Summary and comments to my list of publications

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[1] G. N. Arzhantseva, M. Bridson, T. Januszkiewicz, I. Leary, A. Minasyan, J. Swiatkowski, *Infinite groups with fixed point properties*, Geometry & Topology, (2009), to appear.

We construct finitely generated groups with strong fixed point properties. Let $\mathcal{X}_{ac}$ be the class of Hausdorff spaces of finite covering dimension which are mod-p acyclic for at least one prime p. We produce the first examples of infinite finitely generated groups Q with the property that for any action of Q on any $X \in \mathcal{X}_{ac}$, there is a global fixed point. Moreover, Q may be chosen to be simple and to have Kazhdan's property (T). We construct a finitely presented infinite group P that admits no non-trivial action by diffeomorphisms on any smooth manifold in $\mathcal{X}_{ac}$. In building Q, we exhibit new families of hyperbolic groups: for each $n \geq 1$ and each prime p, we construct a non-elementary hyperbolic group $G_{n,p}$ which has a generating set of size $n + 2$, any proper subset of which generates a finite p-group.

[2] G. N. Arzhantseva, C. Druţu, and M. Sapir, *Compression functions of uniform embeddings of groups into Hilbert and Banach spaces*, Journal für die Reine und Angewandte Mathematik, [Crelle's Journal], (2008), in press.

We construct finitely generated groups with arbitrary prescribed Hilbert space compression $\alpha \in [0, 1]$. This answers a question of E. Guentner and G. Niblo. For a large class of Banach spaces $\mathcal{E}$ (including all uniformly convex Banach spaces), the $\mathcal{E}$–compression of these groups coincides with their Hilbert space compression. Moreover, the groups that we construct have asymptotic dimension at most 2, hence they are exact. In particular, the first examples of groups that are uniformly embeddable into a Hilbert space (moreover, of finite asymptotic dimension and exact) with Hilbert space compression 0 are given. These groups are also the first examples of groups with uniformly convex Banach space compression 0.

[3] G. N. Arzhantseva, V. S. Guba, M. Lustig and J.-Ph. Préaux, *Testing Cayley graph densities*, Annales mathematiques Blaise Pascal, (2008), in press.

We present a computer-assisted analysis of combinatorial properties of the Cayley graphs of certain finitely generated groups: Given a group with a finite set of generators, we study the density of the corresponding Cayley graph, that is, the least upper bound for the average vertex degree (= number of adjacent edges) of any finite subgraph. It is known that an m-generated group is amenable if and only if the density of the corresponding Cayley graph equals to $2m$. We test amenable and non-amenable groups, and also groups for which amenability is unknown. In the latter class we focus on Richard Thompson's group F.

[4] G. N. Arzhantseva, A. Minasyan and D. Osin, *The SQ-universality and residual properties of relatively hyperbolic groups*, Journal of Algebra, **315**(1) (2007), 165–177.

We apply methods of generalized small cancellation theory to study the residual properties of relatively hyperbolic groups. In particular, we prove that if a group G is non–elementary and hyperbolic relative to a collection of proper subgroups, then G is SQ–universal. We apply our method to show that if G is finitely presented then it possesses a lot of finitely presented quotients having "monster"-like group-theoretic properties.

[5] G. N. Arzhantseva, Z. Šunić, *Construction of elements in the closure of Grigorchuk group*, Geometriae Dedicata, **124**(1) (2007), 17–26.

The first Grigorchuk group has many remarkable properties (e.g. it is a finitely generated infinite torsion group; it is amenable but not elementary amenable and has intermediate growth). This group can be viewed as a group of automorphisms of the binary rooted tree $\mathcal{T}$. We describe constraints that need to be satisfied "near the top" of the portraits of the elements in Grigorchuk group. These constraints, if satisfied by the portraits of all sections of some binary tree automorphism, guarantee that this automotphism belongs to the closure of Grigorchuk group in the pro-finite group of binary tree automorphisms. This gives an effective way to construct all elements of the closure. This answers a question of Grigorchuk. We also build elements in the closure that do not belong to the group.

