On home grown and borrowed theories in mathematics education research – the example of embodied cognition

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The development over time of a field of inquiry is a complex process. One critical component affecting such development is the communication with external fields more or less related to the home field. A key present concern in mathematics education is the role and use of theory (Silver & Herbst 2007, Sriraman & English 2005), where a distinction made is the one between *home grown* and *borrowed* theories, the former developed specifically to deal with mathematics education phenomena, while the latter already existed within some other field before it was "imported" for use also in mathematics education, often in rather "vague ways" (Niss 2007, p. 101). Examples of such home grown theories are Brousseau's theory of didactical situations and Chevallard's anthropological theory of didactics. The following list of "theoretical perspectives currently on offer" (Cobb 2007, p. 3) may serve as examples of borrowed theories: "radical constructivism, sociocultural theory, symbolic interactionism, distributed cognition, information-processing psychology, situated cognition, critical theory, critical race theory, and discourse theory" (ibid.). Several home grown theories in mathematics education developed on the basis of theories from other fields, e.g. APOS theory.

Among borrowed theories, embodied cognition is a recent example which has influenced mathematics education research (Mamona Downs & Downs 2002, Sriraman & English 2005). In a 'constructive response' to Lakoff and Núñez (2000), Schiralli and Sinclair (2003, p. 80) state that it "has had a significant impact on the mathematics education community. Their claims relate to, and in some cases support, many active, important areas of research". This paper will look through the window of embodied cognition (EC) as an example to discuss the question *What happens when a "new" theory from outside feeds into an existing field of study?* Critical sub-questions relate to how it happens and what happens to the field, but also to what happens to the theory. In particular, the following issues will be discussed with reference to ICMI activities and recent discussions on diversity of theories in mathematics education:

- *Dimensions of relevance* – What does the imported theory account for? This includes what problems can or cannot be studied by the theory, and what problems the theory can account for that had not been accounted for by other theoretical perspectives.
- *Implications* – What has been the influence on scientific discourse, the progression of scientific knowledge and of educational practice in the home field?
- *Compatibility* – How does the result of the application of the imported theory connect to other related results in the home field? And what about its basic assumptions?

Frameworks and theories

Bergsten (2007) makes a distinction between three general approaches used in mathematics education research, an *epistemological*, a *cognitive*, and a *social approach*. The *main* approach chosen largely governs in which domains descriptions and explanations of observed phenomena are sought. What is seen as a problem within one approach may be viewed as a symptom of another kind of problem within another approach. Instead of relying on only one overarching theory as in the case of a *theoretical framework*, a *conceptual framework* can be "based on different theories and various aspects of practitioner knowledge, depending on what the researcher can argue will be relevant and important to address about a research problem" (Lester 2005, p. 460). This relates to the idea of a *networking* strategy for dealing with the diversity of theories (Bikner-Ahsbahs & Prediger 2006). In a similar vein Cobb (2007) suggests that "rather than adhering to one theoretical perspective, we act as bricoleurs by adopting ideas from a range of theoretical resources" (p. 29). Since the development of such perspectives has been initiated by other purposes than those driving mathematics education research, they may serve as "sources of ideas that we can modify for our purposes as mathematics educators" (p. 29). Niss (2007) notes that the notion of theory is essential for mathematics education research, and often used, but a definition of the key term *theory* is seldom or never explicitly given (see pp. 98-99 for a definition). He separates the purpose of using theory form its role in research, where the latter includes providing an overarching framework, organising observations/interpretations of related phenomena into a coherent whole, terminology, and methodology. Similar points are raised in Silver and Herbst (2007), who see the different roles of theory as mediators of connections between the vertices of *the scholarship triangle*, i.e. practices, problems, and research.

Embodied cognition and mathematics

Classical cognitive science (with sub-fields developmental psychology, robotics, linguistics, philosophy of the mind) models cognition by rule-based information-processing systems seeing problem-solving from an input-output perspective, computing solutions via symbolic representations, and accounting for cognition primarily through internal cognitive processes. Criticizing this approach by pointing to its limitations due to negligence of environmental factors, this *isolationist analysis* was replaced by a *relational analysis* by the EC perspective. To understand cognition one need to consider also how cognizing subjects are embodied and how the interactions with the environment are constrained and defined by the form of this embodiment. This perspective is based on three assumptions grounded in empirical research, i.e. the primacy of goal-directed actions in real-time, the type of cognition is determined by the form of embodiment, cognition is constructive. Embodiment is here seen as "the unique way an organism's sensori-motor capacities enable it to successfully interact with its environmental niche".[1] Six claims made by EC are:

