From producing optimal teaching to analysing usual classroom situations. Development of a fundamental concept in the theory of didactic situations: the notion of *milieu*

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Introduction

In the last ICME meeting in summer 2004, I took part in DG10, in a panel where panellists were invited to provide, from their own experience, examples of fashion waves or continuity in theoretical options. Most of them witnessed of shifts in their experience; for my part, in particular referring to the theory of didactic situations (for short TDS), I saw my own theoretical background as a process of enrichment of theories in order to take into account more and more of the classroom complexity. Perhaps, there was a shift or rather an extension in the construction of TDS, but it was a long time ago, about 1980. Indeed, the first versions of the theory, during the seventies, modelled what was called later "adidactical situations", taking no real account of the teacher's action in class. But Brousseau was himself a teacher and his theory could not ignore for a long time the importance of the teacher's action in class: with the concepts of institutionalisation and devolution, the teacher came into the theory. Thus, since about forty years, Guy Brousseau develops and makes more accurate and operative theoretical constructs, most of them existing implicitly or explicitly from the beginning in the model of TDS but not always "visible" enough for other researchers. It is the case for the notion of *milieu*, now considered as fundamental by all researchers referring to this theory. The development of the theory makes it more suitable for addressing questions nearer and nearer the teacher's actual work in relation with mathematics knowledge and students' learning. I will try to explain, from my own perspective, some aspects of this concept, enlightened by a look on its historical development and its epistemological background. For more information, the reader may refer to Brousseau (1997), Herbst & Kilpatrick (1999), Perrin-Glorian (1994, 1999), Salin (2001), and Warfield (2006).

1. A systemic approach focused on the dynamics of didactic situations

First, it is very important to understand that the theory consists in a systemic approach, focused on the didactic relation, nor the teacher, nor the student, neither the mathematical content itself, but the three at the same time, the famous didactic triangle. Many theories in mathematics education consider this triangle but focus on one term or relations between two terms. The first hypothesis behind TDS from my opinion is that we cannot separate the three terms: the focus is on the conditions allowing a didactic system (e.g. the teacher, but it may be some institution bearing an intention to teach some knowledge to somebody) to obtain, using a reasonable time, learning of knowledge considered as useful (by a culture, a society, an institution), by students who did not always decide by themselves to learn it and did not always see this usefulness.
Another fundamental hypothesis is that some pieces of knowledge cannot be transmitted only by explaining them; there are pieces of knowledge that cannot be transmitted in a definitive form and need to be learnt through different contexts, reviewed several times with different senses along school life. It is the case, for instance, for the successive extensions of the concept of number. For this kind of knowledge, the theory considers learning and teaching as dynamics orchestrated by the teacher aiming at students' knowledge growth. This entails that "a learning process can be characterized by a sequence of reproducible *situations*[1] that lead to the students' learning of a particular piece of knowledge, or more concretely to a set of modifications of the students' behaviors which characterize the acquisition of that piece of knowledge." (Brousseau 1975, translated by Warfield[2], 2006). Thus, the theory plays on two

faces of knowledge: its usefulness to solve problems in specific contexts, its universal nature as integrated in an organisation of mathematical knowledge.
These two hypotheses explain why Brousseau referred to the theory of games to elaborate the TDS, with the long term project to create a mathematical model of teaching and learning. There is a stake: students' knowledge growth for some specific mathematical content; there are players: the students and the teacher. In the classroom some different games have to be played, each of them with some specific stake linked to a specific piece of knowledge.
A third important hypothesis, from my opinion, is that the teaching relationship will stop and students have to be able to use knowledge out of the didactic system. Thus learning has to be thought on a long term and with some part of autonomy for the student.
Moreover, the construction of the theory rests on a large experimental design, in the long term, in a school fitted out for observation and research, but submitted to the regular curriculum. Thus, from the beginning, the theory develops with a methodology, didactical engineering, facing the complexity of classroom and with care of concrete questions about teaching and learning mathematics.

2. The notion of *milieu*, a fundamental concept in the theory of didactical situations

The concept of *milieu* is present in the theory from the beginning: the student is supposed to learn by adaptation to a *milieu*; but this concept evolves, grows and becomes more precise along years. As soon as 1977, the teacher is distinguished from the *milieu*: *"il s'agit de décrire les interactions entre 3 régulateurs, le maître (Ma), l'élève (E), le milieu (Mi) à propos d'un système de connaissance C. Les interactions de base sont celles de l'élève avec le milieu."* Thus there are not only three but four systems in interaction: the *milieu* is distinguished from actors, teacher or students: actors may act on the *milieu* or receive information from the *milieu*. Moreover, the theory distinguishes "*savoirs*" (mathematics knowledge) and "*connaissances*" (knowledge to take decisions): if the student learns by adaptation to the *milieu*, the teacher has to organise the *milieu* so that the knowledge produced by this adaptation *(connaissances)* may be recognised as the knowledge to be learnt *(savoir)*.
In a conference in Mexico, Brousseau (2000) explains that teaching is an activity needing to conciliate two processes: acculturation and independent adaptation. Identifying on the one hand the student and the learning subject and, on the other hand *"savoirs"* (target mathematical knowledge) and *"connaissances"* (knowledge developed by action on the *milieu*), he proposes then a four poles diagram (figure 1) to represent these two processes.

