Using multiple theoretical perspectives to connect, clarify and convey research results

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Introduction

Documented teaching qualification becomes increasingly important for academic staff. Besides teaching experience, courses on teaching and learning are required to get an appointment as a lecturer or a promotion. PhD-students and newly appointed lecturers therefore demand opportunities to qualify as university teacher. One possibility to meet this demand at large mathematics departments is to design specific courses in teaching and learning mathematics at university level. During the process of developing and delivering such a course for two Swedish universities, it became clear, that the existence of many competing theories in the areas of mathematics education and in general education tend to obstruct the learning outcomes of the course. One reason is that almost all participants in the course have their background solely from mathematics and science and are unacquainted with the use of theories in social sciences.

In this paper I will briefly describe the course in question and its relative success as well as shortcomings. The difficulties for the students to appreciate and learn about competing theories and theoretical perspectives are discussed. The pedagogical question arises if there are any opportunities to reduce the complexity by connecting different theoretical perspectives. I claim that such possibilities exist and will show by examples the strength of using a connecting strategy. I also argue that such connections – when they exist – will deepen and enhance our interpretation of the research results and therefore are in the interest of the researchers themselves. The main part of the paper is devoted to a couple of examples from the mathematics education literature, both addressing learning obstacles and dealing with tertiary level mathematics (Vinner, 1997; Lithner, 2003). I will identify connections between the analysis presented by Vinner and Lithner and more general theories of learning in higher education. Finally I will discuss these findings in the light of the ongoing discussion about multiple theories in mathematics education research and specifically to the idea of a networking strategy (Bikner-Ahsbahs & Prediger, 2006).

A course in teaching and learning mathematics in higher education

The course in question was a graduate courses offered to doctoral students and young lecturers, first at the Centre for Mathematical Sciences, Lund University in 2005/06 and later at the Department of Mathematics, Stockholm University and KTH Mathematics, Royal Institute of Technology in cooperation in 2006/2007. The latter course was a development of the first version[1]. About 15-20 students participated in each course. The majority of the students in the course were PhD-students, while a minority were newly appointed lecturers. The participants represented several subjects and divisions in their departments: mathematics, mathematical statistics and numerical analysis. The aim of the course was for students to learn to describe and evaluate students' learning on scientific and empirical grounds, and to develop their ability to plan, teach, supervise, assess and examine courses in mathematics at undergraduate level. An orientation of research in mathematics education with emphasis on higher education was included in the course. Theory and praxis were integrated in the course. The workload corresponded to five weeks full time study while the course was stretched out during seven months.

John Mason's book on teaching and learning mathematics at university level (Mason, 2002) was chosen to be the main literature. This book focuses on practical aspects and is partly inspired by investigations among mathematics lecturers. Chapters from other books, scientific articles and papers supplemented the main book. Lectures, seminars, classroom observations, workshops and teaching

[1] The following persons are responsible together with the author, for the course given in Lund, Lena Zetterqvist, Lund university, and June Morita, University of Washington and, for the course in Stockholm, Torbjörn Tambour, Stockholm University, Hans Thunberg and Lars Filipsson, KTH Mathematics, Royal Institute of Technology. The development of the course was mainly the work of the author; June Morita and Lena Zetterqvist.

exercises were used. All students wrote an essay addressing a mathematics education question of their own choice, based either on literature or on data from a small empirical investigation.

The result of the course was positive in several aspects. The interest to take the course was high and very few students left the course, in these cases due to heavy workload from thesis work or teaching. The practical parts and the essay writing were judged by many to be the most interesting and rewarding parts of the course. On the other hand several students had wished to involve teaching practice more. During an evaluation discussion some participants suggested that feedback upon their own teaching under normal conditions would be a valuable addition to other parts of the course.

Some criticism against the more theoretical parts of the course was put forward during the course. One argument was repeated a couple of times in the evaluations. The argument was that the course was not intended for future researchers in the area but for future mathematics lecturers. Hence, there was no need to dig dip into the theories of the field. Some texts arouse frustration because they were actually discussing the empirical grounds for the growth of theories or presented the development and progress within theoretical paradigms such as social constructivism. Those articles were found hard to absorb. Reactions like these are not unknown for mathematics educators working with teacher students who prefer more practical recipes and are less interested in reflecting on how to establish evidence for knowledge in the field.

However, the criticism must be taken seriously. One issue to address was obviously to put more emphasis on coherence. I started to reflect on what possibilities there are to help students connect different theoretical perspectives. Ideally they should appreciate that acquaintance with various theoretical perspectives could help them to better understand complex teaching situations.

Problem solving is a central theme in mathematics education treated by many authors from different perspectives and using different theories. Two examples of theoretical frameworks created by researchers in mathematics education in order to understand students' problem solving will be presented and discussed in the following.

