AFFINE ISOMETRIC ACTIONS OF DISCRETE GROUPS

ABSTRACTS OF TALKS

JUNE 28 - JULY 3, 2009
ASCONA, SWITZERLAND
ORGANIZERS: GOULNARA ARZHANTSEVA AND ALAIN VALETTE

PLENARY TALKS

Bachir BEKKA (Rennes)

On the ergodic theory of groups of automorphisms of nilmanifolds.

Let N be a connected nilpotent Lie group and D a lattice in N. Let Γ be a group of automorphisms of N preserving D. Then Γ acts by measure preserving transformations on the nilmanifold N/D. We discuss the spectral decomposition of $L^2(N/D)$ under the associated unitary representation of Γ, giving conditions under which a spectral gap exists. E-mail: bachir.bekka@univ-rennes1.fr

Marc BOURDON (Lille)

ℓ_p-cohomology and word hyperbolic Coxeter groups.

ℓ_p-cohomology in degree 1 gives rise to equivalence relations on boundaries of word hyperbolic groups. These are of great interest because of their invariance by quasi-isometries. I will present some results obtain with Bruce Kleiner in the special case of word hyperbolic Coxeter groups. E-mail: Marc.Bourdon@math.univ-lille1.fr

Indira CHATTERJI (Orléans)

Cohomology and distortion for central subgroups in Lie groups.

We show that given a Lie group G, all its Borel cohomology with integer coefficients is bounded if and only if the radical of G is linear. We will discuss some ideas used in the proof of this result, as well as related problems. This is joint work with Mislin, Pittet and Saloff-Coste. E-mail: indira.chatterji@math.cnrs.fr

Cornelia DRUTU (Oxford)

Asymptotic median structure of mapping class groups, applications to homomorphisms.

Median spaces are non-discrete versions of CAT(0) cubical complexes. Both Kazhdan and Haagerup properties can be formulated in terms of actions of groups on median spaces. Moreover, it turns out that every asymptotic cone of a mapping class group has a natural equivariant structure of median space. This allows to prove that groups with property (T) have finitely many homomorphisms into mapping class groups of surfaces, up to conjugation. The talk is on joint work with I. Chatterji and F. Haglund, and with J. Behrstock and M. Sapir. E-mail: drutu@maths.ox.ac.uk

Erik GUENTNER (Hawaii)

Metric complexity and topological rigidity.

I shall describe the notion of finite decomposition complexity (FDC), introduced in joint work with Romain Tessera and Guoliang Yu on the Novikov and related conjectures. The talk will focus on the definition of FDC, examples of groups having FDC and applications to rigidity. E-mail: erik@math.hawaii.edu

Nicolas MONOD (EPF Lausanne):

Higher vanishing in bounded cohomology.
E-mail: nicolas.monod@epfl.ch

Assaf NAOR (Courant Inst)

On the role of random walks in quantitative non-embeddability.

In this talk I will describe random walk methods which are useful to show that certain metric spaces do not admit a bi-Lipschitz embedding in other metric spaces. The key idea is to prove that the geometry of the target metric space forces every symmetric random walk in it to wander slowly from its starting point, and then to design a random walk (which is adapted to the space that we wish to embed and the target space) which contradicts this "slow speed" result. Such methods are surprisingly versatile and robust, and can yield sharp results in various contexts. I will explain the general methodology, and work out the full details in a couple of illustrative examples. E-mail: naor@cims.nyu.edu

Andres NAVAS (Univ. de Chile, Santiago):

On the affine isometric actions of groups of 1-dimensional diffeomorphisms of low regularity.

It is still an open question whether a finitely generated Kazhdan group can act faithfully by homeomorphisms of the real line (equivalently, it is left-orderable). This question is very subtle, as is shown by the fact that for every free subgroup $F \subset \mathrm{SL}(2,\mathbb{Z})$, the semi-direct product $F \ltimes \mathbb{Z}^2$ is left-orderable, though it has the relative property (T) with respect to $\mathbb{Z}^2$ when F has finite index.

