

The Development of Mathematics Education as a Scientific Discipline – Some Reflections from a German Perspective

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Introduction

The introduction of the book “Didactics of Mathematics as a Scientific Discipline” edited by Rolf Biehler, Roland Scholz, Rudolf Strässer and Bernhard Winkelmann in 1994 starts with the remark “Since the work of the International Commission for Mathematics Instruction (ICMI) at the beginning of this century, nobody can challenge the fact that scientific work has been done in the field of teaching and learning mathematics” (p.1). The book aims at providing a state-of-the-art picture, it structures the field of mathematics education and attempts to clarify its relationships to other disciplines. The origin of the book was related to two birthdays, the 65th birthday of Hans-Georg Steiner, one of the founding directors of the IDM, the Institute for Didactics of Mathematics at Bielefeld University, and the 20th birthday of the IDM itself. The prominent relationship to ICMI is no accident. The IDM saw itself as active part of the emerging international community where ICMI played an important part and where Hans-Georg Steiner held several positions and functions in the ICMI committees. In a recent special issue of the ZDM, a part of this development was reflected on (Biehler & Peter-Koop, 2007a,b).

Hans-Georg Steiner convincingly argued for an interdisciplinary and transdisciplinary approach to mathematics education / didactics of mathematics integrating

- the philosophy and history of mathematics,
- subject matter specific didactical analyses,
- curriculum research,
- research from cognitive psychology,
- and theories of teaching and learning.

His creation of the TME (Theories of Mathematics Education) group and programme intended to support this development (e.g. see Steiner 1987).

In our contribution to the symposium to celebrate the centennial of ICMI, we will provide three historical snapshots, where developments in Germany were closely related to international developments concerning ICMI and the constitution of didactics of mathematics as a scientific discipline. We will start with looking at Felix Klein’s contribution, turn to the early ICME congresses and highlight the role of the IDM in Bielefeld.

Looking Back 100 Years: International Reforms in Mathematics Education

In a recent article in the newsletter of the German Mathematical Union (DMV), Gert Schubring (2007) pointed out a unique feature of mathematics as a school subject: Only in mathematics major *international* reforms with respect to instruction took place: the first one between 1908 and 1914 related to the ICM in Rome and the second one – a more or less worldwide movement – was closely related to the “new math reform”. While we do not want to go into detail regarding the new math reform and its broad international discussion and curricular implications, it seems to be important to acknowledge the leading role of Felix Klein¹ with respect to early national as well as international reform movements in mathematics instruction in general and within ICMI activities in particular.

¹ Felix Klein (1849-1925) – a German mathematician from the University of Göttingen widely acknowledged for his research in the field of geometry in the 19th century as well as his contributions to mathematical applications and to mathematics education.

One of his major concerns was the perceived gap between school mathematics and university mathematics which from the 1890s on lead him to engage in the improvement of mathematics education. Klein's program for the restructuring of the relationship between secondary and tertiary mathematics dated May 1900 is considered a key document for mathematics education reform (for details see Schubring 1989) and involved the introduction of analytical geometry and calculus to the mathematics curricula of secondary school. Klein's (1902-1908, 2004) "Elementary mathematics from an advanced standpoint" is legendary. The Meran reform included the fundamental idea of "educating functional thinking", where (kinematic-)functional thinking was aimed at relating cognitive processes, mathematical concepts and big ideas for general mathematics education. Although the Meran reform in this respect was not very successful (Krüger 2000), thinking with functions has become a central element in many current curricular reforms.

In 1908 at the ICM in Rome, Klein was not only elected as one of three members of a "Comité Central"² but also chosen to be its president. In this role he developed comprehensive and dynamical as well as organisational and conceptual activities in his strive to improve mathematics education at an international level. His merits include the invitation of more and more countries from all five continents and the integration of all school types from secondary schools to general as well as vocational schools. In order to foster reform in mathematics education, he initiated a number of international surveys on the key problems of mathematics education (see Schubring 2007), which were published and discussed in detail at the international mathematics conferences. With respect to the numerous publications as well as the actual change in mathematics education, the effects of the committee and the related international reform movement had been enormous (ibid). Hence, the fundamental importance of Felix Klein for mathematics education at an international level was formally acknowledged almost one hundred years later in 2002, when ICMI introduced the *Felix Klein Medal* and awarded Guy Brousseau as first medallist in 2004.

