The Contribution of Multiple Disciplines of Influence to Mathematics Education: A Complexity Science Interpretation.

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Mathematics education researchers have recently adopted complexity theories to understand curriculum, teaching and learning (see for example, Davis & Simmt, 2006; Namukasa & Simmt, 2003). In this paper, I attempt a complexity science interpretation of the contribution of multiple disciplines of influence to mathematics education. I focus on the problem of conceptualizing mathematical thinking.

There are a variety of schools of thought on what mathematical thinking is, all depending on theoretical framework espoused. Although the schools of thought might appear distinct, B. Davis, Sumara and Luce-Kapler (2000) observe that some of them share overriding metaphors and basic assumptions about cognition in general and mathematical thinking in particular. In this paper I suggest that schools of thought that share a discipline of influence are likely to share basic assumptions. For instance schools that are influenced by individual psychology may share the assumption that learning is individually based, those influenced by sociology that learning is socially based and those influenced by anthropology that it is culturally based. I have grouped schools of thought that share fundamental assumptions into the: *child* and *structural* (individual and content) *psychology* paradigm, *cognitive* and *information* paradigm, *co-emergent* and *context* paradigm, *coherent* and *post-structural* paradigm and, recently, the *ecological* and *systems* paradigm. In the first five sections I briefly examine how mathematical thinking is construed in each of these paradigms. Due to limitations of space I do not go into the details nor do I give a conclusive list. Views such as the genetic approach (Safuanov, 2007) and the modeling approach that are rising (in English publications) are beyond the scope of this paper. In Section 6, from the view point of complexity science, I outline some contributions of having multiple disciplines of influences in mathematics education.

1 Individual and Content Psychology

The influence of psychology on mathematics education may be traced back to Brownell's (1935/1970) movement toward meaningful arithmetic. This movement later consisted of psychological studies, most of which were based on *Piagetian theory*, though a few arose out of *structuralists'* and *conceptuanalist* studies (e.g. Cuisenaire & Gattegno,1957; Dienes & Goldings,1971; Hadamard, 1945/1996; Polya,1945/1973) and *gestalts* and others remained loyal to Thorndike's (1970/1924) *connectionist* studies. Subject matter was the basis for students' thoughts. Fundamental structures of mathematics were highlighted (Bruner, 1960; Steffe & Kieren, 1994). Mathematical thinking was mainly construed as masterly or proficient performance.

Researchers who turned to experimental and behavioral psychology in the 1960's for a tradition of scientific inquiry considered the mind to be, for matters of analysis, a black box (Schoenfeld, 1994). As well, since they focused on instruction, the structuralists insufficiently took into account the details of children's thinking (Dreyfus, 1990). Teachers found it difficult to understand their analyses (Confrey, 1991). In the early 1970s, Piagetian studies, such as Steffe's (1970), began to draw from Piaget's *developmental* psychology to demonstrate how Piagetian *genetic structures* could, alongside the basic mathematical structures, explain children's thinking in terms of stages (Steffe & Kieren, 1994). Together with Piaget's and Bruner's work on play, Dienes and

Goldings (1971), Cuisenaire and Gattegno (1957) and Gattegno's (1970) modern mathematics work ushered in studies, such as Kieren (1971), that explored the role of children's hands-on activities, story and play in enhancing concept learning (English & Halford, 1995; Steffe & Kieren, 1994).

2 The Cognitive and Information Paradigm

Later in the 1970s and 1980s, the cognitive revolution renewed interest in studying mind. Whereas many researchers in the 1980s only drew metaphorical language from information-processing, the cognitivists were strongly influenced by communication and information processing studies. They construed mathematical thinking as acquisition, processing or representation (Bereiter, 1997). Mental states—the processing unit or software—were key (Dehaene, 1997). Cognitive and developmental psychology studies raised educators' awareness of the significance of the environment and of the thinker's structure—particularly the age and mental structures. Cobb & Bauersfeld (1995) nevertheless observe that these early paradigms were limited in their ability to explain the richness and messiness in classrooms. They were also limited in their assumption of pre-existing fixed mathematical structures (Núñez, Edwards, & Matos, 1999), and in their construal of mathematical thinking as solely an aspect of an individual's processes.

