THEORIES AS INTELLECTUAL RESOURCES IN MATHEMATICS EDUCATION RESEARCH

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Introduction

The field of knowledge production in the community of mathematics education research, as with other curriculum domains, gazes for the most part on the mathematics classroom as its empirical field, although also on other sites of learning and social practices defined as mathematical by observers. Researchers in mathematics education draw on a range of disciplines for explanations, analyses and curriculum designs. The process of adopting theoretical frameworks into a field has been defined as *recontextualization*, as different theories become adapted and applied, allowing space for the play of ideologies in the process. Prescribing teaching strategies and the ordering of curriculum content on the basis of Piaget's psychological studies is a prime example of recontextualization. Psychologists, sociologists, mathematicians, and others might therefore look at work in mathematics education and at educational studies in general, as derivative. At the same time, however, we should also look on the process as *knowledge production*, in that new formulations and frameworks emerge in dialectical interaction with the empirical field and are therefore *produced* in the educational context. The development of radical constructivism as a field in mathematics education research on the basis of Piaget's work is an example of what is more appropriately seen as knowledge production. The adaptation of the ideas of radical constructivism, or any other theoretical framework, into pedagogy, however, is again a process of recontextualization where the play of ideologies is often quite overt.

The range of disciplines on which we draw, which should be seen as *resources* for knowledge production, is wide and one might ask why this is so. The mathematics education research community appears to be particularly open to drawing upon other disciplines, for at least four reasons. First, mathematics as a body of knowledge and as a set of social practices has been and remains of particular interest to other disciplines such as psychology, sociology and anthropology as it presents particularly interesting challenges to their work. It is not surprising, for example, that one of the major challenges for Piaget was to account for the development of logical reasoning, nor that Piaget's account of knowledge schemata used group theory as its fundamental structure. Second, mathematics has stood as exemplar of truth and rationality since ancient times, giving it a unique status in most world cultures and in intellectual communities. That status may account for mathematics being seen as a marker of general intellectual capacity rather than simply aptitude at mathematics. Its symbolic power certainly lays mathematics open to criticisms of its gendered and Eurocentric character, creating through its discursive practices the reasoning logical 'male' norm. Third, mathematics has played a large part in diverse cultural practices including religious life, music, pattern, design and decoration. It appears all around when one chooses to apply a mathematical gaze. Finally, there is the apparent power of mathematics such that its use can enable the building of skyscrapers, bridges, space exploration, economic theories, theoretical physics and so on.

Foundational Theories: Mathematics and Psychology

Until the last 20 years mathematics education tended to draw on mathematics itself, or psychology, as disciplines for the production of knowledge in the field. Analyses of mathematical concepts provided a framework for curriculum design and enabled the study of the development of children's understanding as the building of higher order concepts from their analysis into more basic building blocks. Behaviourism supplied the psychological rationale both for the building blocks metaphor for the acquisition of mathematical knowledge and for the pedagogical strategies of drill and practice, and positive and negative reinforcement. Piagetian psychology called for historical analyses of mathematical (and other) concepts, based on the assumption that the individual's development replays that of the species (ontogeny replicates phylogeny). It was argued that identifying historical and epistemological obstacles would reveal pedagogical obstacles. This again emphasized the importance of mathematical concepts for education. In terms of psychology, the influences of Piaget and the neo-Piagetian radical constructivists are too well known to require documentation here, and I would refer in particular to the detailed studies of children's thinking. Both the disciplines of mathematics and psychology have high status in universities, and locating mathematics education within either group is seen as vital in some countries in terms of its status and therefore funding and respectability. Psychology has well established research methodologies and procedures upon which mathematics education has fruitfully drawn. Evidence can be seen, for instance, in the proceedings of the ICMI's first sub-group, the International Group for the Psychology of Mathematics Education (PME) over the past 30 years. Although its constitution has developed to include other theoretical resources it was certainly 'natural' that psychology would be the choice in 1976.

Interest in the implications of the philosophy of mathematics for mathematics education research was given impetus by Lakatos' *Proofs and Refutations* partly, I suspect, because of the style of the book, which is a classroom conversation between teacher and students. More important, though, is the humanistic image of mathematics it presents, as a quasi-empiricist enterprise of the community of mathematicians over time rather than a monotonically increasing body of certain knowledge. The book by Davis & Hersh which was inspired by Lakatos has become a classic in the community but others have become equally influential. A number of researchers have studied aspects of teaching and learning mathematics from the humanistic, quasi-empirical point of view. That mathematical certainty has been questioned in the absolutism/fallibilism dichotomy is not due directly to Lakatos since he never subscribed to that view. With Popper, Lakatos considered knowledge to be advancing towards greater verisimilitude, but identifying the process of knowledge growth as taking place through refutation, not indubitable deduction, raised the theoretical possibility that all knowledge might be challenged by a future counter-example. In mathematics education the absolutist/fallibilist dichotomy has been used as a rationale for teaching through problem solving and as a challenge to the traditional mathematical pedagogy of transmission of facts. Fallibilism's potential challenge to mathematical certainty has led to mathematical activity being identified by its heuristics, but to a much greater extent in the mathematics education community than amongst mathematicians. This is another illustration of the recontextualizing process from the field of production of mathematics education knowledge, driven perhaps by democratic tendencies for pedagogy amongst some schoolteachers.

