MEANING IN MATHEMATICS EDUCATION
REFLECTIONS FROM VARIOUS PERSPECTIVES

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German philosophers emphasise, that human beings show a desire for meaning in reality. It is further elaborated that it is a specific human desideratum to provide the world with meaning and sense so as to interpret it as being meaningful. In this way, the world can be understood by human beings. This attitude does not stop outside classroom doors but meaning is also – if not especially – sought inside the classroom when students deal with learning contents.

But what do we mean by meaning in mathematics education?

In mathematics education, the term meaning is used to stand for very different concepts. The paper will describe two different studies concerned with the personal meanings students construct when they are dealing with mathematics. One approach coming from a normative perspective on mathematics education (based on the work of Vorhoelter 2007) focuses on the orientation of mathematics education towards applications and modelling as main goal of mathematics education, which shall provide meaning to students in mathematics education, because they can see a reason for doing mathematics (see Maaß 2004). The other approach (based on the work of Vollstedt 2007) takes a descriptive perspective on meaning in mathematics education using the comparison of two different learning cultures, namely in Hong Kong and Germany in order to construct patterns of meaning existing in different learning cultures.

Personal meaning vs. objective meaning

There is a big variety of notions on meaning in mathematics education; Kilpatrick et al. (2005b) show this variety when they write:

> Some students find it pointless to do their mathematics homework; some like to do trigonometry, or enjoy discussions about mathematics in their classrooms; some students' families think that mathematics is useless outside school; other students are told that because of their weakness in mathematics they cannot join the academic stream. All these raise questions of meaning in mathematics education. (Kilpatrick et al., 2005b, p. 9)

One can see that there are very different kinds of meaning dealt with in this passage. On the one hand, meaning is used in a rather personal sense of the student “relating to relevance and personal significance (e.g., ‘What is the point of this for me?’)” (Howson, 2005, p. 18). On the other hand, meaning can also be used in a rather objective way when describing “an agreed, common meaning within a community” (Kilpatrick et al., 2005b, p. 9). It is important to keep this distinction. Therefore, in this paper, the terms *personal meaning* and *objective meaning* are to be used to describe these different aspects of meaning.

This difference between personal and objective meaning comes to the point when the difference between philosophical and non-philosophical interpretations of meaning are considered. Kilpatrick et al. state that “we may claim that an activity has meaning as part of the curriculum, while students might feel that the same activity is totally devoid of meaning” (Kilpatrick et al., 2005a, p. 2). One can, however, even go a step further by saying that although a student might think that a certain activity is totally devoid of objective meaning, she still sees a personal meaning in relation with the activity. This personal meaning, then, can be of different kinds. It may be the case that she might still work on the task so as to fulfil

her teacher's or parents' expectations, because she might after all seek her teacher's meaning, because she wants to get good marks in the next class test, etc.

Within the debate on meaning in mathematics education the distinction between personal meaning and objective meaning is emphasised, but other aspects are important as well. The two already mentioned ongoing empirical studies by Vorhoelter (2007) and Vollstedt (2007) describe the following thesis as basis for theoretical framework necessary for them in order to clarify this difficult construct meaning. The first issue for meaning is that personal meaning is *subjective* and *individual*. This means that every person has to construct her or his own meaning with respect to a certain object. There is no given objective meaning which just has to be applied; meaning cannot just be endowed. Also, as the construction of meaning is not collective but individual, different students sitting in the same lesson can also construct different meanings. However, offerings of meaning can be assimilated. They may be provided in a lesson e.g. in the shape of modelling tasks given by the teacher, or by the context of the learning task which for instance shows a relation to daily life. But still – the individual is involved in the process of constructing a meaning before a certain personal meaning is generated.
The construction of meaning is also context bound. Context hereby means on the one hand the subject context as well as the situation in the classroom. On the other hand, it also embraces the personal context of the students.
Meanings can be reflected on but normally do not have to. This means that the process of meaning-making can in some parts be dominant in the situation so that one is aware of what is going on; the meaning enters consciousness. An 'Aha-experience', for instance, is an example of a meaning which is dominant and conscious in the very situation. On the other hand, meaning does not have to be conscious but can be constructed implicitly so that it is there without being dominant in the situation. From a constructivist perspective, Kilpatrick et al. state that