ICME 3 in Karlsruhe 1976: Didactics of Mathematics as a Scientific Discipline

After World War II, the International Commission of Mathematics Instruction (ICMI) reconstituted itself as part of the International Mathematics Union (IMU) in 1951 and held his first international conference – ICME 1 – in Lyon 1969 followed by ICME 2 in Exeter 1972 and ICME 3 in Karlsruhe in 1976. In retrospective, the 1976 conference turned out to be an important event with respect to the continued emergence of didactics of mathematics as a scientific discipline. Hans-Georg Steiner, a member of the German ICMI subcommittee since 1962, chaired the International Programme Committee (IPC) and had a most significant influence on the scientific structure of the congress.

ICMI had played an important role in the prehistory of this congress. The German mathematician Heinrich Behnke became a member of ICMI's international executive committee, was its president from 1954 to 1958 and served as president of the German ICMI subcommittee from 1966 to 1971. Hans-Georg Steiner, then a research assistant and collaborator of Heinrich Behnke, became involved in international activities very early.

It is important to note that the GDM, the German Association for Didactics of Mathematics, was founded not earlier than 1975 and that the German ICMI subcommittee had an important institutional function in the emerging discipline of didactics of mathematics in Germany. However, the foundation of a separate professional organization for Didactics of Mathematics in Germany and the founding of the IDM in Bielefeld in 1973 as a central research institute are milestones in constituting an interdisciplinary discipline of didactics of mathematics as a science separate from mathematics. These organizational developments were not free of tensions with regard to the mathematicians and their organizations, to say

² The international committee was originally implemented for only four years and then had been extended for another four years until 1916. However, World War I interrupted its activities and in 1920 Klein had to accept its dissolution, while the German chapter, called IMUK (i.e. Internationale Mathematische Unterrichtskommission) remained active.

the least. Separate institutions were most important for constituting a new scientific discipline. Whereas today ICMI covers all school types and levels, originally there were some biases. The new institutional developments in Germany also included people as well as research and development traditions from other sources, namely from teacher training colleges and from universities of education responsible for training teachers who are not teaching at the German *Gymnasium* (grammar school) but at primary and lower secondary schools. In this respect, it is very remarkable that in 2007 the annual conferences of the German Mathematical Union (DMV) and the German Association of Didactics of Mathematics (GDM) were held together in Berlin as one overall conference for the first time which also signals a new step in the collaboration of two scientific communities sharing common interests.

Let us switch back to ICME 3 in Karlsruhe in 1976. A major outcome of the conference was the book "New Trends in Mathematics Education" (Christiansen & Steiner 1979) which was published and disseminated by UNESCO. Other than traditional conference based volumes with many single but rather unrelated papers, the book edited by Christiansen and Steiner contains 13 chapters with survey reports attempting to summarize international developments and to identify major trends in mathematics education. These chapters were based on group work that had started one year prior to the 1976 congress.

The Institute for Didactics of Mathematics (IDM) at Bielefeld University

In the following years the collaboration with UNESCO continued on the basis of the shared goal of the global development of mathematics education as a practice as well as a scientific discipline. In particular Steiner's analysis of the failure of the new math reform in the 1960s led him to two major consequences: (1) recognising a need for more fundamental research and (2) a recognition of mathematics education from a systems perspective including the school system, the teacher education system and didactics of mathematics at university level. Steiner and his colleagues (at the IDM and beyond) became convinced that any curriculum development had to be based on fundamental research, addressing questions such as

- What is mathematics, what is mathematical thinking?
- What is the role of mathematics in other sciences or in society, for people in everyday life and at their workplace?
- What were determinants of the development of mathematical curricula in the past?
- What are conditions for innovation in educational systems?
- What knowledge of mathematics and of didactics of mathematics do teachers need?
- How can mathematics education contribute to the general goals that were set up for education at upper secondary level?
- How can goals for mathematics education be justified?"