3 The Co-emergent and Context Paradigm

Radical constructivists focus on the individual child actively constructing knowledge. Drawing from disciplines such as sociology, activity theory, anthropology, cultural studies and linguistic studies, social theorists contend that mathematical thinking is an aspect of social practices and discourses. The classroom situation does not only influence mathematical thinking but it determines what is accepted as thinking mathematically (Balacheff, 1990). I refer to these schools of thought that seek explanations from either the: (a) material context (including *radical constructivists* e.g. Confrey, 1994; Steffe & Wiegel, 1992; von Glasersfeld, 1995; Steffe & Thompson, 2000), (b) social context (including *social constructivists* e.g. Balacheff, 1990; Boaler, 2000a; Bauersfeld, 1995; Cobb, Yackel & Wood, 1992; Driver et al. 1994; Ernest, 1994; (c) cultural and communal context (including *situated cognitionists* e.g. Greeno,1991; Lave & Wenger, 1991 or (d) from connectionism and distributed learning in cognitive science as the *co-emergent* and *context* schools of thought. I also include in this paradigm attempts to juxtapose the various perspectives (e.g. Cobb 2000).

I use the term *co-emergence* for three reasons. Firstly, most researchers in this paradigm view cognition as a property that occurs within and through—emerges from—individual or social activity or community and expert practices. The other two reasons have origin in complexity science. I will return to these later. Mathematical thinking in this paradigm is considered to be synonymous with mathematical reflection, communication or participation. It is the sense that emerges through individual, inter-individual or community experiences (Sfard, 2001).

4 The Coherent and Post-structuralist Paradigm

A more recent paradigm is what I refer to as the coherence and post-structural paradigm. Researchers in this view uniquely draw from anthropology, sociology, political and cultural studies. In particular (a) Socio-cultural theorists (e.g. Boaler, 2000; Lerman, 2001; van Oers, 2001) draw from social, literal, cultural and political theories to examine the coherence or dialectic between thinking, language, culture and (b) the *interaction theorists* (e.g. Bauersfeld, 1995; Voigt, 1994) draw from sociologist particularly from Blumer's *symbolic interaction*. (c) The *political-critical* theorists (e.g.

Apple, 2000; Lerman, 2001; Walkerdine, 1990) draw from critical education scholars to understand macro aspects that economically, anthropologically and politically subjugate or emancipate people. In this paradigm the whole discourse of mathematical thinking as it is constructed around the fear of the marginalized other to be positioned as a non-mathematical thinker (Walkerdine, 1990) is questioned. Thinking happens between, not inside or outside, individuals, and in relation to conventions (Voigt, 1994) or cultural capital (Walkerdine, 1990). Whereas to constructivists and social theorists, thinking is about circumnavigating social or cognitive conflicts to politico-critical theorists it is about pose hard and critical questions about political injustices and social inequalities (Skovsmose, 2001). Critical theorists are critiqued for underplaying the influence of the biological structures that shape knowing, as many social and cultural theorists did (Núñez et al., 1999), and for overemphasizing social determinism.

5 Ecological and Systems Paradigm

Researchers now draw from juxtapositions of disciplines. Some of the perspectives that have been taken on by mathematics educators in this paradigm are eco-feminism (Confrey, 1995), enactivism (biology-activity) (Davis et al., 2000; Gordon, 2002; Kieren, 2000; Simmt, 2000; Towers 1998), neuro-biology (Butterworth, 1999), mathematical idea analysis—cognitive-structuralism (Núñez', 2000); and social-cultural semiotics (Radford, 2003), to mention but at few. I have also included in this paradigm researchers who draw metaphors from complexity and other newer science theories such as chaos theory to investigate cognition. *Complexity theorists* maintain that cognition is a complex, dynamic and adaptive phenomenon (see Davis & Simmt, 2002; Kieren & Simmt, 2002; Towers & Davis, 2002). It is from the complexity perspective that I have undertaken this brief review. I find the complexity metaphor of emergence particularly helpful.

