NOTES ON THE AESTHETIC DIMENSION OF MATHEMATICS EDUCATION

Nathalie Sinclair
Simon Fraser University

Some of the most important changes in mathematics education over the past century have evolved in response to perceptions of the unacceptable differences between mathematics, as conceived of and practiced by research mathematicians, and school mathematics, which is embodied in textbooks and classrooms. For example, Pólya's work inspired many mathematics educators to lessen the gap between mathematics and school mathematics by infusing the latter with "problem solving," which Pólya saw as the defining activity of the research mathematician. A few decades after that, Brown and Walters persuasively argued that posing problems was just as important–and sometimes more so—as solving problems, and called for increased opportunities for student problem posing. Both activities (posing and solving problems) were meant to attenuate differences, but they also offered an attractive humanistic dimension to the world of the mathematics learner. Of course, there remain many differences between mathematics and school mathematics that few would ever consider eliminating—for example, no one has ever suggested that students should publish new research results or review articles for publication.

The aesthetic dimension of mathematics has also remained more or less on the side of the research mathematicians, though with intermittent forays into the school mathematics. In one sense, it seems natural to leave it in its place, given its tight connection with published mathematics, that is, with the production of theorems and proofs that characterize the mathematician's work. We know, from the writings of a few great mathematicians, that proofs should be elegant (Gauss, 1865), and that (good) mathematicians have a special aesthetic sensibility (Poincaré, 1956). We are even told that the primary goals of the mathematician are aesthetic, and not epistemological (Krull, 1930/1987).

It may be tempting to leave aesthetic with the mathematicians, but it would also be wrong, and this for two reasons—neither of which has to do with attenuating differences. First, mathematics education is one of the few disciplines that can (and sometimes does) play an interpretive role in explaining the nature, purpose and goals of mathematics to the 'outside' world, which are infused with aesthetic values. This interpretive role is crucial to the health of mathematics as a discipline (see Corfield, 2003) and to the monitoring of the impact of mathematics on society (see Illich, 1994). Second, and more closely related to immediate pedagogical issues, it can be (and has been) argued that the aesthetic is deeply connected to learning, and not just to artistic production or appreciation. I therefore propose a stronger two-way interaction in which the discipline of aesthetics has much to contribute both to understanding the rationality of mathematics itself, and to enriching existing theories in mathematics education.

Mathematics Educators as Mathematics Critics

In Lakatos' (1976) *Proofs and Refutations*, Gamma and Alpha are discussing the question of how far they are willing to go in providing a generalization of 'Euler's formula.' Gamma notes that it's a question of what's interesting or exciting. Alpha responds that interest is a matter of taste. So Gamma asks "Why not have mathematical critics just as you have literary critics, to develop mathematical taste by public criticism?" (p. 98). Indeed, it is difficult to identify the arbiters of taste in mathematics, even though questions of taste are central to the growth of the discipline. They are central for the simple reason that mathematicians must choose their objects of study; they are not somehow dictated by the real world (as is the case with physics). Further, as Steiner (1998) points out, "There is no objective criterion for a structure to be mathematics—and not every structure counts as a mathematical structure" (p. 6). Whereas the criteria of the past may have included utility, contemporary mathematicians have adopted other criteria, including beauty. Hardy (1940) spoke to this in his famous quotation "Beauty is the first test: There is no permanent place in the world for ugly mathematics" (p. 85). Indeed Euler's formula is a structure worth attending to because of its beauty, that is, its simplicity and generativity. Csiszar (2003) notes that "aesthetic sensibility becomes a measure of value itself" (p. 251).

Aesthetics are involved not only in the choice of structures (and, thus, the choice of mathematical objects, relationships and problems), but also in the communication of results about these structures. Style is crucial in mathematical communication, as can be seen in the various guides provided by journals and professional societies (see AMS (1990)). Mathematicians such as Krull (1930/1987) claim, for example, that the rhetorical nature of mathematical writing depends strongly on aesthetic devices: "Mathematicians are not concerned merely with finding and proving theorems, they also want to arrange and assemble the theorems so that they appear not only correct but evident and compelling. Such a goal, I feel, is aesthetic rather than epistemological" (p. 49).

Given the importance of aesthetics, Gamma rightly wonders how taste is defined and communicated in mathematics. In the world of literature, writers produce the works of art, but literary critics assess and analyse these works in a way that is interpretive, formative and summative. One might think that mathematicians themselves take on this role, but they rarely do, and they probably shouldn't—such questions, which relate to the image of mathematics in society, cannot be decided within the body of mathematics, which is what mathematicians concern themselves with (see Corry, 2006). Yet, there are few people who can fill the role of the mathematical critic in part because mathematical communication is designed to be exclusive (Burton and Morgan, 2000; Csiszar, 2003).