Extending the Resources

In this section I examine the theories that have emerged and been taken up over the past 20 years. I will not attempt a chronological account nor, for lack of space, elaborate to any great extent on each of these theories.

Psychology

From within the general field of psychology some researchers have drawn on psychoanalytical theories, either Freudian or Lacanian. Powerful insights are offered by these theoretical fields into links between emotion and students' errors and into power/knowledge relationships.

Perhaps the major development has arisen from the emergence of Vygotsky's cultural psychology into mathematics education research, a rich theory in itself but providing also an opening into situated cognition and sociology. The first appearance of this work into our field, according to my research, was in 1985. Vygotsky's psychology is a method for interpreting how persons become social beings. Vygotsky's work is generally taken to be about the *individual learning* in a *social context*, but his notion of the zone of proximal development (zpd) offers more than that. First, in that consciousness is a product of communication, which always takes place in a historically, culturally and geographically specific location, *individuality* has to be seen as emerging in social practice(s). Second, all *learning* is from other persons-in-practices, and as a consequence meanings signify, they describe the world as it is seen through the eyes of those socio-cultural practices. In his discussion of inner speech Vygotsky argues that the process of the development of internal controls, metacognition, is the internalisation of the adult voice. Again, these are mechanisms that are located in *social contexts*. Finally, the zpd is a product of the learning activity, not a fixed 'field' that the child brings with her or him to a learning situation. The zpd is therefore a product of the previous network of experiences of the individuals, including the teacher, the goals of teacher and learners, and the specificity of the learning itself. Individual trajectories are key elements in the emergence, or not, of zpds.

Anthropology

Ethnomathematics was introduced as a new direction by Ubiritan D'Ambrosio at the Fifth International Congress on Mathematical Education in Adelaide in 1984. It argues that academic mathematics is just one of a whole range of social practices that engage mathematically, in meaningful ways, with the world. Academic mathematics has enormous hegemonic authority of course but in doing so denies the worldviews and needs of traditionally underprivileged and exploited peoples. Typically studies reveal how these groups perform in mathematical ways and either aim to empower them or (possibly and) include those methods in mainstream schooling. As such I classify Ethnomathematics as anthropological.

Situated theories have generated great interest and received much critical attention in recent years. Lave and Wenger have given radically different meanings to knowledge, learning, transfer and identity. Lave's studies of the acquisition of mathematical competence within tailoring apprenticeships in West Africa led her to argue that knowledge is located in particular forms of situated experience, not simply in mental contents. Knowledge has to be understood relationally, between people and settings: it is about competence in life settings. One of the consequences of this argument is that the notion of transfer of knowledge, present as decontextualized mental objects in the minds of individuals, from one situation to another, becomes perhaps untenable but at the very least requires reformulation. Another is that research should focus on the communities of practice within which people operate and within which activities and concepts gain their meaning.

Sociology

Whilst there is a substantial body of literature in social studies of scientific knowledge, there has been much less written about mathematical knowledge. Science education research draws heavily on social studies of scientific knowledge: in mathematics education that resource is still in an early stage. Perhaps the leading body of work in this sub-field is that of Skovsmose and colleagues, under the umbrella of critical mathematics. As such, it draws on a range of sociological theory including Habermas and emancipation theorists such as Freire. It has connections with ethnomathematics in its uncovering of the hegemonic effects of the academic discipline of mathematics and the potentially liberating outcomes of working with school students in real world problem solving with a critical theory perspective.

Mainstream sociology of education (Bernstein, Bourdieu, Apple) has enabled mathematics education researchers to engage with questions of who, in terms of which social groups, are disadvantaged in school mathematics. Research indicates that social background is the major determinant of success or failure in mathematics and sociological theory offers explanations by drawing links between social movements and social capital one the one hand and the social relationships in the classroom on the other. One aspect of these sociological studies can provide a language for examining the phenomenon and effects of multiple theories in our research field, as I discuss in the final section.

Finally, poststructuralist theories, offering a focus on discourse and discursive practices, the power that is carried in language and associated social practices, provide researchers in our field with theoretical resources to examine aspects of teaching and learning. As an illustration of these ideas I quote from a paper I wrote in 1998: from this perspective "learners come to the classroom as persons of multiple, overlapping subjectivities. Different aspects of those subjectivites are called up by different aspects of the practices of the classroom, and are expressed through identities of powerfulness or powerlessness. At the same time, new subjectivities are constituted in the social relationships and forms of communication which make up the activities of the classroom. Rather than the intension of teaching mathematics as the handing over, or the individual construction, of ultimately decontextualised mathematical concepts by the teacher or by the pupil respectively, teaching might be conceived of as enabling pupils to become mathematical actors in the classroom and beyond. The goals and needs of pupils, and the ways of behaving and speaking as mathematicians, become the focuses of the teacher's intentions."