Groups of $C^{3/2+}$ diffeomorphisms of the circle or the interval carry a natural affine action on a Hilbert space. For lower regularity, this can be modified to an actions L^p-spaces (for large p). However, as I will explain, this does not allow dealing with the problem of orderability of Kazhdan groups.

Despite the above, I will show (by rather different methods) that $G \ltimes \mathbb{Z}^2$ does not embed in the group of C^1 diffeomorphisms of neither the circle nor the real line for any non-solvable subgroup $G \subset \mathrm{SL}(2,\mathbb{Z})$. E-mail: andnavas@uchile.cl

Graham NIBLO (Southampton)

On Yu's property A.

Yu's property A is a non-equivariant generalisation of amenability. We show how to establish it for several classes of groups, including groups acting properly on $CAT(0)$ cube complexes of finite dimension and outline Campbell's proof of property A for groups acting on certain 2 dimensional Euclidean buildings. E-mail: G.A.Niblo@soton.ac.uk

Piotr NOWAK (Texas A & M)

Controlled coarse homology and isoperimetric inequalities.

We will introduce a controlled homology theory for discrete metric spaces. This homology is a quasi-isometry invariant and generalizes the uniformly finite homology of Block and Weinberger. We will be interested in vanishing of a certain homology class and we will prove a homological Burnside theorem, namely that in linearly controlled homology the fundamental class vanishes for every finitely generated infinite group. We also characterize the vanishing of this class via an isoperimetric inequality on the group. Finally, we will present applications of controlled coarse homology. This is joint work with Jan Spakula (Universitaet Muenster). E-mail: pnowak@math.tamu.edu

Jesse PETERSON (Vanderbilt): *Virtual $W*E$-superrigidity*

Rigidity results, in their most common form is when a "weak equivalence" between two objects must come from an identification of the objects. Associated to a countable group acting by measure preserving transformations on a probability space is a finite von Neumann algebra (W*-algebra). Two group actions are said to

be W*-Equivalent if the corresponding von Neumann algebras are isomorphic. We will present a rigidity result showing that in in some cases W*-Equivalence implies a stronger form of equivalence (Orbit Equivalence). Combining this with results of Ioana, Ozawa, and Popa we will produce examples of group actions for which the von Neumann algebra virtually remembers both the group and the action. E-mail: jesse.d.peterson@Vanderbilt.Edu

Florin RADULESCU (Roma II): *Operator Algebra representations for Hecke operators and Ramanujan Peterson conjectures modulo operator algebra ideals*

Using a complete positive representation for Hecke operators we determine an operator algebra ideal on which the Ramanujan Peterson conjectures depend. E-mail: radulesc@mat.uniroma2.it

Mark SAPIR (Vanderbilt)

Lacunary Hyperbolic Groups.

This is a joint work with A. Yu. Olshanskii and D.V. Osin. We consider a class of lacunary hyperbolic groups. Those are the groups some of whose asymptotic cones are R-trees. Examples of such groups, as well as solutions of several problems about asymptotic cones and amenable groups will be presented. If I have time, I'll also talk about our new results joint with Conner and Kent about fundamental groups of asymptotic cones. E-mail: m.sapir@vanderbilt.edu

INTRODUCTORY LECTURES FOR PhD STUDENTS

Bachir BEKKA (Rennes)

Affine isometric actions of groups of diffeomorphisms of the circle.

The Pressley-Segal-Reznikov cocycle gives rise to an affine isometric action of the group of diffeomorphisms of the circle. Using this cocycle, Navas showed that a Kazhdan group of diffeomorphisms of the circle is finite. E-mail: bachir.bekka@univ-rennes1.fr

Tsachik GELANDER (Hebrew University)

A shorter proof for the triviality of the first L_p-cohomology for higher rank groups.

This talk, connected with Monod's mini-course, will be about a shorter proof for a result by Bader-Furman-Gelander-Monod. E-mail: gelander@math.huji.ac.il

Nicolas MONOD (EPF Lausanne)

T_{Ba}.

The lectures will be devoted to analogues of Kazhdan's property (T) when unitary representations on Hilbert spaces are replaced by isometric representations on more general Banach spaces. E-mail: nicolas.monod@epfl.ch

CONTRIBUTED TALKS

Florent BAUDIER (Besancon)

Free groups and metric invariants.