[6] G. N. Arzhantseva, A. Minasyan, *Relatively hyperbolic groups are C^*-simples*, Journal of Functional Analysis, **243**(1), (2007), 345–351.

A countable group G is C^*-simple if its reduced C^*-algebra is simple. The C^*-simplicity can be regarded as a strong form of non-amenability. In 1975 Powers established the C^*-simplicity of non-abelian free groups. Later, many other examples of C^*-simple groups where found in geometry and in group theory. In this paper, we characterize relatively hyperbolic groups whose reduced C^*-algebra is simple as those, which have no non-trivial finite normal subgroups. More precisely, we show that in a relatively hyperbolic group G without finite normal subgroups, given any finite subset F, there exists $g_0 \in G$ of infinite order such that, for any $f \in F$, the subgroup of G generated by g_0 and f is isomorphic to the free product of the cyclic subgroups $\langle g_0 \rangle$ and $\langle f \rangle$. An interesting corollary of the main result is that the amenable radical of non-elementary properly relatively hyperbolic group coinsides with it maximal finite normal subgroup $E(G)$ and the quotient $G/E(G)$ is C^*-simple with a unique normalized trace.

Every non-elementary Gromov hyperbolic group is relatively hyperbolic with respect to the family consisting of the trivial subgroup. Therefore our result also describes all C^*-simple Gromov hyperbolic groups.

[7] G. N. Arzhantseva, P. de la Harpe and D. Kahrobaei, *The true prosoluble completion of agroup: examples and open problems,* Geometriae Dedicata, **124**(1) (2007), 5–17.

The true prosoluble completion $P\mathcal{S}(\Gamma)$ of a group Γ is the inverse limit of the projective system of soluble quotients of Γ. Our purpose is to describe examples and to point out some natural open problems. We discuss a question of Grothendieck for profinite completions and its analogue for true prosoluble and true pronilpotent completions.

[8] G. N. Arzhantseva, V. S. Guba, M. V. Sapir, *Metrics on diagram groups and uniform embeddings in a Hilbert space*, Commentarii Mathematici Helvetici, **81**(4) (2006), 911–929.

We give first examples of finitely generated groups having an intermediate, with values in $(0, 1)$, Hilbert space compression (which is a numerical parameter measuring the distortion required to embed a metric space into Hilbert space). These groups include certain diagram groups. In particular, we show that the Hilbert space compression of Richard Thompson's group F is equal to 1/2, the Hilbert space compression of $\mathbb{Z} \wr \mathbb{Z}$ is between 1/2 and 3/4, and the Hilbert space compression of $\mathbb{Z} \wr (\mathbb{Z} \wr \mathbb{Z})$ is between 0 and 1/2. In general, we find a relationship between the growth of H and the Hilbert space compression of $\mathbb{Z} \wr H$.

[9] G. N. Arzhantseva, *A dichotomy for subgroups of word hyperbolic groups,* Contemporary Mathematics, **394**, Amer. Math. Soc., Providence, RI, 2006, 1-11.

Let H be a subgroup of a word hyperbolic group G. Our main result gives a sufficient condition for H to be free and quasiconvex in G. It is an improvement of a result due to Gromov (see [5.3.A] of "Hyperbolic groups").

Given $L > 0$ elements in a word hyperbolic group G, there exists a number $M = M(G, L) > 0$ such that at least one of the assertions is true: (i) these elements generate a free and quasiconvex subgroup of G; (ii) they are Nielsen equivalent to a system of L elements containing an element of length at most M up to conjugation in G.

The constant M is given explicitly. The result is generalized to groups acting by isometries on Gromov hyperbolic spaces. For proof we use a graph method to represent finitely generated subgroups of a group. The technique is of independent interest. In particular, transformations of a labelled graph defined in the paper can be viewed as a generalization of free reductions and Nielsen reductions of tuples of group elements.

[10] G. N. Arzhantseva and I. G. Lysenok, *A lower bound on the growth of word hyperbolic groups,* Journal of the London Mathematical Society, (2) **73** (2006), 109-125.