Pseudo-conceptual and pseudo-analytical thinking

Shlomo Vinner describes learning or problem-solving situations in mathematics that are not truely such from the student's perspective even if they are considered as true learning situations by the teacher (Vinner, 1997). A student may not be involved, or may not want to acquire knowledge about the mathematical reality even if she/he feel forced to learn something. However, the student does not show her/his disintrest but tries to please the system (and the teacher) by giving answers that are not based on reflection or argument but are found by other procedures. Vinner argues that the strategies applied by the student may be better understood if one takes other aspects than cognitive in consideration. Based on the analysis of such situations Vinner identifies and describes *pesudo-conceptual* behaviour as opposed to *conceptual* behavior and *pseudo-analytical* behavior as opposed to *analytical* behavior. Vinner's examples in the article are drawn from all levels, some from higher education.

Vinner describes conceptual behavior and thought processes with reference to relational understanding (Skemp, 1976), meaningful learning (Ausubel, 1968) and conceptul knowledge (Hiebert, 1986).

> Conceptual behavior is based on meaningful learning and conceptual understanding. It is the result of thought processes in which concepts were considered, as well as relations between concepts, ideas in which the concepts are involved, logical connections, and so on. (Vinner, 1997, p 100)

Pseudo-conceptual behavior on the other hand is described as

> …a behavior which might look like conceptual behavior, but which in fact is produced by mental processes which do not characterize conceptual behavior.
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> ….
>
> In mental processe that produce conceptual behaviors, words are associated with ideas, whereas in mental processes that produce pseudo-conceptual behaviors, words are associated with words; ideas are not involved.
>
> …
>
> A dominant feature of the pseudo-conceptual thought processes is the uncontrolled associations which fail to become a meaningful framework for further thought processes. (op cit p 101, 103)

Pseudo-analytical behavior is defined through examples of students' inadequate behavior in problem solving situations. The behavior differs from analytical behavior by the lack of control, by identifying similarities and by using imitation of procedures that are not suitable in the context.

> The person is responding to his or her spontaneous associations without a conscious attempt to examine them. (op cit p 114)

Vinner argues that students may refrain from pseuo-conceptual and pseudo-analytical thought processes if teachers encourage them to ”construct awareness” and get ”cognitively involved” (op cit p 126).

Reasoning types

Johan Lithner (2003) has studied three first year university students with the aim of identifying their problem solving strategies. A set-up problem solving session during which the student is asked to think aloud was documented by video recording. By analysing the transcripts Lithner can discriminate between three types of reasoning: plausible reasoning (PR), reasoning based on established experience (EE) and reasoning based on identifications of similarities (IS). PR is founded on intrinsic mathematical properties of the mathematical components involved while EE is based on experiences from earlier learning situations not related to intrinsic mathematical properties. The student in both cases aims for a solution that ”…probably is the truth, without necessarily having to be complete or correct” (Lithner, 2003, p 33). The third strategy is based on similar surface properties in a known example or rule and implemented by mimicking a procedure. The main finding is that students use a mix of strategies and that the IS and EE dominate strongly over the PR strategy.

In the final discussion Lithner addresses the question of what the students may learn by using the strategy chosen and concludes that

> There are no signs that they attempt to learn general ideas by considering global properties. It seems that they aim at learning or memorising how to do particular, limited, exercise types, instead of learning general ideas. (op cit p 53)

General courses in teaching and learning in higher education

Most of the students taking the course described in the first part of this paper did not study education as a subject during their undergraduate studies. Only very few had gone through teacher education. However, many had already studied one, two or several short courses about university teaching and learning during their time as doctoral students. All Swedish universities offer such courses run by special units for development of teaching and learning at the university. PhD students often get a possibility to enter a first course before starting to teach. During the doctoral studies a PhD-student may choose to take other pedagogical courses. The courses offered are relatively short and in total correspond to five – ten weeks' full time studies.

Theories on learning and teaching presented in these general courses on education differ in various places and courses. However, among specialists in higher education at Lund and Stockholm universities, as well as several other Swedish universities, the ideas of Paul Ramsden, John Biggs and Ference Marton are most influential. Especially the SOLO taxonomy (Biggs & Collis, 1982) has become widespread during the Bologna process these last years as a support to redesign all central course documents with emphasis on learning outcomes according to the Bologna model.

SOLO-taxonomy

John Biggs and K.F. Collis introduced the SOLO-taxonomy (SOLO is an acronym for Structure of the Observed Learning Outcome) in 1982 (Biggs & Collis, 1982). It provides a model for describing the levels of complexity of students' answers to tasks. The five categories (pre-structural, uni-structural, multi-structural, relational and extended abstract) are content-free and are assumed to apply to all subjects. The SOLO-taxonomy may also be used for curriculum design and assessment (Biggs, 2003). It has been applied by a number of researchers to various educational stages and subjects. The use of the SOLO-taxonomy in higher education is described by Ramsden (1992), Boulton-Lewis (1998) and Biggs (2003).