In the past, philosophers have taken played the role of the mathematics critics, but this is no longer the case (Corfield, 2003). There's a sense in which mathematics educators are perhaps best positioned to replace the philosopher. Unlike philosophers, mathematics educators have a natural entry into the body of mathematics. But more importantly, mathematics educators are extremely sensitive to certain aspects of the interface between body and image: the cultural narratives about mathematics, the historical and epistemological aspects of mathematics, and the demarcations between mathematics and school mathematics.

Aesthetic Perspective Informing Mathematics Education

Enzensberger (1998) points out that the mathematics has a certain archaic aesthetic sensibility that has changed very little in the past two thousand years, one that speaks more to the Baroque or the Classical than to any more contemporary or post-modern sensibilities. Le Lionnais (1948/1986) wouldn't agree. He sees at least two aesthetic preferences expressed in mathematics. The classical, on the one hand, which tends toward harmony and order, and the romantic, on the other, which seeks the bizarre and the extreme. Whatever the case may be, recent work in a wide variety of disciplines suggests that the particular tendencies one has are much less important than the fact that one has aesthetic tendencies at all, and that one uses these pervasively and centrally in making sense of the world. Using anthropological insights, Dissanayake (1995) writes convincingly about the satisfactions that come from the successful manifestation of the basic aesthetic motivation of "making special". Other scholars interested more in a cognitive science approach, such as Wilson (1998), argue that humans have predictable, innate aesthetic preferences they use in making sense of their environments.

Closer to education, and from a more philosophical perspective, Dewey (1933) argues that the aesthetic is a pervading quality of human reasoning and experience. He begins his argument by pointing out that the categories used to characterise and communicate different modes of reasoning and experience (i.e. emotional, intellectual, aesthetic) may make our descriptions and analyses of them more tractable, but that these categories impose artificial boundaries and dualisms that do not exist. In other words, not only is it not sufficient enough to focus on either the cognitive or the affective in our study of mathematical activity, but one must also strive to understand the way in which the cognitive, affective and aesthetic intermingle at the most basic level in human inquiry.

The Deweyian approach to aesthetics cares less about whether or not a given theorem is elegant, or what criteria of mathematical beauty might look like. Instead, such an approach focuses on the role that the aesthetic plays in human inquiry. As I have proposed elsewhere (see Sinclair, 2006b), the aesthetic plays a motivational, generative and evaluative role in mathematical inquiry in the sense that it affects the problems mathematicians pose, the process through which they go about solving those problems, and the way they communicate their results. To the extent that school mathematics engages in the kind of inquiry found in research mathematics, it seems possible that the aesthetic can play similar roles for students (see Sinclair, 2006a). It thus appears that the close relationship between aesthetics and inquiry may further inform current theories of problem solving and problem posing.

However, if we stand back from the role of the aesthetic in research mathematics, and focus more on mathematics learning, there seem to be some fruitful connections to make with existing theories and frameworks in mathematics education. One example relates to Goldin's (2000) work on the affective domain of mathematics education. As I have tried to show (see Sinclair, 2004), it is possible to take the elements of the affective domain (emotions, attitudes, beliefs, and values) as highly intertwined with aesthetic responses. Indeed, an aesthetic lens helps to illuminate the way in which emotion and cognition work together in the learning process. Aesthetics thus seems very pertinent to important questions in mathematics education regarding motivation and anxiety.

Another arena in which the aesthetic considerations might help inform mathematics education theories relates to Cobb and Yackel's (1996) work on the development of socio-mathematical norms in the classroom. They conceptualize this development in terms of the ways in which students learn what is mathematically different, sophisticated, efficient and elegant. Although they focus primarily on norms around difference, the norms of efficiency and elegance quite distinctly involve aesthetic values. If students are going to be comparing solutions, and if the teacher has to mediate the discussion around this comparison, it will be difficult to avoid aesthetic values, especially when the solutions are all correct. At the moment, however, mathematics educators do not seem well-equipped to handle issues of aesthetics within school mathematics, whether it's in terms of students comparing solutions, students posing problems, teachers posing problems or teachers assessing student work. One reason for this lacuna in current theories in mathematics education may be related to the long-held belief (especially among mathematicians) that students are unable to appreciate beauty and elegance in mathematics (see Dreyfus and Eisenberg, 1986). Such beliefs may unnecessarily constrain both the nature and role of aesthetics in human thinking.