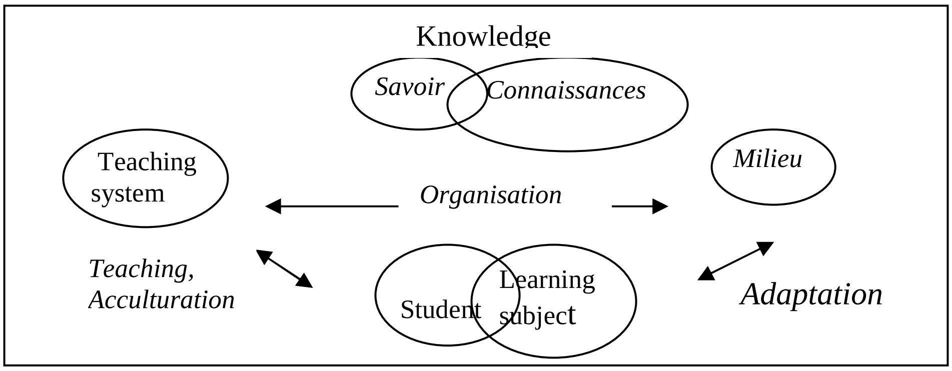

Figure 1

Moreover, he claims that the fourth system, the *milieu*, is the most important to study in order to understand how the student can learn in a didactic system: the students' behaviour reveals how the *milieu* works; the "black box" is the *milieu*.
The *situation* models the interaction of a subject with a *milieu* by a game (e.g. a problem to solve) where players have to take decisions: some states of the game are more favourable than others to win; thus the *situation* defines a piece of knowledge as a means for the subject to reach or maintain a favourable state (for the game) in this *milieu*.

During the seventies, the model develops mainly the right part of this schema, what will be called later (1982) *adidactic situation*, corresponding to an adaptation to a *milieu*, in accordance with Piaget's theory; however the milieu is not a natural *milieu*; it is organised in order to provoke a specific knowledge by adaptation. As soon as this time, he identified three kinds of *situations*, or dialectics with the *milieu*: *action* (to win), *formulation* (some communication between players is needed to win), *validation* (an argumentation is needed). In this first development of the theory, the teacher's role is mainly to organise the *milieu* so that the knowledge to win be the knowledge to be learnt and the prior knowledge of students may help them to play the game and interpret the feedback of the *milieu*. These conditions can be expressed by three constraints on the *milieu* (Salin, 2002): 1) to provoke contradictions, difficulties for the students so that they have to adapt their knowledge; 2) to allow them to work autonomously; 3) to help them to learn some specific mathematical content (by learning to win the game).

During the eighties, the teacher enters more explicitly in the theory: in order that the students' game with the *milieu* can work satisfactorily, the teacher himself has to play a game with two complementary stakes: *devolution* so that the student plays the game to win and not to please the teacher, and *institutionalisation*, to help the student to recognise the knowledge gained in the game and to transform it in knowledge usable to solve other problems. At the same time, Brousseau identified the *didactic contract* and the word "*situation*" takes a second sense, thus he introduces a distinction between *didactic situation* including the teacher and the *didactic contract*, and *adidactic situation* deprived from its didactic intentions. Theses concepts are introduced between 1980 and 1982 but their translation in a structured view of the *milieu* is only explicit in 1986. We will come back on this issue after clarifying two aspects of the notion of *milieu*.

3. Different scales in the concept of *milieu*

One of the difficulties to understand the concept of *milieu* comes from the fact that it recovers two different aspects, complementary for the theory: the first one, related to the notion of *fundamental situation*, corresponding mainly to an epistemological analysis of knowledge and the second one to understanding the action of students and teacher in class.

The notion of fundamental situation.

The *fundamental situation* corresponds to the search of a *milieu* or a little set of *milieus* able to provoke the learning of some key piece of mathematical knowledge. Salin (2002) calls them "*milieux viviers*" (breeding ground). It is not a situation directly for classroom but it is a set of conditions defining all (or most) such possible situations, including classic ones, to learn the target knowledge. Such a *milieu* is represented by a model problem and didactic variables of this problem such that the values of these variables can generate all the problems of this family. Of course, the search of a *fundamental situation* has first an epistemological dimension: the problem must be representative of most aspects of the target knowledge. It is a very strong hypothesis to suppose that it is possible to find such a problem (or a small number of such problems) to represent key pieces of mathematical knowledge. Moreover an epistemological perspective is not sufficient in didactics: there are also conditions such that students can understand the problem and imagine what could be a solution with their previous knowledge. Such *milieus* are very difficult to find but the mere search of them is very productive from a didactic perspective. Warfield (2006) develops the example of statistics; Berthelot & Salin (1998) explore such *milieus* to teach geometry as a model of space and I am now studying myself geometrical drawing with usual tools (ruler, set square, compass…) as a *milieu* to learn geometry in primary school.

