HISTORY OF MATHEMATICS AND THE FUTURE OF MATHEMATICS EDUCATION

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History of Mathematics and ICMI. In 1905, D. E. Smith published a paper in *L'Enseignement Mathématique* proposing an international organization for the study of mathematics teaching. Three years later, at the 1908 meeting of the *International Congress of Mathematicians* in Rome, the proposal was taken up: the establishment of ICMI was set into motion, and Smith remained at the center of its formation. Smith's role is telling because, besides his being an important figure in the history of mathematics education, he was also a historian of mathematics and, generally, a man for whom the cultural aspect of our world was of great moment.[1] That history of mathematics, in his view, was not separate from mathematics education is clear from his classic book, *The Teaching of Elementary Mathematics* (Smith, 1904), which contains three entirely historical chapters and several others in which history has a part. And at the Michigan State Normal School in Ypsilanti, where D. E. Smith held the mathematics chair, he designed a course on the history of mathematics for teachers that "...became the foremost distinguishing characteristic of Smith's program: the importance of a historical perspective" (Donoghue, 2006, p.562). So, with Smith as a founding father, one might say that, from the start, ICMI had history of mathematics in its genes.

D. E. Smith was not alone in thinking that the history of mathematics had something to contribute to mathematics education, and especially mathematics teachers education. Felix Klein, who was the first president of ICMI, also thought history of mathematics could inform the teaching of mathematics. Historical sections appear in both parts of his *Elementary Mathematics from an Advanced Standpoint* (Klein, 1908/1939), which were published in the same year as the founding of ICMI and in which Klein tried to set out what mathematical content he deemed necessary for mathematics instructors. In the geometry part (part II), he states quite explicitly that "...I shall draw attention, more than is usually done...to the *historical development of the science*, to the accomplishments of its great pioneers. I hope, by discussions of this sort, to further, as I like to say, your general *mathematical culture*: alongside of

[1] His doctorate, for example, was not in mathematics, nor even in the history of mathematics, but in the history of art (Swetz, 1987).

knowledge of details, as these are supplied by the special lectures, there should be a grasp of subject-matter and of historical relationship [emphases in the original]" (Klein, 1908/1939, II, p.2).

That mathematical culture, as embodied in the study of the history of mathematics, should be counted among the aims of mathematics education[2] was given concrete expression in ICMI with the initiation of the *International Study Group on the Relations between the History and Pedagogy of Mathematics* (HPM) at the 2nd ICME conference in 1972 and its consolidation as an official affiliate of ICMI in 1976 (Fasanelli & Fauvel, 2004). Through its newsletter, conferences, and books (which have been largely elaborated proceedings of the conferences), the HPM has done much to make the case for the educational potential of history of mathematics.

So in the activities and personalities connected with ICMI, one discerns a deeply rooted interest in history of mathematics in the mathematics education community. Conversely, the connection with ICMI has legitimized the history of mathematics as a subject for the mathematics curriculum and educational research.

Theoretical Difficulties in Incorporating History in Mathematics Education. But is the incorporation of history of mathematics in mathematics education, particularly secondary school education, truly unproblematic? Naturally, bringing history of mathematics into the classroom, requires choosing relevant historical subjects, producing learning materials, and finding room for historical study in existing mathematics curricula. These are, on first sight, only practical problems, which is not to belittle them of course; but the question at hand is whether the incorporation of history of mathematics in mathematics education is unproblematic *in principle*. This was the question raised in Fried (2001). The premise of that paper was, first of all, that the history of mathematics and mathematics education are disciplines, each with its own aims and its own conception of the subject. The question, then, was whether those aims are shared and that conception common: in short, is the history of

[2] The cultural aims of mathematical education, in general, were already highlighted in the 1923 report by the *National Committee on Mathematical Requirements* written under the auspices of the *Mathematical Association of America*. There, it was written that, among other things, the fulfillment of such aims involved appreciating "...the role that mathematics and abstract thinking, in general, have played in the development of civilization" (in Bidwell & Clason, 1970, p.394). D. E. Smith was on the committee that prepared the report, and it would not be surprising if he wrote the section in question: the ideas there, as Bidwell & Clason point out (ibid., note 3) parallel closely views in other writings by Smith, for example, his "Religio Matematici" published in the *American Mathematical Monthly* (vol.28, pp. 339-349) in 1921.

mathematics in perfect harmony with mathematics education? It was here Fried (2001) found a difficulty.