Cayley graphs of free groups are trees. In this talk we will show how they occur as metric invariants for isomorphic properties of Banach spaces. In 1986 J. Bourgain characterized the superreflexivity of a Banach space X in terms of Bi-Lipschitz embedding of hyperbolic dyadic trees (B_n, ρ). This result is a concrete occurrence of what M. Mendel and A. Naor called "the Ribe program" in their famous paper on metric cotype. Bourgain's theorem can be restated as follows: $D(X) > \omega$ (or $D(X^*) > \omega$) iff every (B_n, ρ) Bi-Lipschitz embeds into X with a universal positive constant distortion C, where ω is the first infinite countable ordinal and $D(X)$ denotes the dentability index of X. This is also equivalent to the embedding of the infinite dyadic tree. The talk will focus on a similar result when dealing with the relation between Szlenk index and countable branching trees. If time permits we will give applications. This is a joint work with Nigel Kalton and Gilles Lancien. E-mail: florent.baudier@univ-fcomte.fr

Jacek BRODZKI (Southampton)

Operator algebras and translation structures on metric spaces.

A subspace of a discrete group can be regarded as a space with partially defined symmetries. For an arbitrary metric space, a sufficiently rich family of such partial symmetries is called a partial translation structure. A partial translation algebra is a separable subalgebra of the uniform Roe algebra which arises from a natural representation of this structure that is constructed as an analogue of the left regular representation of a group. We will present examples of partial translation algebras, including some well-known classical algebras. We will also explore the question of computing the K-theory of partial translation algebras, and for this we will introduce a Mayer-Vietoris sequence associated with a subspace of a discrete group. E-mail: J.Brodzki@soton.ac.uk

Talia FERNOS (UCLA)

Reduced 1-cohomology and relative property (T).

The celebrated theorems of Delorme (1977) and Guichardet (1972) establish the equivalence between property (T) and the vanishing of 1-cohomology, where the coefficients are taken in a unitary representation. In 2000 Shalom proved that the (a priori) weaker condition of the vanishing of reduced 1-cohomology is in fact equivalent to property (T) for the class of compactly generated groups. In 2005-2006 de Cornulier, Jolissaint, and Fernos independently showed that the vanishing of the restriction map on 1-cohomology is equivalent to relative property (T). One may ask if the relative version of Shalom's theorem is true. In a joint work with Valette we exhibit a large class of non-compact amenable group-pairs where the restriction map on reduced 1-cohomology always vanishes. Since amenable groups can not have relative property (T) with respect to non-compact subgroups, our result gives a strong negative answer to the above question. E-mail: talia.fernos@gmail.com

Alex FURMAN (Chicago)

Measure-theoretic notion of imbedding between countable groups.

We consider a generalization of the relation: "group G can be embedded as a subgroup in group H" in the measure-theoretic context. This notion relates to uniform imbeddability between groups in the same way as Measure Equivalence relates to Quasi-Isometry. We shall discuss the resulting partial order between classes of groups, which starts as *Finite* $<$ *infinite Amenable* $<$ *Free non Abelian* $<...$, introduce monotone invariants and some rigidity results. Based on a joint project with Uri Bader and Yehuda Shalom. E-mail: furman@math.uic.edu

Damien GABORIAU (ENS Lyon)

Free products and Bass-Serre rigidity in Orbit Equivalence.
E-mail: damien.gaboriau@umpa.ens-lyon.fr

Alexander GAMBURD (Northwestern)

Uniform spectral gap bounds.

We present a new approach to establishing spectral gap for finitely generated subgroups of SL(d, Z) and SU(d) based on using tools from arithmetic combinatorics, in particular sum-product estimates (joint work with Jean Bourgain). E-mail: agamburd@ucsc.edu

Tsachik GELANDER (Hebrew University)

Rank and volume of lattices.
E-mail: gelander@math.huji.ac.il

Gabriella KUHN (Milan)

Representations of currents taking values in a totally disconnected group.