These fundamental questions were related to the *Zeitgeist* of those years and of the IDM in particular: People felt the need of a deeper research base for supporting a scientification of the practice of curriculum design, development, implementation and evaluation.

The idea of the 1994 book on "Didactics of Mathematics as a Scientific Discipline" was to structure the field and join authors from the IDM and its national and international collaborators. It was not aimed as a comprehensive handbook but as some kind of state-of-the-art picture embedded in IDM's international relations.

The volume has the following chapters:

1. Preparing mathematics for students
2. Teacher education and research on teaching
3. Interaction in the classroom
4. Technology and mathematics education
5. Psychology of mathematical thinking
6. Differential didactics

7. History and epistemology of mathematics and mathematics education
8. Cultural framing of teaching and learning mathematics

Let us comment on some of the topics. Preparing mathematics for students (chap. 1) includes the reflection on eclectic approaches to curriculum development, the French approach to didactical engineering and the question of underlying goals of mathematics education and their cultural justification, an aspect which is related to reflecting on cultural framing (chap. 8). The aspects of goals and justifications of mathematics at school is further developed by Heymann (1996, 2003) a former researcher at the IDM. Teacher education and research on teacher development was a very early focus of the work at the IDM. Michael Otte, the second founding director of the IDM, and his working group were focussing on this aspect. Related to this was his co-founding of the BaCoMET group in 1980 (Basic components of mathematics education for teachers)³. Interaction in the classroom (chap. 3) was the focus of the group of the third founding director of the IDM, Heinrich Bauersfeld. Qualitative-interpretative studies of classroom interactions drawing on social science theories and methodologies were in the focus and co-shaping the emergence of various constructivist and socio-cultural approaches in mathematics education. In the 1980s, this approach started predominantly analytical and not yet very specific with regard to mathematics but took up more and more specific aspects and influenced the design of mathematics teaching and learning (Bauersfeld & Cobb 1995). A further reconciliation, extension or shifted focus in relation to epistemological aspects in mathematics education (chap. 7) was already emerging and later on led to approaches such as that of Heinz Steinbring (2005), a former researcher at the IDM in the group of Michael Otte.

History and epistemology of mathematics was a big strand in IDM's work, originally starting from Rene Thom's (1973) critique of the new math reform: „In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics“ (p. 204). This was reflected in the volume as well as in various later books and activities (e.g. Jahnke, Knoche & Otte 1996). In analyzing the tools of thinking and communicating in mathematics, the problem of representations was one of the important aspects of epistemological research at IDM, starting with an early reception of social-cultural approaches (Vygotsky) and lead to an increased interest in semiotic approaches, e.g. see the volume edited by Hoffmann, Lenhard and Seeger (2005).

After the retirement of all three founding directors, the IDM was re-founded as a more usual research group in mathematics education comparable to the growing number of research groups in mathematics education at other German universities. PISA and TIMSS have currently stimulated certain types of research in mathematics education in Germany, either closely related or in contrast and distance to these studies. We see the didactics of mathematics as a dynamically changing network of growing and abating research groups where various theoretical frameworks and methodologies are emerging.

Looking Ahead: Theoretical Frameworks in Mathematics Education

While the international mathematics education community is looking forward to ICME 11 to be held in Monterrey, Mexico in July 2008, it is very obvious that our discipline has grown much further. The First Announcement of ICME 11 identifies a total of 38 different Topic Study Groups showing a complex picture of the discipline and the growing international participation in ICMI activities with TSGs focussing on

- new developments and trends in mathematics education from preschool to tertiary level,
- research and development in the teaching of various content areas from arithmetic to discrete mathematics as well as calculus,
- various dimensions of curriculum research,

³ http://www.mathematik.uni-kassel.de/didaktik/HomePersonal/biehler/home/abgeschlossene_projekte/bacometh.htm

- teacher pre-service and in-service education,
- reasoning, proof and proving / problem solving / visualisation / mathematical applications and modelling / new technologies / assessment / gender / language and communication / students' motivation and affective factors,
- new trends in mathematics education research as well as the history of teaching and learning mathematics.

This plurality of topics and research areas more than ever before draws our attention to the need of developing theoretical frameworks that guide, structure and inspire current and future research in our field.

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