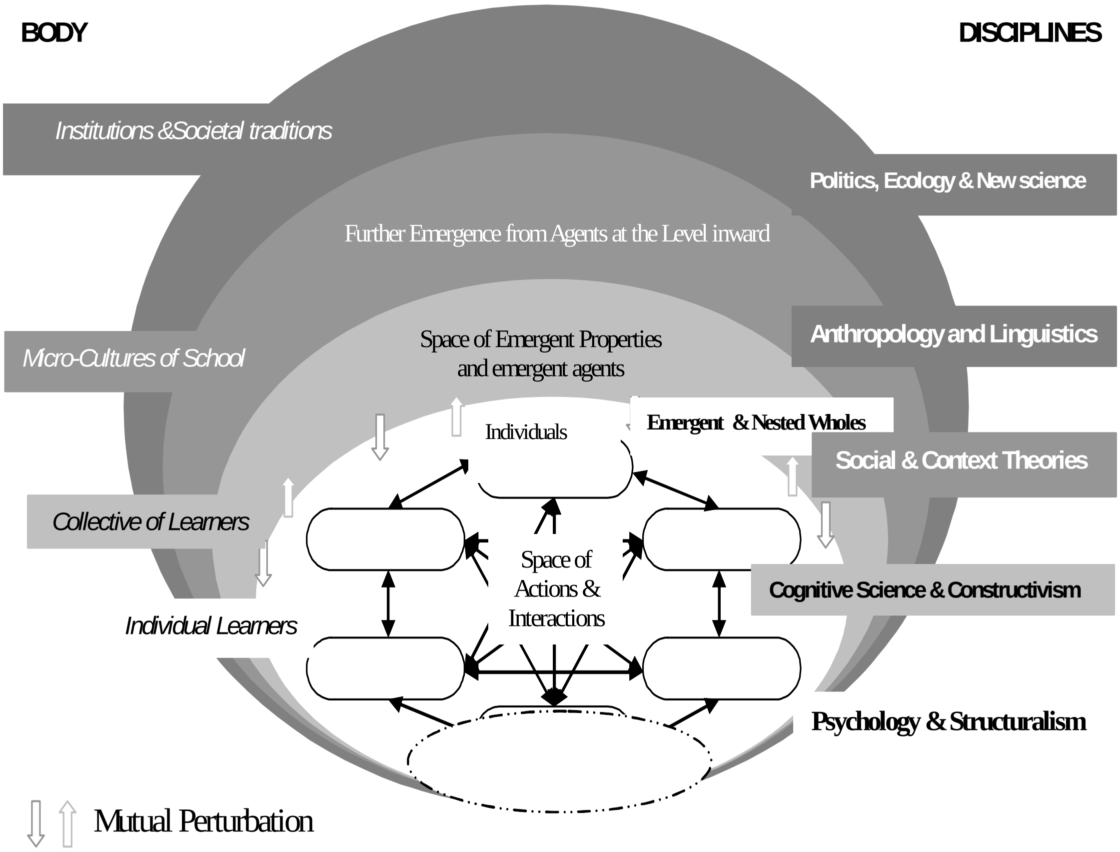
Emergence illustrates the budding of new structures from existing ones and is at the core of exploration of mathematical thinking as a higher order activity that emerges from lower order activity (Namukasa, 2005; Stanley, 2005). Social collectives bud from individual learners, micro-cultures from social collectives and societal traditions from micro-cultures as shown in Figure 1. In terms of influence of other disciplines, a psychological influence paves way to a sociological influence and sociological to a cultural influence. This is the second connotation of emergence as I used it to refer to the context paradigm.