> (1) cognition is situated; (2) cognition is time-pressured; (3) we off-load cognitive work onto the environment; (4) the environment is part of the cognitive system; (5) cognition is for action; (6) offline cognition is body based. Of these, the first three and the fifth appear to be at least partially true, and their usefulness is best evaluated in terms of the range of their applicability. The fourth claim, I argue, is deeply problematic. The sixth claim has received the least attention in the literature on embodied cognition, but it may in fact be the best documented and most powerful of the six claims. (Wilson 2002, p. 265)

I would add that this sixth claim is the most crucial part of the message put forward by proponents of EC in mathematics education, where metaphorical projection is the key mechanism regulating the links between abstract thinking and embodied experience.

The main theorists behind the use of EC in mathematics and mathematics education have been G. Lakoff and R. Núñez, where Núñez has been personally engaged in ICMI-related activities such as ICME and PME conferences. They claim that their *cognitive science of mathematics* "has important things to say about the nature of mathematics, which in turn can have a positive impact in the enterprise of mathematics education" (Núñez, in press, p. 130). The human and mind-based mathematics that comes out of this combined cognitive / epistemological approach, is structured by our everyday functioning with our environment. This goes for mathematical concepts and thought processes, the abstract concepts conceived in concrete terms by using precise modes of reasoning based on bodily experience. Núñez (2000) claims that conceptual metaphor "constitute the very fabric of mathematics", present in all its subfields, and that "an extremely important feature of image schemas is that their *inferential structure* is *preserved* under metaphorical mappings". Having both perceptual and conceptual aspects, image schemas also link discursive reasoning to visualisation. The method of study used is called *mathematical idea analysis* (Lakoff and Núñez 2000), interpretations of conceptualisations of mathematical ideas within individual thinkers as bodily based by way of metaphorical projection, seen as unconscious, non-arbitrary and culturally shared. Some consequences for education of this approach are listed in Núñez (2000). I will here limit the perspective to the mathematics education community, with a focus on ICMI-related activities, not relating to cognitive science.

Embodied cognition in mathematics education

Research in mathematics education has from its early stages had a close tie to psychology. Influence from cognitive psychology has been dominated by Piaget-based research, cognitive science with an information-processing approach, and sociocultural perspectives. The influence on mathematics education from the emerging field of EC, as grown out from cognitive science, can historically be traced back to two 'waves'.

The *first wave* built on the EC constructs *image schemata* and *conceptual metaphors*, based mainly on Lakoff and Johnson (1980), Johnson (1987), and Lakoff (1987). Plenary presentations at PME14 (Davis 1990) and PME15 (Dörfler 1991) implemented a discussion about the relevance for mathematics education of this new interpretation of the relations between bodily based experiences and the mind, where especially the 1991 plenary caused an intense debate on site about non-mentioned connections to Piaget. The key role of metaphors and image schemata for visualisation were further discussed by Presmeg (1992), who the same year also made the work of M. Johnson visible to the mathematics education community with her review in ESM of *The body in the mind*. A focus on metaphor is found in many papers in the 1990s. The construct of image schemata in relation to mathematical formulas was discussed by Dörfler (1991), and also at PME16 by Bergsten (1992; see also 1999). A Discussion Group at PME19 in 1995 on *Embodied cognition and the psychology of mathematics education*, organised by R. Núñez and L. Edwards, continued at PME20. The background text invited to a discussion on this "new paradigm in cognitive science", referring in particular to the work of Lakoff and Johnson, with the overall purpose "to begin to build a bridge from mathematics education to these new paradigms". At PME22 it continued as a Working Group and at PME24 as a Discussion Group, renamed to *Theory of embodied cognition*, indicating a shift of focus. Since 2001 a Working Session on embodiment in mathematics focused on gesture and metaphor has been running at PME.

The second wave built on *mathematical idea analysis*, based mainly on Lakoff and Nuñez (1997, 2000), including constructs such as conceptual metaphor further developed for mathematics (grounding, linking, and definitional), metonymy, conceptual blend, and image schema. Even if the method of mathematical idea analysis was presented to

[1] This description is based on www.iep.utm.edu/e/embodcog.htm [available 2007-07-21]

the mathematics education community both in Lakoff and Núñez (1997) and in Núñez, Edwards, and Matos (1999), it was the launch of the book *Where mathematics comes from* (Lakoff and Núñez 2000) that made the perspective well known and much debated, both in the mathematics and the mathematics education communities (e.g. Presmeg 2002, Schiralli & Sinclair 2003). This "second wave" of the EC perspective on mathematics was broadly introduced to the ICMI community at the plenary by Núñez (2000) at PME24, followed up by his regular lecture at ICME10 in 2004, and has enhanced an increased interest in the role of gesture as one key component in the complex coordination of different semiotic resources used in students' construction of mathematical knowledge. In the European community a working group on imagery and metaphors (including EC) has been active at the CERME conferences since 2001.