I conclude by pointing to a number of questions that emerge in considering the aesthetic dimension of student learning and the presence of aesthetics in theoretical developments in mathematics education. These questions span a wide conception of aesthetics that include the aesthetic as a theme in human experience (as a way that human organize and derive meaning from everyday situations) and the aesthetic as a field of study, which includes the nature of perceptually interesting aspects of phenomena and artifacts.

(1) How might (and should) aesthetic considerations in mathematics differ from those in school mathematics?
(2) How deeply is aesthetic sensibility tied to mathematical motivation and curiosity?
(3) Can aesthetic sensibility be taught? If so, how?
(4) How do theories of embodied cognition relate to aesthetic perception?
(5) How can mathematicians help educators gain entry into the aesthetic values that guide their work?

References

American Mathematical Society. (1990). *A Manual for Authors of Mathematical Papers.* Providence, RI: American Mathematical Society

Brown, S. I. & Walter, M. I. (1983). *The art of problem posing*. Hillsdale, NJ: Lawrence Erlbaum Associates
Burton, L. and Morgan, C. (2000). Mathematicians Writing. Journal for Research in Mathematics Education, 31(4), 429-453.
Corfield, D. (2003). *Towards a philosophy of real mathematics*. Cambridge University Press.
Corry L. (2006). Axiomatics, Empiricism, and Anschauung in Hilbert´s Conception of Geometry: Between Arithmetic and General Relativity, in: J. Gray and J. Ferreirós (eds.) *The Architecture of Modern Mathematics: Essays in History and Philosophy*, Oxford: Oxford University Press.
Csiszar, A. (2003). Stylizing rigor; or, Why mathematicians write so well. *Configuration,* 11, 239 – 268.
Dewey, J. (1934). *Art as experience*. New York: Perigree
Dreyfus, T. and Eisenberg, T. (1986). On the aesthetics of mathematical thought. *For the Learning of Mathematics* 6(1), 2-10.
Gauss, C. (1863) *Werke* (Schering, E., ed.), II, Leipzig-Berlin, Königliche Gesellschaft der Wissenschaften zu Göttingen.
Goldin, G. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning, 2*, 209–219
Hardy, G. H. (1940). *A mathematician's apology*. Cambridge, UK: Cambridge University Press.
Illich, I. (1994) *The Wisdom of Leopold Kohr,* Fourteenth Annual E. F. Schumacher Lectures, New Haven, CT, Yale University. (Available from: www.smallisbeautiful.org/lec-ill.html.)
Krull, W. (1987).The aesthetic viewpoint in mathematics. *The Mathematical Intelligencer, 9*(1),48–52.
Lakatos, I. (1976). *Proofs and refutation: The logic of mathematical discovery*. Cambridge: Cambridge University Press.
Le Lionnais, F. (1948/1986). La beauté en mathématiques. In F. Le Lionnais (Ed.), *Les grands courants de la pensée mathématique* (pp. 437–465). Paris: Editions Rivages.
Pimm, D. (2006). Drawing on the Image in Mathematics and Art. In N. Sinclair, D. Pimm and W. Higginson (Eds.). *Mathematics and the aesthetic: New approaches to an ancient affinity* (pp. 160-189). New York: Springer.
Poincaré, H. (1956). Mathematical creation. In J. Newman (Ed.), *The world of mathematics* (pp. 2041–2050). New York: Simon & Schuster. (Original work published in 1908)
Pólya, G. (1957). *How to solve it.* Garden City, NY: Doubleday.
Rota, G. (1997). *Indiscrete thoughts.* Boston: Birkhauser.
Sinclair, N. (2004). The roles of the aesthetic in mathematical inquiry, *Mathematical Thinking and Learning*, *6*(3), 261-284.
Sinclair, N. (2006a). *Mathematics and beauty: Aesthetic approaches to teaching children*. Teachers College Press.
Sinclair, N. (2006b). The aesthetic sensibilities of mathematicians. In N. Sinclair, D. Pimm and W. Higginson (Eds.). *Mathematics and the aesthetic: New approaches to an ancient affinity* (pp. 87-104). New York: Springer.

Steiner, M. (1998). *The applicability of mathematics as a philosophical problem*. Cambridge: Harvard University Press.

Tymoczko, T. (1993). Value judgements in mathematics: Can we treat mathematics as an art? In A. M. White (Ed.), *Essays in humanistic mathematics* (pp. 67–77). Washington, DC: MAA.

Wilson, E. (1998) *Consilience: the Unity of Knowledge*, New York, NY, Knopf.

Yackel, E., and Cobb, P. (1996). Sociomathematical norms, argumentations and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27,* 458–47.