Making Sense of Multiple Theories

In this final section I will draw on an aspect of the sociology of Basil Bernstein to examine the phenomenon of multiple (increasing?) theories in our field, and I will end with some comments on the effects on the research in mathematics education.

Bernstein proposes a notion, verticality, that describes the extent to which a sub-field grows by the progressive integration of previous theories, what he calls a vertical knowledge structure, or by the insertion of a new discourse alongside existing discourses and, to some extent, incommensurable with them. He calls these horizontal knowledge structures. Bernstein offers science as an example of a vertical knowledge structure and, interestingly, both mathematics and education (and sociology) as examples of horizontal knowledge structures. He uses a further distinction that enables us to separate mathematics from education: the former has a strong grammar, the latter a weak grammar, i.e. with a conceptual syntax not capable of generating unambiguous empirical descriptions. Both are examples of hierarchical discourses in that one needs to learn the language of linear algebra or string theory just as one needs to learn the language of radical constructivism or embodied cognition. It will be obvious that linear algebra and string theory have much tighter and specific concepts and hierarchies of concepts than radical constructivism or embodied cognition. A major obstacle in the development of accepted knowledge in mathematics for teaching may well be the strength of the grammar of the former and the weakness of the latter. Whilst we can specify accepted knowledge in mathematics, what constitutes knowledge about teaching is always disputed.

As a horizontal knowledge structure, then, it is typical that mathematics education knowledge, as a sub-field of education, will grow both within discourses and by the insertion of new discourses in parallel with existing ones. Thus we can find many examples in the literature of work that elaborates the functioning of the process of reflective abstraction, as an instance of the development of knowledge within a discourse. But the entry of Vygotsky's work into the field in the mid-1980s with concepts that differed from Piaget's did not lead to the replacement of Piaget's theory (as the proposal of the existence of oxygen replaced the phlogiston theory). Nor did it lead to the incorporation of Piaget's theory into an expanded theory (as in the case of non-Euclidean geometries). Indeed it seems absurd to think that either of these would occur precisely because we are dealing with a social science, that is, we are in the business of interpretation of human behaviour. Whilst all research, including scientific research, is a process of interpretation, in the social sciences, such as education, there is a double hermeneutic since the ‘objects’ whose behaviour we are interpreting are themselves trying to make sense of the world.

Education, then, is a social science, not a science. Sociologists of scientific knowledge might well argue that science is more of a social science than most of us imagine, but social sciences certainly grow both by hierarchical development but especially by the insertion of new theoretical discourses alongside existing ones. Constructivism grows, and its adherents continue to produce novel and important work; models and modelling may be new to the field but already there are novel and important findings emerging from that orientation.

I referred above to the incommensurability, in principle, of these parallel discourses. Where a constructivist might interpret a classroom transcript in terms of the possible knowledge construction of the individual participants, viewing the researcher’s account as itself a construction, someone using socio-cultural theory might draw on notions of a zone of proximal development. Constructivists might find that describing learning as an induction into mathematics, as taking on board concepts that are on the intersubjective plane, incoherent in terms of the theory they are using (and a similar description of the reverse can of course be given). In this sense, these parallel discourses are incommensurable. I conjecture, however, that the weakness of the grammars in mathematics education research is more likely to enable communication and even theory-building across different discourses, although I emphasise the term ‘building’. It is no easy matter to join together different theories and it is done unsatisfactorily rather too often, I feel.

Finally, I will comment on concerns about the effectiveness of educational research in a time of multiple and sometimes competing paradigms, described here as discourses. ‘Effectiveness’ is a problematic notion, although one that certainly figures highly in current discourses of accountability. It arises because by its nature education is a research field with a face towards theory and a face towards practice. This contrasts with fields such as psychology in which theories and findings can be applied, but practice is not part of the characteristic of research in that field. Research in education, in contrast, draws its problems from practice and expects its outcomes to have applicability or at least significance in practice. Medicine and computing are similar intellectual fields in this respect.

However, what constitutes knowledge is accepted or rejected by the criteria of the social field of mathematics education research. Typically, we might say necessarily, research has to take a step away from practice to be able to say something about it. Taking the results of research into the classroom calls for a process of recontextualisation, a shift from one practice into another in which a selection must take place, allowing the play of ideology. To look for a simple criterion for acceptable research in terms of ‘effectiveness’ is to enter into a complex set of issues. Indeed ‘effectiveness’ itself presupposes aims and goals for, in our case, mathematics education. To ignore the complexity is to lose the possibility of critique and hence I am not surprised by the multiplicity of theories in our field and the debates about their relative merits, nor do I see it as a hindrance. I am more troubled by how those theories are used. Too often theories are taken to be unproblematically applied to a research study. I am particularly troubled by the attacks on educational research as an inadequate shadow of a fetishised image of scientific, psychological or medical research, as we are seeing currently in the USA, increasingly in the UK, imminently in Australia and, I expect in other countries too. Finally, I consider that equity and inclusion are aspects of mathematics education that should be of great concern to all of us, given the role of a success in school mathematics as a gatekeeper to so many fields. I believe that the social turn and the proliferation of social theories have enabled us to examine and research equity issues in ways that our previous theoretical frameworks did not allow.