Areas in mathematics education where theoretical tools borrowed from EC have been used in research studies include mathematical symbolism, visualisation, modelling with and without technology, inequalities, linear equations, graphing functions, limits and continuity, vectors, mathematical thinking, mathematical reasoning, understanding and cooperative work, understanding numbers, transformational geometry, fraction talk, metaphorical language in geometry, gesture and other semiotic means of knowledge construction.

Discussion

Of the roles played by theory as mediator between *problems and research*, as described by Silver and Herbst (2007, p. 50), most can be observed in mathematics education studies referring to EC, in particular "as a means to analyze data and produce results" and "as the means to transform a commonsensical problem into a researchable problem" (ibid.). Of the categories concerning *research–practice*, i.e. prescription, understanding, prediction, and generalization, arguing with Cobb (2007, p. 28) the focus of individual cognition makes EC useful along the three first categories mainly at specific task levels, while its value is limited at classroom level. Mediating between *problems and practice*, EC can help identify some possible aspects of what is problematic in a practice, though limited to the level of conceptualizations of mathematical ideas, as well as propose solutions to such problems for specific concepts.

Dimensions of relevance

Theories are used in research at different levels, which can be related to the kind of research framework chosen. In mathematics education elements of EC theory often form parts of conceptual frameworks, using specific theoretical constructs such as conceptual metaphors or image schemas. These often play the role of providing *explanatory power*.

> When you explain some mathematical relations in R^n, say, you almost always draw pictures or use images from 'similar' ideas in R^2 or R^3. You talk about the *volume* of the unit *ball* in R^n, and about the *angles* between two vectors in R^n. This way of using metaphorical projection from more familiar objects to 'new' abstract objects is a way (the only?) to give a sense of meaning to the abstract objects, and is ubiquitous in mathematics. This can be explained by the human way of using (mental) image schemata, originated in the experienced world, as the structural basis of abstract thinking and construction of subjective meaning (Lakoff, 1987, Johnson, 1987), and suggests that imagery could facilitate understanding. (Bergsten, 1993, p. 133)

This is an example of how an increased awareness of the language used may reveal metaphors implicitly in play in mathematical reasoning. These kinds of phenomena, researchable in the EC approach, occur in classroom discourse. It also relates to how the gap between intuitive and formal mathematics can be investigated and understood (Núñez et al., 1999). However, effects of didactic phenomena at different institutional levels, due to didactic transpositions and organisations, may give alternative accounts for problems as explained by this cognitive approach (Bosch et al. 2006).

The world of conceptual metaphors in mathematics, as outlined in *Where mathematics comes from*, is described as *the* grounding and linking metaphors in play, needing to be exploited in teaching. In her review of the book, Presmeg (2002) raised the critical issue of the power also of idiosyncratic metaphors for an individual's mathematical reasoning, not accounted for in the educational implications of the theory. A related point was made by Schiralli and Sinclair (2003), who based on a distinction between concepts and conceptions find the method of mathematical idea analysis "insufficient to adequately describe the nature of mathematical concepts; for this, further empirical research is necessary, especially research that can probe the very idiosyncratic nature of students' individual conceptions" (p. 80). They also argue for the need to also take into account interactions between the a-cultural cognitive predispositions with "various cultural and symbolic variables to produce mathematical cognitive structures" (p. 90). To this end, Radford (2003) takes a semiotic-cultural approach to embodied experience rather than a physical-biological.

Regarding the range of the EC perspective for mathematics, one issue discussed is the 'distance' between formal mathematics and the grounding metaphors for basic mathematical ideas. Tall (2001) points to limitations inherited in the EC model as regards "the abstract thoughts of axiomatic thinking" (p. 206). Metaphors in language express linguistically the embodiment of abstract thinking in mathematics, and "the intimate link between oral and gestural production" (Núñez, in press, p. 148) has also increased the interest in gesture in relation to mathematical knowledge. This has linked EC to semiotic perspectives in mathematics education, with a renewed analysis of the role of embodiment in meaning production. In Radford (2005) thinking is outlined as mediated by *and* located in "signs, artifacts, and body" and "the object that the body encounters is more than a mere thing: it is a cultural object" (p. 119). The embodiment of the mind is thus only one aspect of a learning game which cannot be fully comprehended without accounting also for the cultural embodiment of the object of learning.