Apart from several simple examples, no classes of groups are known for which the exponential growth rate achieves its infimum on some generating set. In this paper, we try to do a step towards the proof of the conjecture that, for non-elementary word hyperbolic groups the infimum of the exponential growth rate is achieved on some of its generating sets. Namely, we give a linear lower bound on the exponential growth rate of a non-elementary subgroup of a word hyperbolic group, with respect to the number of generators for the subgroup. As an immediate consequence of this we get another results (see Corollary and Theorem 2) which restrict generating sets to check the property of a group to reach its uniform exponential growth rate.

As observed in [13], an affirmative answer to the conjecture would imply that a non-elementary word hyperbolic group is Hopfian. Note that word hyperbolic groups which are *torsion free* are known to be Hopfian by a result of Z. Sela.

[11] G. N. Arzhantseva, V. S. Guba, L. Guyot, *Growth rates of amenable groups,* Journal of Group Theory, **8** (2005), no. 3, 389–394.

Let F_m be a free group with m generators and let R be its normal subgroup such that F_m/R projects onto $\mathbb{Z}$. We give a lower bound for the growth rate of the group F_m/R' (where R' is the derived subgroup of R) in terms of the length $\rho = \rho(R)$ of the shortest nontrivial relation in R. It follows that the growth rate of F_m/R' approaches $2m - 1$ as ρ approaches infinity. This implies that the growth rate of an m-generated amenable group can be arbitrarily close to the maximum value $2m - 1$. This answers question 15.4 by P. de la Harpe from the "Kourovka Notebook" (Unsolved problems in group theory). In fact we prove that such groups can be found already in the class of abelian-by-nilpotent groups as well as in the class of finite extensions of metabelian groups.

[12] G. N. Arzhantseva, J. Burillo, M. Lustig, L. Reeves, H. Short, E. Ventura, *Uniform non-amenability,* Advances in Mathematics, **197** (2005), no. 2, 499–522.

For any finitely generated group G an invariant $\text{Føl}\, G \geq 0$ is introduced which measures the "amount of non-amenability" of G. If G is amenable, then $\text{Føl}\, G = 0$. If $\text{Føl}\, G > 0$, we call G *uniformly non-amenable*. We study the basic properties of this invariant; for example, its behavior when passing to subgroups and quotients of G. We prove that the following classes of groups are uniformly non-amenable: non-abelian free groups, non-elementary word-hyperbolic groups, large groups, free Burnside groups of large enough odd exponent, and groups acting acylindrically on a tree. Uniform non-amenability implies uniform exponential growth. We also exhibit a family of *non-amenable* groups (in particular including all non-solvable Baumslag-Solitar groups) which are not uniformly non-amenable, that is, they satisfy $\text{Føl}\, G = 0$. Finally, we derive a relation between our uniform Følner constant and the uniform Kazhdan constant with respect to the left regular representation of G.

[13] G. N. Arzhantseva and P.-A. Cherix, *On the Cayley graph of a generic finitely presented group*, Bulletin of the Belgian Mathematical Society, **11** (2004), no. 4, 589–601.

We prove that in a certain statistical sense the Cayley graph of almost every finitely presented group with $m \geq 2$ generators contains a subdivision of the complete graph on $l \leq 2m+1$ vertices. In particular, this Cayley graph is non planar and the genus of a generic finitely presented group (that is the minimum of genus of its Cayley graphs taken over all generating sets) is the infinity. We also show that some group constructions preserve the planarity.

[14] G. N. Arzhantseva and I. G. Lysenok, *Growth tightness for word hyperbolic groups,* Mathematische Zeitschrift, **241** (2002), no. 3, 597–611.

We show that non-elementary word hyperbolic groups are growth tight. This means that, given such a group G and a finite set A of its generators, for any infinite normal subgroup N of G, the exponential growth rate of G/N with respect to the natural image of A is strictly less than the exponential growth rate of G with respect to A. This gives an affirmative answer to the question about growth tightness of word hyperbolic groups, posed by R. Grigorchuk and P. de la Harpe.

