't work, how would you use it to compute 9393 – 1439?* Apparently the task was aimed at applying properties of numbers and operations using the calculator, but the teacher did not allow the students to use it. The task lost its meaning and became boring and tedious for the students, leading them to do the calculations with paper and pencil. Obviously, the teacher didn't restrict the use of paper and pencil. These devices have become transparent in the act of doing mathematics. No one questions their use to solve a mathematical exercise. It seems that the paper and pencil have always been present together with mathematics. Authors like Lévy (1993) pointed out that orality and writing, as well as informatics, are technologies of intelligence that condition the production of knowledge. In Borba & Villarreal (2005) we have developed the notion that knowledge is produced by collectives of humans-with-media. This notion brings two main ideas: that cognition is not an individual enterprise but rather social/collective in nature; and that cognition includes tools, media with which knowledge is produced. The media are components of the epistemic subject, being neither auxiliary nor supplementary, but a necessary part of it. They are so relevant that if they are absent, different knowledge is produced. The media we work with alter, redefine and reorganize the practices, the contents and the ways of knowing.

We always have new media, but some, like the blackboard and paper and pencil, seem to have gained acceptance in the schools. In this paper, I will continue the project we have summarized in Borba & Villarreal (2005) studying how other devices which were novelties a hundred years ago became accepted in schools. We used history of education papers from Argentina and Brazil to analyze how such media were introduced in schools[1].

The use of media in the production of mathematics as well as in mathematics education has usually been treated as transparent. For instance, Davis & Hersh (1989) stress that: *"There is no lack of mathematicians who like to think that even a solitary being closed up on a dark room would be able to create mathematics, appealing so alone to the resources of a brilliant Platonic intellect*" (p.28). These authors talked about the auxiliary tools or equipment that are needed to produce mathematics. They assert that, in the past, primitive mathematics, as well as religion and great epopees, were probably transmitted via oral tradition, but it later became evident that the use of devices for writing, record-keeping, and duplication were essential for doing mathematics. Davis & Hersh (1989) go further when they highlight that the ruler and compass are intrinsic parts of the axioms that lay the foundations for the Euclidean geometry.

In educational contexts, the use of manipulative materials has been a frequent recommendation for teaching and learning mathematics at different times in the history of mathematics education. We can count or solve arithmetic calculations using pebbles, abacus, counters, fingers, paper and pencil, mechanical adding machines, or handheld electronic calculators. According to Davis & Hersh (1989): *"Each one of these methods has, in effect, a different understanding of the integers, and a different form of relating to them"* (p.40). Such words seem to be in resonance with the notion that media are essential for knowledge production. However, we can also find other points of view related to the use of different manipulative materials with children (logical blocks, matches, etc.). For instance, in the Proceedings of ICME 2 (Howson, 1973) we found a warning saying that *"generalisations about material and methods could be dangerous"* (p. 24) due to the complexity of the development of children's thinking and the fact that little was known about it. Another section of these proceedings was devoted to the mathematics workshop or laboratory where the use of games, apparatuses, or manipulative materials was analysed in terms of advantages and disadvantages for the teaching of mathematics. In this case, the materials were considered as means to evoke the students' interest or lead them to discover a mathematical fact, concept or generalisation. At that time, this method appeared more widely accepted at the primary level, and recommendations regarding their use included the following: *"Each lesson should have an underlying framework of objectives: both subject oriented and student oriented. <u>The hardware and the equipment are not the essence</u>; more important than the physical facilities are the attitudes of the children, the atmosphere of the classroom, and the objectives of the teacher"* (p. 54, my emphasis). While the attitudes of the children, the atmosphere of the classroom, and the objectives of the teacher are considered paramount in the classroom, the teaching materials are not seen as main actors in the knowledge production, and they become transparent media.

Another transparent medium that is characterized by its ubiquity in schools is the blackboard. We cannot imagine a classroom without a blackboard, but this device was not always present. Different sources indicate that the blackboard was used for the first time in schools around 1800. An instructor at West Point Military Academy (Mr. George Baron) is considered to be the first American instructor to use a large blackboard to teach mathematics in 1801. Contemporary with Baron, the Headmaster of the Old High School of Edinburgh (Scotland), James Pillans, is also credited for inventing the blackboard and colored chalk which he used to teach geography. In France, since 1882, the blackboard was considered a teaching material that every teacher should have in primary school, and a dogma of the modern school was: *"the best teacher is who uses chalk the most"* (M.P, 1901, p.186, cited in Bastos, 2005, p. 136). The arrival and definitive adoption of the blackboard in Latin American schools occurred to the end of the 19th Century (Bastos, 2005),

[1] This paper is part of a project I have developed with Marcelo C. Borba, from São Paulo State University– UNESP – Rio Claro – Brazil. I would like to thank him for his comments and collaboration in parts of this paper.

shortly before the ICMI was created, when public systems of elementary education were being consolidated.