Let us notice however that, even taking into account the cognitive perspective, it is not yet enough for a classroom situation: we must consider also curriculum, time available…

Vertical structure of the milieu in a didactic situation.

Another aspect is the structure of the *milieu*, introduced by Brousseau at the end of the eighties and developed later by other researchers. This structure explains how the student may learn from his action on the *milieu* and how the teacher may regulate this action and this learning. Like the three dialectics (action, formulation, and validation) are embedded one inside the other, the different levels of *milieu* are embedded one inside the other, a *situation* at one level becoming a *milieu* for a *situation* at the next higher level: action at an upper level supposes reflection on the previous level (figure 2)[3].

M1 didactic milieu	E1 universal subject	P1 teacher preparing the class	S1 Metadidactic Situation
M0 learning milieu	**E0 Student**	**P0 Teacher**	**S0 Didactic Situation**
M-1 reference milieu	E-1 epistemic subject		S-1 Learning Situation
M-2 objective milieu	E-2 acting subject		S-2 Reference Situation
M-3 material milieu	E-3 objective actors		S-3 Objective Situation

Figure 2

Those levels must not be seen as successive but simultaneous: they correspond to positions the teacher or the students may take. At the level M-3, there is no didactic intention; objective actors act in a material *milieu*, this action will be the object of the problematic *situation* S-2; E-2 is the student acting with his prior knowledge, he has to understand the rules of the game (possible states and final state to reach) and to play; E-1 is the student reflecting on his action and learning: he has to elaborate a strategy to win.

Let us notice that a game may be an individual game or a game with several actors, cooperating (for instance in a *situation* of formulation) or playing one against another. Thus some social interactions are considered in the *milieu* and in the model of didactic *situations*, the ones having an effect on the knowledge involved to solve the problem (or to win).

4. What may be the contribution of this theory to analyse regular lessons?

Up to now we considered theoretical *situations;* we can imagine that such a model gives means to product classroom situations trying to fulfil the conditions and to analyse them but may this model be used to analyse regular lessons prepared by a teacher without any reference to the model? How to use it such that regular teaching does not seem only unsatisfactory?

Some researchers tried to do (e.g. Hersant & Perrin-Glorian, 2005). The first important issue is to identify the target knowledge (it is not always explicit and not always the one expressed by the teacher) and how it appears in the problem to solve. The second one is to identify what could be the *milieu*: data and all actual givens usable by students without any intervention of the teacher. The third one is to identify prior knowledge of students to foresee actions students may undertake on this *milieu* and how they could interpret feedback coming from it. Doing this, we can elaborate an *a priori* analysis of the class situation (even yet carried out). This *a priori* analysis helps for instance to identify some possible insufficiencies in the *milieu*, some issues on which the teacher have to give himself a feedback in case of errors of students. However, actual knowledge of students may be different from the one expected by the teacher. Thus they may be unable to interpret some feedback of the *milieu* to invalidate their action: only a part of the *milieu* is activated for some students. Other causes may intervene without any relation with mathematical knowledge; the theory does not take theses causes into account even if, obviously, they may have a very important effect on students' learning.

5. Concluding remarks

Some recent research works use the notion of *milieu* to analyse teacher's learning through teaching Margolinas et al., 2005). They add positions for teacher (e.g. as observer) in the lower levels and extend the model of *milieu* in the upper levels to take into account interactions of teachers out of the class, inside professional world.

This analysis considers that the teacher is in a natural situation (no-didactic), interacting with a double *milieu*: the first one coming from lower levels, linked to his experience in class and students' work; the other one coming from upper levels: his contacts with professional world. It shows how the teacher may learn from his class practice and it helps to draw up conditions for that. This kind of analysis, in terms of relationships to mathematical knowledge and to students' knowledge may be compatible and articulated with other analyses of the teacher's role from psychological or social perspectives. This articulation is of real importance for research in mathematics education.
By way of conclusion, I would also say that TDS is quite compatible with Vygotski's theories and with most research works about social interactions. For instance, the ZPD may be put in relation with the articulation between an epistemological analysis of knowledge to teach and an analysis of prior knowledge of students in order to elaborate a *milieu*; social interactions between students are considered: a situation is not only a problem but it includes also an organisation of students' work on this problem. Moreover, the TDS don't entail that the teacher do not intervene in students' work: it gives means to recognise some different functions in the teacher's interventions. In the devolution game, the teacher encourages students, focuses them on the target problem, and helps them to avoid dispersion in too far directions, especially if the *milieu* cannot give a sufficient feedback. In the institutionalisation game, the teacher gives information, helps students to give a status to knowledge involved to solve the problem and to place them in cultural knowledge among previous knowledge. The theory and especially the notion of milieu, helps to anticipate what part of knowledge may be produced by students, what part will stay in the charge of the teacher.

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[1] From now, except in titles and tables, I will use italics for the term situation in the model.
[2] Warfield (2006) gives a good introduction to this theory, with several detailed examples, including the historical one of "race to 20" about which Brousseau clarified the three dialectics in 1970.
[3] I use here a presentation in a table proposed by Margolinas in 1993 and used by many researchers from then but I fill only the boxes identified by Brousseau in 1986.