Thinking mathematically is about *expanding possibilities* as well as bringing forth individual, social, cultural and political worlds of mathematical significance (Kieren, 2000; Simmt, 2000). Two other complexity notions are helpful in understanding emergence in a non-linear manner: mutual perturbation and nested wholes. Mutual perturbation illustrates how there is two-way interaction among the many layer or nesting spheres of cognition—psychological, social, technological and cultural. By using bigger outer spheres, the figure also illustrates that latter paradigms address the deficiencies of prior ones. The figure is limited at showing that latter paradigms ignore some of the focus of earlier paradigms. For instance it may not show that constructivists focused on actions but brushed aside the structural emphasis on content (Ginsburg & Seo, 1999)

6 Contributions of Multiple Influences

Psychology, communication studies, sociology, anthropology, political science and the other disciplines that have influenced mathematics education seek to understand human action. Davis & Sumara (2000) assert that these paradigms share a metaphoric commitment to a single body—biological, social, cultural, epistemological or political. From a complexity point of view each of the schools seems to be focusing on cognition at a different scale or emergent level—knowledge, activity, interactions, culture, and the like.

Figure 1. Theorizing the Problem of Learning: An Emerging but Partial Whole[a]



a. Adopted from Davis et al. (2000) and Namukasa (2005)

Evolutionally speaking, research in the co-emergent and context paradigm was necessary for further paradigm shifts toward the ecological and systems theories. Future disciplines of influence are motivated by critiques of today's dominant paradigms. Kieren and Steffe (1994) maintain that constructivism was implicit in the critiques to some of the perspectives in the structuralists and cognitive paradigm. As well, attempts to understand incompatibility of schools of thought (e.g. by Confrey (1995); Kieren (2000); Lerman, 2001 & Sfard, 2000) evoke deeper understandings of learning.

Lerman (1999, 2001) insists on the impossibility of theoretical conflation of the individualistic psychological views with the discursive, sociological views. Rightly so, the overemphasis on the individual person as the only cognizing system does not go away by conflating radical constructivism with social constructivism. Each emergent body has unique patterns and behaviors (Davis & Sumara, 2005; Namukasa, 2005). Any two emergent bodies—individual learner and collectives of learners in this case—might not usefully be studied from one paradigm

When the varied perspectives are considered in light of each other, they produce dramatic iterations to bring forth hybrid perspectives (Davis & Sumara, 2000; Kieren, 2000). That novel metaphors for understanding thinking are born from the interaction of insights from varied schools of thought, say in the context and co-emergence paradigm, is the third connotation of co-emergence.

Latter paradigms have expanded perceived limits of cognitive activities away from strictly head-based structures, passive and linear modes of thinking (Confrey, 1994; Núñez et al., 1999). Practically speaking, they have offered better[1] metaphors of thinking. For example, understanding thinking as making sense guides teaching in more practical ways than understanding thinking as processing. Kieren (2000), in his paper entitled "Binoculars or Dichotomies", calls on researchers to view their theories as partial truths in order to "occasion for new and perhaps different ways of thinking/acting" (p. 231). For example construing mathematical thinking as the sense that arises, say through individual activity, may still not elaborate on how students' thinking, given two-way, mutual perturbation among complex systems, turns back to influence the individual activity.

References

Apple, M. W. (2000). Mathematics reform through conservative modernization? Standards, markets, and inequality in education. In J. Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning* (pp.171-200). London: Ablex.

Balacheff, N. (1990). Beyond a psychological approach: The psychology of mathematics education. *For the Learning of Mathematics, 10(3),* 2-8.

Bauersfeld, H. (1995). Language games. In P. Cobb & H. Bauersfeld (eds.), *The emergence of mathematical meaning: interactions in mathematics cultures,* 277-. Hillsdale: Lawrence Elrbaum.

Bereiter, C. (1997). Situated Cognition and how to overcome it. In A. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp.281-300).