> the problem of construction of meaning itself is not really tackled. This is an evasive problem: It is difficult to know what each partner [i.e. student and teacher] thinks; we can only hypothesise this by interpreting what they do and say (Kilpatrick et al., 2005c, p. 137)

Things and events have no implicit meaning. This implies that everyone has to construct his or her own personal meaning so that it is possible that students develop different kinds of meaning concerning the same mathematical task or problem.
Although everyone has to construct his or her own meaning, the kind of meaning students construct is not arbitrary. It depends on the one hand on offers of meaning given by teachers, parents and society as well as, on the other hand, on the students' personal experiences, abilities, dispositions, their wishes and intentions.
The kind of personal meaning students develop when dealing with a situation can differ from the one they construct after dealing with the situation. So the kind of personal meaning may change.
To summarise the theoretical description of meaning in mathematics educations depends on concepts coming from mathematics education such as mathematical thinking style or their mathematical beliefs, other influential variables such as learning motivation come from educational psychology and pedagogy is providing the guiding norms. So, the theoretical framework developed by Vollstedt (2007) and Vorhölter (2007) refers to these different relational disciplines and make use of them.
Although the importance of meaning for learning is well known (Krapp, 2003) hardly any empirical studies exist on the students' personal meaning in certain mathematical issues and the conditions which can influence this process. In the following two case-studies will be highlighted, which try to clarify the fuzzy concept of meaning and examine patterns of meaning in mathematics education.

Methodology

Both studies used the same approach: several classes of 15-16 olds were visited, either only in Germany in the study of Vorhoelter and in Germany and in Hong Kong within the study of Vollstedt. The classes were visited for one week. Every mathematics lesson the students had in this time was videotaped with a two-camera-design. After every lesson a sequence of five to ten minutes was cut from the material. These situations were chosen as new processes of construction of personal meaning are very likely to occur. During free lessons, lunch break or after school, volunteering students were interviewed for 45 minutes in average. The interviews always started with the video sequence of the last lesson. The students were asked to tell what was in their minds when they were sitting in class and what came to their minds when they were watching the sequence. After this stimulated recall (cf. Gass & Mackey), a guided interview was done, which was structured by the interviewee. The guide contains questions with relation to the different aspects influencing the meaning the students developed connected to the lesson, the content taught and so on. In addition Vorhoelter supplied material for a modelling problem to the classes, which were done in one lesson. The students could choose between several problems such as the amount of money one has to save each month in order to have a certain amount of pension after retirement or the noise a snorer is making. The students worked in groups on these problems. She videotaped this lesson and as contrast a rdinary mathematics lesson as well. She then interviewed the students on their opinions of this kind of application, feelings and so on in order find out the differences between applied and non-applied lessons from the perspectives of the students.
The data is being evaluated with the help of Grounded Theory (Glaser/Strauss, 2005; Strauss/Corbin, 1996).

Preliminary results

The preliminary results point to the high relevance of experiencing competence, either in the already taught mathematical topic or in new topics as well. This distinct aspect holds for both group of students, either the students from Hong Kong and the students from Germany. In addition it shows the effects of modelling tasks, which even as singular event enable special groups of students being disappointed from mathematics education to construct meaning in these particular examples.

Case William from Hong Kong

William is 15 years old and attends a private EMI-school in Hong Kong. He is very good in mathematics and likes the subject very much. According to his own judgement, he primarily does mathematics as it is a subject at school. Had it not been a subject, he would not have come into contact with it. Therefore, he acknowledges the school's and curriculum's great importance for learning mathematics.
William's dominant personal meaning constructed in the context of learning mathematics can be described as perception of his own competence. His own achievement, e.g. being the 'faster one to finish' as he puts it, is very important for him. It is, however, astonishing that he hardly speaks of competition although competition is implicitly and explicitly a very important factor in Hong Kong lessons. It is therefore probable that it is merely important for William to experience his own competence rather than to experience that he is better than his classmates. This may also result from the fact that William is a high-achieving student who knows his position among his classmates. Comparison with them may therefore take a back seat. When asked when he is pleased with himself in a mathematics lesson, he therefore answers among other things: 'answering a questions from my classmate eh … because they have difficulties and I can explained to them'.