[15] G. N. Arzhantseva and D. V. Osin, *Solvable groups with polynomial Dehn functions*, Transactions of the American Mathematical Society, **354** (2002), 3329–3348.

Given a finitely presented group H, finitely generated subgroup B of H, and a monomorphism $\psi : B \to H$, we obtain an upper bound of the Dehn function of the corresponding HNN-extension $G = \langle H, t \mid t^{-1}Bt = \psi(B) \rangle$ in terms of the Dehn function of H and the distortion of B in G. Using such a bound, we construct first examples of non-polycyclic solvable groups with polynomial Dehn functions. The constructed groups are metabelian and contain the solvable Baumslag-Solitar groups. In particular, this answers a question posed by Birget, Ol'shanskii, Rips, and Sapir. (This question goes up to the problem of simulating of Turing mashines in groups with a good control of Dehn functions using ideas of the Novikov-Boone construction.)

Note that all finitely presented solvable groups that were known to have polynomial Dehn functions are nilpotent (by results of Gromov, Pittet) or polycyclic (by results of Drutu).

First examples of finitely presented groups $H(q)$ with the Dehn functions $\delta_{H(q)}(n) \simeq n^{10}$ containing the solvable Baumslag-Solitar group $BS(1,q)$, $q > 1$, were obtained by Ol'shanskii and Sapir. However, the groups constructed by them are very distant from solvable ones.

[16] G. N. Arzhantseva, *On quasiconvex subgroups of word hyperbolic groups*, Geometriae Dedicata, **87** (2001), 191–208.

We prove that a quasiconvex subgroup H of infinite index of a torsion free word hyperbolic group can be embedded in a larger quasiconvex subgroup which is the free product of H and an infinite cyclic group. Since this larger quasiconvex subgroup can be chosen of infinite index we obtain in fact a method for constructing quasiconvex subgroups of word hyperbolic groups.

The result was formulated by M.Gromov in [5.3.C] of "Hyperbolic groups". The main technical statement is that a quasiconvex subgroup of a word hyperbolic group is of finite index if and only if the number of double cosets of the group modulo this subgroup is finite.

[17] G. N. Arzhantseva, *A property of subgroups of infinite index in a free group*, Proceedings of the American Mathematical Society, **128**(11) (2000), 3205–3210.

Let F be a finitely generated free group. It is known (by results of Greenberg, Karrass and Solitar) that if H is a finitely generated subgroup of F then H is of infinite index if and only if there is a normal subgroup K of F such that $K \cap H = \{1\}$. In the present paper, we study this property of subgroups of free groups from a statistical point of view. We prove that if H is a finitely generated subgroup of F of infinite index, then a randomly chosen normal subgroup K of F has trivial intersection with H with the probability tending to 1 as the lengths of the elements whose normal closure is K tend to infinity. In other words, this gives a "generic property" of normal subgroups of a free group: for a fixed H, a generic normal subgroup of F trivially intersects with H.

[18] G. N. Arzhantseva, *Generic properties of finitely presented groups and Howson's Theorem*, Communications in Algebra, **26**(11) (1998), 3783-3792.

The main result in this paper is a generalized version of Howson's theorem. Given $d > 0$, consider the class of finitely presented groups such that the intersection of any two subgroups with $\leq d$ generators is finitely generated. We prove that this class of groups is generic. The main step is to show that every subgroup with $\leq d$ generators is quasiconvex with respect to any finite presentation of a generic group. This is also used to prove that the class of groups with the property that every nontrivial normal subgroup with $\leq d$ generators has finite index is generic.

[19] G. N. Arzhantseva, *On the groups all of whose subgroups with fixed number of generators are free*, Fundamental and Applied Mathematics, **3**(3) (1997), 675-683 (in Russian).