Let $G = \mathcal{A}ut(\mathcal{T})$ be the group of all automorphisms of a homogeneous tree $\mathcal{T}$ of degree $q+1 \geq 3$ and (X, m) a compact metrizable measure space with a probability measure m. We assume that μ has no atoms. The group $\mathcal{G} = \mathcal{A}ut(\mathcal{T})^X = G^X$ of measurable currents is the completion of the group of step functions $f : X \to \mathcal{A}ut(\mathcal{T})$ with respect to a suitable metric. Continuous functions form a dense subgroup of $\mathcal{G}$. Following the ideas of I.M. Gelfand, M.I. Graev and A.M. Vershik we shall construct an irreducible family of representations of $\mathcal{G}$. The existence of such representations depends deeply from the non-vanishing of the first cohomology group $H^1(\mathcal{A}ut(\mathcal{T}), \pi)$ for a suitable infinite dimensional π. E-mail: mariagabriella.kuhn@unimib.it

Masato MIMURA (Tokyo)

A fixed point property and the second bounded cohomology of universal lattices.

Let $n \geq 4$ be an integer, and B be any L^p space or any Banach space isomorphic to a Hilbert space. In this talk, we will show that the universal lattice $SL_n(\mathbb{Z}[X_1, \ldots, X_k])$ has a fixed point property for affine isometric actions on B. We will also verify that the comparison map in degree 2

$$\Psi^2 \colon H_b^2(SL_n(\mathbb{Z}[X_1, \ldots, X_k]), B) \to H^2(SL_n(\mathbb{Z}[X_1, \ldots, X_k]), B)$$

from bounded to ordinary cohomology is injective. For our proof, we establish a certain implication from the relative Kazhdan property to the relative fixed point property on uniformly convex Banach spaces.

The preprint is available at http://arxiv.org/abs/0904.4650 . The work is supported by JSPS research fellowships. E-mail: mimurac@ms.u-tokyo.ac.jp

Yves STALDER (Clermont-Ferrand)

Affine actions of wreath products.

I will report on some recent results about Kazhdan and Haagerup properties for wreath products and about the L^1-compression of a wreath product of a free group. This is joint work with Yves Cornulier and Alain Valette.
E-mail: yves.stalder@math.univ-bpclermont.fr

Troels STEENSTRUP (Odense)

Herz–Schur multipliers and non–uniformly bounded representations of second countable, locally compact groups.

It is shown that, for second countable, locally compact groups, all Herz–Schur multipliers may be written as coefficients of (not necessarily uniformly bounded) representations. For countable, discrete groups this was shown by [Bozejko–Fendler '91]. By a theorem of [Struble '74], second countability guarantees the existence of a proper, left invariant metric. Moreover, according to [Haagerup–Przybyszewska '06], this metric may be chosen to have at most exponential growth, i.e., the Haar measure of a ball of radius n is bounded by a function which is exponential in n. This metric becomes central in the construction of the representations. E-mail: troelssj@imada.sdu.dk

Andreas THOM (Göttingen)

Cocycles, L^2-invariants and the ring of affiliated operators.

In joint work with Jesse Peterson (Berkeley) we obtained strong results about the subgroup structure of a group with positive first ℓ^2-Betti number. These results have numerous applications in geometric and combinatorial group theory. In particular, many results which were known to hold for free groups or one-relator groups extend by our results to arbitrary groups with a positive first ℓ^2-Betti number. E-mail: thom@uni-math.gwdg.de

Alina VDOVINA (Newcastle)

Cayley graph expanders and groups of finite width.

We present a new family of expander graphs of vertex degree, 4. These graphs define a tower of coverings (with powers of 2 as, covering indices) and are the Cayley graphs of particular finite, quotients of a group G of infinite Toeplitz matrices over the ring, $\mathrm{M}(9, \mathbb{F}_2)$. We present explicit vector space bases for the, finite abelian quotients of the lower exponent-2 groups of G by, upper triangular subgroups and encounter a particular, 3-periodicity of these quotients. We also discuss properties of, the pro-2 completion of the group G. MAGMA-computer calculations, suggest that the group G has finite width 3 and finite average, width 8/3. E-mail: alina.vdovina@ncl.ac.uk

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