The blackboard in the school gave rise to new educational practices. Prior to its presence in the classrooms, students used a handheld slate where they wrote the assignments. The teacher had to go from one student to another copying the tasks on each student's individual slate. The use of these slates meant that there were no permanent records of the school tasks. According to Bastos (2005) *"It may be affirmed that the pedagogical centrality of the blackboard is due to the absence of school manuals and other visual learning resources, and also results in centralizing the pedagogical process in the figure of the teacher"* (p. 133). One of the main practices that the large blackboard enabled was the simultaneous teaching of reading and writing lessons for the whole class. Since that time, the blackboard and the teachers became central actors in classroom life, and copying from the blackboard was the main activity of the students. However, the blackboard is also used to share the students' solutions to a given problem and, in this sense, can also become a medium for hearing the students' voices.

The individual handheld slates were the only devices that students used in the school before the arrival of notebooks, when the production of paper increased and the costs decreased. Certainly the historical data about the use of notebooks in school differ from place to place. In France, for example, use of the notebook became common in high school (10 to 14 year-olds) in the 16th Century, and was obligatory in the teaching of calligraphy in the 17th Century, but its generalized use in the elementary schools dates from the first third of the 19th Century (Hébrard, 2001). The cost of paper limited its use until the more advanced grades, restricting children's learning in the area of literacy to reading. Hébrard reports that, around 1833, the use of the notebook in elementary education was considered by the Ministry of Elementary Instruction to be a sign of pedagogical modernity. In Argentina, the notebook was introduced into the classroom around 1920, and was closely linked to the so-called new school movement, which recommended the use of a single class notebook as an organizing tool for school work (Gvirtz, 1999).

When notebooks were introduced into the schools, everyday life inside as well as outside the school was transformed. These transformations affected the activities of the students. The use of the notebook implied not only knowing how to copy, write dictation, do arithmetic exercises, or solve arithmetic problems, but also how to organize and present them, making the notebook a *"small theatre of school knowledge"* (Hébrard, 2001, p. 137). According to Hébrard (2001), the notebook permits the student to give himself *"the means of having a proper instrument for organizing the encyclopaedia of his knowledge"* (p. 115). The notebook thus constituted a daily register and chronology of all school activity, turning it into a notebook-appointment calendar, and was also transformed into a link between school and the family: parents could follow the progress of their children. It was also a medium for the school inspectors to supervise the teacher's performance. The presence of the notebook in the school introduced a series of changes into the day-to-day activities in the classroom, whose importance equals that of broader administrative and curricular changes. This aspect was raised by Gvirtz (1999), who asserts that: *"The notebook is not a mere physical support ... On the contrary, it is a device whose articulation generates effects: in more concrete terms, the notebook constitutes, together with other elements, a shaper of the classroom"* (p. 160). According to Gvirtz, considering the changes introduced by the notebook, its use cannot be seen as a simple change in the technology used to report school activities, but as a re-organizer of life in the classroom.

Humans-with-notebooks were foreign to schools at the beginning of the 20th Century, similar to the way that humans-with-computers are absent in many educational settings even in the 21st Century. The introduction and use of computers or new information and communication technologies in educational settings have been extensively discussed and researched in mathematics education, and although many authors have reported on their benefits, we still find dichotomous positions among mathematicians and educators. For instance, Devlin (1997) refers to the transformations that new technologies have brought to mathematicians' activities: *"This rapid transformation of mode of working has changed the nature of* ***doing*** *mathematics in a fundamental* *way. Mathematics done with the aid of a computer is qualitatively different from mathematics done with paper and pencil alone. The computer does not simply 'assist' the mathematician in doing business as usual; rather, it changes the nature of what is done"* (p. 632, emphasis in original). Devlin (1997) believes that the computer can play a significant role in the mathematician's reasoning process. But many authors indicate that this trend, when associated with mathematical proof, is still highly polemic inside the mathematical community: the computer is seen as a "stranger in the nest" (Domingues, 2002) and "the pure mathematical community by and large still regards computers as invaders, despoilers of the sacred ground" (Mumford[2] -quoted in Horgan, 1993- who is critical of that position among mathematicians, p. 76). The previous references are just a sample of different positions on the use of new technologies inside the mathematical community. There is obviously fear of attributing a relevant role to computers in mathematics, and that fear is not insignificant, since we are talking about the nature of mathematics itself.[3]