Boaler, J. (2000a). Exploring situated insights into researching and learning. *Journal of Research in Mathematics Education, 31,* 113-119.

Brownell, (1970). Psychological consideration in the learning and the teaching of arithmetic. In J. K. Bidwell & R. G. Clason (Eds.) *Readings in the history of mathematics education* (pp. 504-530). Washington, DC: NCTM. (Original work published in 1935)

Bruner J. (1960). *The Process of Education*. Cambridge: Harvard University.

Butterworth, B. (1999). *What counts: How every brain is hard wired for math*. New York: The free press.

Cobb, P. (2000). The importance of a situated view of learning to the design of research and instruction. In J. Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning* (pp. 45-82). A series in International perspectives on mathematics learning, series editor L. Burton. London: Ablex.

Cobb, P. (1994). data analysis. *Mathematical Thinking and Learning, 1*(1), 5-43. education. *For the Learning of Mathematics, 9(2),* 32-42.

Cobb, P., & Bauersfeld, H. (1995b). The emergence of mathematical meaning: Interaction in classroom cultures. Hillsdale, NJ: LEA.

Cobb, P., Wood, T., & Yackel, E, (1993). Discourse, mathematical thinking and classroom practice. In E. A. Forman, N. Minick & C. A., Addison. (Eds.), *Contexts for learning: Socio-cultural dynamics in children's development,* (pp. 93-119). New York: Oxford University.

Confrey, J. (1987). *The constructivist.* In J.C. Bergeron, N. Herscovics, and C. Kieran. Proceedings of the 20th annual general meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, 3 (pp. 307-317).

Confrey, J. (1991). Steering a course between Vygotsky and Piaget. *Educational Research, 20* (8), 28-32.

[1] They are better in that they enable us to see through the myths of the old (Rorty, 1982), also to the extent that they allow us to see, to do and to say a lot of things not possible before.

Confrey, J. (1994). Voice and perspective: Hearing epistemological innovations in students' words. In N. Bednarz, M. Larochelle, and J. Desautels (Eds.), *Revue des Sciences de l'education, Special Issue: Constructivism in Education 20 (1)* (pp. 115-133).
Confrey, J. (1995). The theory of intellectual development, part III. *For the Learning of Mathematics,* 15(2), 36-45.
Cuisenaire, G., & Gattegno, C. (1957). Numbers in colors: A new method of teaching arithmetic in primary schools. London: Heinemann Educational.
Davis, B., & Simmt, E. (2002). The complexity science and the teaching of mathematics. In Denise Mewborn (Ed.), *Proceedings of the PME-NA XXIV October* (pp.831- 840), Georgia Center for Continuing Education, Athens, GA.
Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. Educational Studies in Mathematics, 61(3), 293–319.
Davis, B. A., Sumara, D. J., & Kieren, T. E. (1996). Cognition, co-emergence, curriculum. *Curriculum Studies, 28(2),* 151-169.
Davis, B., Sumara, D., & Luce-Kapler, R. (2000). *Engaging minds: Learning and teaching in a complex world.* New Jersey: Lawrence Erlbaum.
Dehaene, S., (1997). *The number sense: How the mind creates mathematics.* New York: Oxford University Press.
Dienes, Z. P., & Goldings E. W. (1971). *Approach to modern mathematics.* New York: Herder and Herder.
Doll, W. E. (2005). The culture of method. In W. E. Doll, M. J. Fleener, D. Trueit, and J. St. Julien (Eds.), *Chaos, complexity, curriculum and culture: A conversation.* New York: Lang. (pp. 21-76).
Dreyfus, T. (1990). Advanced mathematical thinking. In ICM study series, *Mathematics and cognition: a research synthesis by the international group for the psychology of mathematics education* (pp.113-134). New York: Cambridge University.
Driver, R., Asoko, H., Leach, J., Mortimer, E., & Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher*, 23(7), 5-12.
English, L. & Halford, G. (1995). *Mathematics education: Models and process.* New Jersey: Lawrence Erlbaum.
Ernest, P. (1994). Varieties of constructivism: Their metaphors, epistemologies and pedagogical implications. *Hiroshima Journal of Mathematical Education 2,* 1-14.
Ernest, P. (1999). Forms of knowledge in mathematics and mathematics education: Philosophical and rhetorical perspectives. *Educational studies in Mathematics, 38,* 67-83.
Freudenthal, H. (1991). *Revisiting Mathematics Education: China Lectures.* Dodrecht: Kluwer Academic.
Furinghetti, F. & Radford, L. (2002). Historical conceptual developments and the teaching of mathematics: from phylogenesis and ontogenesis theory to classroom practice. In: L. English (Ed.), Handbook of International Research in Mathematics Education (631-654). New Jersey: Lawrence Erlbaum.
Gattegno, C. (1970). *What we owe children: The subordination of teaching to learning*. New York: Avon Books.
Ginsburg, H. P. & Seo, K. H. (1999). Mathematics in children's thinking. *Mathematical Thinking and Learning, 1*(2), 113-129.
Gordon-Calvert, L.M (2001). Mathematical conversations within the practice of mathematics. New York: Peter Lang.
Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22(3),* 170-218.
Hadamard, J. (1996/1945). *The mathematician's mind: The psychology of invention in the mathematical field*. Princeton: Princeton University.
Hoy, C. D. (1991). Is hermeneutics ethnocentric? In D. R. Hiley, J. F. Bohman & R. Shusterman (Eds.) *The interpretive turn: Philosophy Science and culture* (pp. 155-175). London: Cornell University