We prove that in a generic finitely presented group on $m \geq 2$ generators all $\leq L$-generated subgroups of infinite index are free (L is an arbitrary preassigned bound, possibly $L \gg m$) and all subgroups of finite index are not free. For proof we give a condition on defining relations which guarantees that all subgroups of infinite index with fixed number of generators are free in a finitely presented group. This condition is formulated by means of the finite labelled graphs. The result is a strong generalization of the main result of [19].

[20] G. N. Arzhantseva and A. Yu. Ol'shanskii, *Generality of the class of groups in which subgroups with a lesser number of generators are free*, Mathematical Notes, **59**(3-4) (1996), 350-355.

We show that in a generic m-generated finitely presented group every subgroup with $m-1$ generators is free. This solves question 11.75 from "Kourovka notebook" (Unsolved problems in group theory).

[21] G. N. Arzhantseva, *Generic properties of finitely presented groups,*
PhD thesis, Moscow Lomonosov State University, December 1998.

[22] G.N. Arzhantseva, A.Valette (eds.), Limits of graphs in group theory and computer science, Fundamental Sciences, EPFL Press, Lausanne, 2008, in press.

The research articles and survey papers of this volume focus on three fields and on the interactions between them: geometric combinatorics, theoretical computer science, and geometric group theory. They highlight modern state of the art, current methods and open problems, that will be of interest both to experts and to graduate students.

[23] G. N. Arzhantseva, L. Bartholdi, J. Burillo, and E. Ventura (eds.), *Geometric group theory*, Trends in Mathematics, Birkhuser Verlag, Basel, 2007, 253 p.

The volume assembles research papers in geometric and combinatorial group theory. The contributions range over a wide spectrum: limit groups, groups associated with equations, with cellular automata, their structure as metric objects, their decomposition, etc. Their common denominator is the language of group theory, used to express and solve problems ranging from geometry to logic.

[24] G. N. Arzhantseva and T. Delzant, *Examples of random groups*, 2008, submitted.

We present Gromov's construction of a random group with no coarse embedding into a Hilbert space.

[25] G. N. Arzhantseva, *An algorithm detecting Dehn presentations,* preprint, University of Geneva, 2000.

A Dehn presentation of a group G leads to a known Dehn's algorithm solving the word problem for G. The existence of a Dehn presentation is equivalent to the word hyperbolicity of a finitely generated group. Because being word hyperbolic is a Markov property of groups there cannot exist an effective procedure for determining if a finitely presented group admits a Dehn presentation. However, there may exist an algorithm to decide whether a finite presentation of a group is a Dehn presentation. In this article we prove a result in this direction. We give an algorithm determining whether or not a finite presentation of a group is an α-Dehn presentation for some $\frac{3}{4} \leq \alpha < 1$. Note that a Dehn presentation in the traditional sense is an α-Dehn presentation with $\alpha = 1/2$. Observe also that any α-Dehn presentation is a Dehn presentation.

[26] G. N. Arzhantseva, J. Díaz, J. Petit, J. Rolim, M. J. Serna, *Broadcasting on networks of sensors communicating through directional antennas,* Ambient Intelligence Computing, 1–12, Proceedings, CTI Press and Ellinika Grammata, 2003.

We propose the use of random scaled sector graphs as the basis for a model for networks of sensors communicating through radio frequency using directional antennas. We propose two broadcasting algorithms, and compare empirically their performance.

[27] G. N. Arzhantseva and J. D. P. Rolim, *Considerations for a geometric model of the web,* Approximation and Randomization Algorithms in Communication Networks, Rome, 2002, 1–11, Proceedings, Carleton Scientific.

We suggest a new geometric viewpoint on the world-wide web graph. Namely, we regard the web as a space with negative (or hyperbolic) curvature. This gives a finer information on the hyperlinked structure of the web. We also outline potential applications of this analytical approach to improve on algorithms that search and mine the web.

[28] G. N. Arzhantseva and J. D. P. Rolim, *Computability and Complexity,* e-learning theoretical course of Virtual Logic Laboratory (a project of Swiss Virtual Campus), 90 pp.

This is an extensive electronic tutorial: A complete theoretical course supported by interactive quizzes and exercises.