Few recent applications of the SOLO-taxonomy to mathematics at tertiary level have been published. Helen Chick studies research mathematics according to the SOLO classification in view of a Piagetian model of cognitive progression (Chick, 1998). Otherwise the most advanced mathematics topics addressed seem to be functions (Coady & Pegg, 1994), polynomials (Chick, 1988), and statistics (Groth & Bergner, 2006).

Deep and surface approach

The idea of two *approaches to learning* in higher education – the *deep* approach and and the *surface* approach – has proved to be seminal in the research of higher education. These concepts grew out of research studies during the 70-ties and rely on an original study by Ference Marton and Roger Säljö of students' reactions to reading academic texts as a learning task (Marton & Säljö 1976). The concepts have been developed since then and applied to all types of tasks (Marton et al, 1984). The two approaches can not be used to characterise an individual, but a student's work in a specific learning situation. How does the student organise the learning task and what is the focus of the student's attention? Ramsden (1992) presents the two approaches in the following way:

> **Deep approach**
> *Intention to understand. Student maintains structure of task.*
> Focus on 'what is signified' (e.g. the author's argument, or the concept applicable to solve the problem).
> Relate previous knowledge to new knowledge.
> Relate knowledge from different courses.
> Relate theroretical ideas to everyday experience.
> Relate and distinguish evidence and argument.
> Organise and structure content in a coherent whole.
> *Internal* emphasis: 'A window through which aspects of reality become visible, and more intelligble' (Entwhistle & Marton, 1984)
> **Surface approach**
> *Intention only to complete task requirments. Student distorts structure of task.*
> Focus on the 'signs' (e.g. the words and sentences of the text, or unthinkingly on the formula needed to solve the problem).
> Focus on unrelated parts of the task.
> Memorise information for assessments.
> Associate facts and concept unreflectively.
> Fail to distinguish principles from examples.
> Treat the task as an external imposition.
> *External* emphasis: demands of assessments, knowledge cut off from everyday reality.
> (Ramsden, 1992/2002, p 46)

It is important to observe that what consitutes a deep approach is depending on the discipline and the traditions of designing tasks for students within that discipline. Ramsden (1992) summarises science students' descriptions of a deep or a surface approach from interview studies. An initial concentration on details and logical connections and a later move to generalities may be part of a deep approach to learning while a surface approach is often characterised by a narrow focus on techniques, procedures and formulas. However, these characteristic are not applicable to humanities.

According to Ramsden (1992) there is a robust relation between approaches to learning and outcomes of learning based on evidence from many studies. A deep approach is often what teachers in higher education describe that they wish for their students. The approach is a function of the student's earlier experiences of learning and the learning context in question. Teachers ought to take this into consideration when designing tasks and learning environment in general if they wish to improve outcomes.

> Good teaching implies engaging students in ways that are approriate to the development of deep approaches. (Ramsden 1992/2002, p 61)

Surface approach to learning, pseudo-analytical thougt processes and superficial reasoning

From the description above it is clear enough that the empirical data from the studies by Vinner (1997) and Lithner (2003) or similar studies may be analysed from a perspective of a deep versus surface approach to learning. Already in formulations found in the articles and cited above analysis along the lines of deep/surface approach to learning seems to be a possibility. Such an analysis would enrich our understanding of what it is in the situations that could provoke a surface approach from the student in that context.

One example may illustrate the possibilities. One of the students in Lithner's study apparently has access to a complete set of solutions (a solutions manual intended for instructors). A common experience is that access to such a manual among some students promotes a special working style that is heavily based on the manual, easily promoting a surface approach as long as the manual is at hands. So research questions to pose from that perspective is the following. Do the students use the manual in different ways? Is there a connection between their problem solving strategies and their style of making use of the manual?

Multiple perspectives

In a discussion about the question of how to deal with diversity and richness of theories in mathematics education Angelika Bikner-Ahsbhs and Susanne Prediger (2006) elaborate on four ideas: unifying, integrating, competing and comparing and finally networking. They argue that networking is perhaps the most promising strategy.

> The main idea of networking is to exploit the diversity of approaches constructively by first analyzing the same phenomena from complementary perspectives. This allows comparing and competing as well as integrating local theories. The complementary or integrative understanding of a phenomenon generates a deeper or more comprising understanding of the phenomenon… (Bikner-Ahsbahs & Prediger, 2006, p 56)

I argue that research on problem solving in mathematics at tertiary level may benefit from an additional perspective of deep/surface approach to learning. Adding this theory could turn out to become a networking process as described in the citation.

Furthermore a connection to the widespread literature on learning approaches in higher education would make research in mathematics education more accessible for students with a background similar to the participants in the course described in this paper, i.e. mathematics PhD students and young lecturers in mathematics with no special interest in mathematics education research in advance but some knowledge about general theories of students' learning in higher education.

Likewise more frequent use of the SOLO taxonomy to enrich the study of mathematics education at tertiary level would be useful for teachers in view of the attention that this theory gets within institutions of higher education.

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