For mathematics education—at least as it is usually conceived (and this qualification is not trivial)—aims towards modern mathematics, but treats mathematics as it is conceived today as if it were mathematics *tout court*; thus, in the classroom, although one might refer to modern algebra to mean the theory of algebraic structures, one would generally see the addition of "modern," if one considered it at all, as supererogatory. In a way, this is the projection of the position of working mathematicians or working scientists who, to use Kuhn's famous terms, must, in a normal period, work within a paradigm, a set of concepts, procedures, and approaches fixed, as if eternal, in textbooks. Regardless of what mathematics may be in fact or even what mathematicians may think mathematics is when they are philosophizing, at work, mathematicians treat their problems and their objects as having real and independent existence, for which reason Hersh points out that "an inarticulate, half-conscious Platonism is nearly universal among mathematicians" (Hersh, 1998, p.11).

The history of mathematics, by contrast, always treats mathematics as modern or ancient, Greek or Egyptian. Historians of mathematics are like anthropologists who study mathematical cultures very different from our own; at work, historians must consider mathematics as ever changing and having no eternal, fixed, reference. Thus Sabetai Unguru (1979) has written:

> The history of mathematics is history not mathematics. It is the study of the idiosyncratic aspects of the activity of mathematicians who themselves are engaged in the study of the nomothetic, that is, of what is the case by law. If one is to write the history of mathematics, and not the mathematics of history, the writer must be careful not to substitute the nomothetic for the idiosyncratic, that is, not to deal with past mathematics as if mathematics had no past beyond trivial differences in the outward appearance of what is basically an unchangeable hard-core content (p.563)

For this reason, historians of mathematics assiduously try to keep modern notions of mathematics away from the mathematics of the past, that is, to keep anachronism at bay. More generally, even though it is interested in understanding how the present has come to be as it is, the practice of history studiously avoids measuring the past according to modern conceptions of what mathematics is and modern standards of what is mathematically significant. Failure to do this leads one to what is known in historiography as "Whiggism," after the tendency of certain British historians to see

history as marching ever towards the liberal values and aspirations of the Whig party (Butterfield, 1931/1951). Needless to say, Whig history is not history at all.

In light of this, the theoretical problem of combining history of mathematics and mathematics education really begins with those practical problems with which I began this section. For, in choosing and presenting historical material, mathematics educators must accommodate curricular needs, which are generally directed towards learning the kind of mathematics necessary for science and industry, that is, modern mathematics; they must filter history to suit predetermined needs, making history a tool to be used rather than a subject to be studied. However well intentioned, then, mathematics educators are forced into a Whiggist position—a position that has less to do with historical accuracy than with historical approach.

The Future of Mathematics Education: The upshot of all this seems to be that, with regards to the history of mathematics, mathematics education is faced with a dilemma: be true to the commitments of mathematics education and be forced to adopt a Whiggist brand of history, or be true to the commitments of the history of mathematics and be forced to spend time on mathematical and philosophical ideas which may not be relevant to the general mathematics curriculum. Yet, this dilemma should not be taken as an argument repudiating any effort to incorporate history of mathematics in mathematics education, or, worse, as an attempt by historians of mathematics to protect their own special bailiwick. For the dilemma really turns on the phrase "mathematics education, as it is usually conceived," or, I should say, "as it *has been* conceived." Rather than a disavowal of history of mathematics in mathematics education, the dilemma should be taken as a challenge to reconsider the nature of mathematics education so that the history of mathematics as a serious pursuit becomes an integral part of what it means to be mathematical educated.

Such reconsideration does not necessarily involve revolution. Indeed, although it is has tended in certain directions in the past, the mathematics education community has in fact been almost continually engaged in a process of self-definition (see Kilpatrick, 1992). In recent years, the shift in the community's attention towards socio-cultural aspects of mathematics education promises to move the field in directions making a non-perfunctory approach to history of mathematics in mathematics education not only possible, but also necessary. These socio-cultural studies, though spurred, in part, by the practical challenges of large immigrant populations in Europe and

elsewhere and the general globalization of mathematics education, imply a view of mathematics itself as a cultural system (to quote the title of Wilder's well known book (Wilder, 1981)). Consistent with this view are currents in research such as D'Ambrosio's ethnomathematics (e.g. D'Ambrosio, 1995) and semiotic investigations, which put emphasis on mathematical knowledge and thinking as arising within a culturally engendered system of signs (e.g. Radford, 2003). And these currents, when they do not invite historical analysis directly, are consistent with the history of mathematics as the historian pursues it. In a way, what has happened is that the view originally reflected in the ICMI, the view held by Smith and Klein, that mathematics has cultural value—and, therefore, a history to be taken seriously—is finally being taken seriously. And, surely, the ICMI will, in the future, have a major role in developing it and clarifying its implications.

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