Kieren, T. E. (1971). Manipulative activity in mathematics learning. *Journal for Research in Mathematics Education, 2(3)*, 228-234.
Kieren, T. E. (2000). Dichotomies or Binoculars: Reflections on the papers by Steffe and Thompson and by Lerman. *Journal for Research in Mathematics Education, 31*, 228 - 223.
Kieren, T., & Simmt, E. (2002). Fractal Filaments: A simile for observing collective mathematical understanding. In D. S. Mewborn, P. Sztajn, D. Y. White, H. G. Wiegel, R. L. Bryant & K. Nooney (Eds.), *Proceedings of the 24th annual meeting of the North American Chapter if the International Group for the Psychology of Mathematics Education, vol. 2. PME-NA XXIV, Athen*s, GA, pp. pp.865- 874).
Lakoff, G., & Núñez, R. E. (2001). *Where mathematics comes from: How the embodied mind brings mathematics into being.* New York: Basic Books.
Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: University press.
Lerman, S. (2000). A case of interpretations of social: A response to Steffe and Thompson. *Journal of Research in Mathematics Education, 31,* 210-227
Lerman, S. (1999). Culturally situated knowledge and the problem of transfer in learning mathematics. In L. Burton (Ed.), *Learning mathematics, from hierarchies to networks* (pp. 3-20). London: Falmer Press
Lerman, S. (2001). A cultural/discursive psychology for mathematics teaching and learning. In B. Atweh, H. Forgasz, and B. Nebress (Eds.) Social cultural research on mathematics education: An alternative perspective (pp. 3 -17). Mahwah: Lawrence Erlbaum
Namukasa, I. (2005). Attending in mathematics: A dynamic view about students' thinking. In E. Simmt & B. Davis (Eds.), Proceedings of the 2005 Annual Meeting of the Canadian Mathematics Education Study Group Meeting (pp 111-118). University of Ottawa, May 27-31.
Namukasa, I., & Simmt, E. (2003). Collective learning structures: Complexity science metaphors for teaching mathematics. In N. A. Pateman, B. J. Dougherty, & J. T. Zilliox (Eds.), *Proceedings of the 27th Conference of the International Group for the Psychology pf*
Núñez, R. E. (2000). *Mathematical idea analysis: What embodied Cognitive Science can say about the human nature of Mathematics*. In proceedings of the conference of the international Croup for the Psychology of Mathematics Education (PME), 24th, Hiroshima, Japan, July 23-27, Vol. 1. Also available as Eric Document ED 466 734.
Núñez, R. E., Edwards, L. D., & Matos, J. F. (1999). Embodied cognition as ground for situatedness and context in mathematics education. *Educational Studies in Mathematics, 39(1),* 45-65.
Polya, G. (1945/1973). *How to solve it: A new aspect of mathematics (2nd ed.*) New Jersey: Princeton University Press.
Radford, L. (2003). Gesture, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning, 5(1),* 37-70.
Rorty, R. M. (1982). Hermeneutics, general studies, and teaching. In Gras, V. W. (General Ed.), *Hermeneutics, general studies, and teaching* (pp 1- 15). Selected papers from Synergos Seminars. George Mason University, Fairfax, Virginia.
Safuanov, I. (2007). Genetiv approach to teaching geometry. In J-H. Woo, H-C. Lew, K-S. Park and D-Y. Seo (Eds.), *Proceedings of the 31th Conference of the International Group for the Psychology pf Mathematics Education, Seoul National University, Korea, vol. , pp.145-152*.
Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Shoenfeld (Ed.) *Mathematical thinking and problem solving* (pp. 53-69).
Sfard, A. (2001b). *Learning mathematics as developing discourse*. A paper presented at PME-NA XXXIII, Snowbird Utah vol.1, 23 – 41.
Simmt, E. (2000). *Mathematical knowing in action*. Unpublished doctoral dissertation, University of Alberta, Alberta.