Inside the field of mathematics education, research has provided evidence of the transformation that the use of computers brings to the teaching and learning of mathematics. One such transformation was the creation of environments where mathematics could be experienced as an experimental science, through tools to generate conjectures and check their validity; a mathematical laboratory where "educated trial and error" was permitted and visualization was an ally to understanding mathematics. These environments make some old practices obsolete, uninteresting and boring. For instance, making the graph of the function $y = 3x^2 - 5x + 8$ using a plotter is a mechanical exercise, but exploring the effects of parameters *a*, *b* and *c* in the graphs of equation $y = ax^2 + bx + c$ can generate many interesting conjectures, as shown in research presented in Borba & Villarreal (2005).

In spite of such research findings, there continues to be resistance to the use of technology in educational environments. For example, in my own university, where pre-service mathematics teachers are studying, the computers are used only in a "domesticated way", acting as an auxiliary to show a graph or a dynamical geometrical construction; students have no opportunities to learn mathematics with the computer because that is not part of mathematical contents.

The ubiquity of technology in our society constantly renews the concern with the use of technology in educational settings among mathematics educators. In 2006, this topic was revisited when a new ICMI Study Conference held in Hanoi, Vietnam: *Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain*. Among the 72 contributions that were presented, I could find only five abstracts related to the use of the Internet. I would like to make a short reference to the Internet as being a communication technology that has grown considerably in recent years. Particularly, the Internet has created new possibilities for distance education, and this situation constitutes a new research scenario for mathematics education. Again, in this case, the media play a fundamental role. For instance, the proposals of teacher education activities in on-line courses have fostered the development of communities that discuss issues related to mathematics and mathematics education. There is a variety of types of virtual learning environments depending on the technological resources they count on. I would like to refer to an experience with distance education carried out since 2000 by members of the Brazilian research group, called GPIMEM[4] (Borba, Malheiros & Zulato, 2007). They have developed on-line courses aimed at mathematics teachers where issues of mathematics and mathematics education are discussed. All these courses were structured with the conception that interaction, dialogue and collaboration are factors that condition the nature of learning and the quality of the distance education. The diverse technological possibilities available in the virtual learning environments

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[2] David Mumford was granted the Fields Medal in 1974 for his research in pure mathematics.

[3] The analysis regarding the history of the notebook in the school and the divergent opinions of mathematicians about the use of computers in mathematics was adapted from Borba & Villarreal (2005).

[4] Grupo de Pesquisa em Informática, outras Mídias e Educação Matemática: Technology, other media and Mathematics Education Research Group

enabled the appearance of different styles of mathematical knowledge productions. The learning possibilities in an environment where the interactions occur through the chat alone are different from those using videoconference to communicate.

Although the use of the Internet in online classes is fundamental for communication, and the attempt to prohibit it from entering the class is almost impossible, the situation is quite different in the case of face-to-face classes. Borba (2007) have questioned whether the Internet will be accepted in the classroom. According to him, *"access to Internet may not be allowed based on arguments such as: you can find answers for the problems given, students may get distracted, or it will privilege students who know how to navigate better on the Internet"* (p.4). In spite of these arguments, or maybe because of them, he argues that maybe the Internet and its overwhelming role as information supplier will end up changing what will be considered a mathematical problem in schools. The collectives of humans-with-Internet transform the very notion of problem. A problem also depends on the media available to solve it. For instance, the possibility of studying the graphs of functions by varying a set of parameters using a graphing calculator shows a different facet of mathematics when compared to making the graph of a particular function with paper and pencil.

It is not quite clear whether calculators will be accepted or not in classrooms around the world; but it is quite likely that the Internet will participate in one way or another in schools. I believe that disagreement concerning the use of the Internet or other technologies in mathematics classrooms will continue, but there is a major concern that, as educators, we cannot elude the students' right to have access to information and communication technologies as new tools in our culture and as new actors in the knowledge production. I believe it is important to examine the history of the use of technical devices throughout the history of ICMI, and also to look at the history of media in education in order to recognise the crucial role of media in the production of knowledge.

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