Due to his high achievements in mathematics and his desire for experiencing his competence, William is looking for challenges in the lessons. He for instance wants to find out the relation between mathematics and daily life on his own. He says: ‘I don't want they [the teachers, MV] told us. Because ehm I'm I’m th- I think that … they should think it by m- ourselves. This can increase our thinking logic thinking ability.’ He, however, understands that the teacher has to show this relation for lower achieving students to enable them to participate in the lessons. William also provides himself with another challenge with his general refusal to use the calculator. The calculator enables him to quickly come to the results needed but denies the feeling of success which is so desperately sought by William. He says: ‘I don't like using calculator eh because ehm using calculator is ... although it's fast but eh it's not ... success ... eh not there is not a feeling of successful so I like calculating by myself.’
Similarly, William wants to deeply understand the content taught in the lessons. It is not enough for him to memorize formulae or facts as it is done in subjects like history or geography. He says: ‘Doing the … formula, solving the f- the formula eh is … make me feel … confidence. … Eh increase myself on this’, and further: ‘Mathematic lesson: no need to […] to remember all the things eh is ... just calculating and … observation to to the graph eh ... and is more eas- I think is more easier eh but ... interesting.’ He wants to understand mathematics on a deeper level so that, when he does, the subject seems to be comparatively easy for him. This is also the case why dealing with formulae and reading information from graphs gives him more trust in his own mathematical skills. It also shows him how interesting mathematics can be and how good it is to train the ability to think logically. This is why William has an apparently good feeling after a mathematics lesson: ‘after the mathematics ... lesson I go out to the corridor I feel very ... happy and ... ehm (2 sec) I have ... confidence. Yes, because ... eh maybe the logic thinking is ... eh I can do for the questions.’ (case description from Vollstedt 2007, p. 7f)

Case Larissa from Germany

Larissa seems to be a student who can be found quite often in German classrooms. Her mathematics marks are relatively bad as well as her relationship with her mathematics teacher. For her, the meaning for learning and doing mathematics consists in pretending to be competent, not necessarily being competent. The reason thereof is her strong desire to move up to the next class level and to get a new mathematics teacher. The more astonishing (especially for her teacher) were the enthusiasm and the involvement with which she, together with the members of her group, tried to solve the chosen modelling task. the 'Noisy Snorer'. Her decision for this task was influenced by different factors: One was the assumed low level of difficulty, the other was the task context: Larissa had just heard a report about a comparison between the volume of an mp3-player and an aircraft turbine which she could not believe. Furthermore an eardrum of some of her friends had burst some time afore and the friend claimed that it had burst just because he was boxed on his ear. She could not believe neither the report not the description of her friend.
For her enthusiasm for and involvement in working on the task, Larissa gave the following reasons: After a long time of feeling incompetent during mathematics lessons, she got the impression of being able to solve a task, because she did not just have to find the right formula and use it in the correct way. Concerning this task it was necessary to find a way for solving the task and to discuss it with others. Hence, not only the high-achieving but also the poor-achieving students got the possibility of finding a suitable answer to the problem. Furthermore, because of the task context and the given information, Larissa was enabled to value the report and the description of her friend in a new way.
So Larissa's personal meaning was influenced by the modelling problem at least temporarily: Pretending to be competent was not that important any more, but, on the contrary, getting the feeling of being competent was way more important. The experience to be able to value

reports and descriptions with the help of mathematics pointed out a new kind of personal meaning for learning and doing mathematics to her. Whether these different kinds of personal meaning are permanent or only temporary can, however, not be said on the basis of the existing data (case description from Vorhölter 2007, p. 8f)

To summarise these two cases show remarkable similarities between two highly different school systems such as the ones from Germany and Hong Kong. Furthermore the two cases show that even for weak students modelling examples can offer meaning to mathematics learning. We need more knowledge about patterns of meaning, sources of meaning, where does it come from in order to establish learning environments, which allow the students to construct personal meaning in their mathematics lessons.

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