Implications

The notion of embodiment has today a paradigmatic stance in research on thinking and learning, also supporting an increased interest in *multimodality* in mathematics education as a basis for theory development (e.g. Arzarello & Robutti 2007). The influence from the 'first wave' of EC into mathematics education seems to have been more significant in this respect than the 'second wave'.

One explicit claim by EC in mathematics education is that it offers a new understanding of 'misconceptions', since the method of analysis shows that "there are no wrong conceptions as such, but rather variations of ideas and conceptual systems with different inferential structures" (Núñez 2000). A remediate instructional design for "inducing students to operate with the appropriate conceptual mappings" (ibid.) still needs to be conceived, as well as how to help students to "become aware of the organization, limitations, and potentials of their own conceptual systems, making explicit (and conscious) what in everyday life is implicit (and unconscious)" (ibid.), and its learning pay off. Other educational 'implications' of EC expressed in Núñez (2000), such as the emphasis on the human nature of mathematics and the need to teach it along with its cultural history and struggles for meaning, seem to require a deeper cultural analysis and a widening of the embodiment paradigm beyond the focus on physical-biological real-time interactions with the environment that are claimed to structure the basis for conceptual metaphors to become operative. How can the two classic dichotomies, body–mind and mind–culture, if merged by EC and sociocultural theory, respectively, meet in a merged triangular unit of analysis with vertices body–mind–culture?

One impact of EC theory has been to provide a new perspective on discussions about epistemological foundations of mathematics, and its relevance for education. A questioning of the 'scholar' view of mathematics within education plays a key role in anthropological theory of didactics, and is also a starting point for the 'cognitive science of mathematics' (Núñez, in press, p. 135), where it has, however, replaced the "popular" scholar view with an alternative view which is not being further questioned. Problems to include formal mathematics in this 'mind-based mathematics' have been discussed by Tall (2001, p. 206), who concludes that "their theoretical position is better at describing 'where mathematics comes from' rather than 'where mathematics goes to'." But it seems to have influenced the analysis of formal mathematical thinking (as in Tall 2001, p. 228).

Compatibility

The diversity of theories within the research field of mathematics education can be seen as a strength rather than as a problem (Bikner-Ahsbahs & Prediger 2006, Lerman 2006). The role of imported theories in mathematics education relates to its *horizontal knowledge structure*. According to Lerman and Tsatsaroni (2004, p. 4) "we view new theories as, in general, positioned alongside other theories and not replacing them, as one might expect to happen in the development of theories in science". In line with this Lerman (2006, p. 9) further states that "mathematics education knowledge will grow both within discourses and by the insertion of new discourses in parallel with existing ones". This also leads to the need of making the relations to other discourses in the field explicit, an example of which is Núñez et al. (1999) where EC is described as compatible with a sociocultural approach and an attempt to "network" the two perspectives for use in mathematics education is presented. Another remark refers to the dominant borrowing of EC perspectives and theoretical tools as separate components within conceptual frameworks, while seldom explicating the basic assumptions, issues of compatibility are not discussed. In mathematics education research studies EC has thus been more applied than discussed or questioned (with some of the exceptions mentioned above). One could also argue that the normative language sometimes used, and the missionary task seemingly taken on (as in Núñez, in press, p. 153) do not invite to a discussion of compatibility with other perspectives or not.

Concluding remarks

The 'mind-based mathematics', as presented in Lakoff and Núñez (2000), may be seen as primarily a home grown theory within the field of philosophy of mathematics based on an imported theory from another filed (i.e. EC), rather than primarily a mathematics education theory. It could thus, following Niss (2007, p. 107), be seen as a sub-theory of an imagined fully comprehensive theory of mathematics education. The role of this home grown sub-theory for mathematics education would then mainly be, from the discussion above, to provide a terminology (conceptual metaphor, etc.) and a method (mathematical idea analysis). Since this sub-theory is based on an imported theory from another field (EC), not developed to account for the specific phenomena of mathematics and mathematics education, its scope and explanatory specificity will by necessity be limited, and an induced process of recontextualisation opening up for influences from ideological preferences and community related interpretations is likely to follow its epistemic and pragmatic development (Lerman, 2000). By the nature of the home field, this sub-theory will add to the stream of parallel sub-theoretical perspectives, each contributing to model different aspects of its complexity. Elements of these phenomena have been observed in the case studied here.

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