Skovsmose, O. (2001). Mathematics in action: A challenge for social theorizing. In E. Simmt, & B. Davis (Eds.), *Proceedings of the 2001 Canadian Mathematics Education Study Group*, (pp3-17). University of Alberta.

Stanley, D. (2005). Paradigmatic complexity: emerging ideas and historical views of the complexity sciences. Doll, W. E. (2005). The culture of method. In W. E. Doll, M. J. Fleener, D. Trueit, and J. St. Julien (Eds.), *Chaos, complexity, curriculum and culture: A conversation.* New York: Lang. pp. 1333-151.

Steffe, L. P. (1970). Differential performance of first-grade children when solving arithmetic addition problems. *Journal for Research in Mathematics Education*, *1*(3), 144-161.

Steffe, L. P., & Kieren, K. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education, 25(6),* 711-733.

Steffe, L. P. & Thompson, P. W. (2000). Interaction or intersubjectivity? A reply to Lerman. *Journal for Research in Mathematics Education, 31*(2), 191-209

Steffe, L. P., & Wiegel, H. G. (1992). On reforming practice in mathematics education. *Educational studies in Mathematics Education, 23,* 445-465.

Towers, J. (1998). Telling Tales. Journal of Curriculum Theorizing, 14(3), 29-36.

Thorndike's (1970). The Psychology of Arithmetic. In J. K. Bidwell & R. G. Clason (Eds.) *Readings in the history of mathematics education* (pp. 461-477). Washington, DC: NCTM. (Original work published in 1924)

Towers, J., & Davis, B. (2002). Structuring Occasions. *Educational Studies in Mathematics, 49,* 313-340.

van Oers, B. (2001). Education forms of initiation in mathematical culture. *Educational Studies in Mathematics, 46,* 59-85.

Voigt, J. (1994). Negotiations of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26,* 275-29

von Glasersfeld, E. (1995). *Radical constructivism a way of knowing.* A series in studies in mathematics education. London: Falmer.

von Glasersfeld, E., & Steffe, L. P. (1991). Conceptual models in education research and practice. *Journal of Educational thought, 25(2),* 91-103.

Walkerdine, V. (1990). Difference, Cognition, and Mathematics education. *For the Learning of Mathematics, 10(3),* 51-55.