

The 135 heptagon spaces

$$b_i = \dim_{\mathbf{F}} H^{2i}(\mathcal{N}_3^7(\alpha); \mathbf{F}) = \dim_{\mathbf{F}} H^{8-2i}(\mathcal{N}_3^7(\alpha); \mathbf{F})$$

$$\text{or } :b_i = \dim_{\mathbf{F}_2} H^i(\mathcal{N}_2^7(\alpha); \mathbf{F}_2) = \dim_{\mathbf{F}} H^{4-i}(\mathcal{N}_2^7(\alpha); \mathbf{F}_2).$$

$$r_{\cup} = \text{rank}(x \rightarrow x \cup x), \text{ from } H^2(\mathcal{N}_3^7(\alpha); \mathbf{F}_2) \text{ to } H^4(\mathcal{N}_3^7(\alpha); \mathbf{F}_2)$$

$$\text{or } :r_{\cup} = \text{rank}(x \rightarrow x \cup x), \text{ from } H^1(\mathcal{N}_2^7(\alpha); \mathbf{F}_2) \text{ to } H^2(\mathcal{N}_2^7(\alpha); \mathbf{F}_2)$$

$$s = \text{sol}(\alpha)$$

$$\sigma = \text{signature of } \mathcal{N}_3^7(\alpha) \text{ (computed from formula of [HK2, Cor. 4.5]).}$$

The following 7-gon spaces have been sorted by lexicographic order of the numbers b_1, b_2, r_{\cup} and s , showing that these invariants distinguish the 7-gon spaces.

	code	a "min."	p	b_1	b_2	r_{\cup}	s	σ
1	$\langle \rangle$	(0, 0, 0, 0, 0, 0, 1)	1	0	0	0	0	0
2	$\langle 7 \rangle$	(1, 1, 1, 1, 1, 1, 5)	11	1	1	1	1	1
3	$\langle 71 \rangle$	(0, 1, 1, 1, 1, 1, 4)	9	2	2	2	2	0
4	$\langle 72 \rangle$	(1, 1, 2, 2, 2, 2, 7)	17	3	3	3	3	-1
5	$\langle 721 \rangle$	(0, 0, 1, 1, 1, 1, 3)	7	3	4	3	4	0
6	$\langle 73 \rangle$	(1, 1, 1, 2, 2, 2, 6)	15	4	4	4	4	-2
7	$\langle 721, 73 \rangle$	(1, 1, 2, 3, 3, 3, 8)	21	4	5	4	5	-1
8	$\langle 74321 \rangle$	(0, 0, 0, 0, 1, 1, 1)	3	4	6	0	0	0
9	$\langle 7321 \rangle$	(0, 0, 0, 1, 1, 1, 2)	5	4	6	3	0	0
10	$\langle 731 \rangle$	(0, 1, 1, 2, 2, 2, 5)	13	4	6	4	6	0
11	$\langle 732 \rangle$	(1, 1, 1, 3, 3, 3, 7)	19	4	7	4	8	1
12	$\langle 74 \rangle$	(1, 1, 1, 1, 2, 2, 5)	13	5	5	5	5	-3
13	$\langle 721, 74 \rangle$	(1, 1, 2, 2, 3, 3, 7)	19	5	6	5	6	-2
14	$\langle 7432 \rangle$	(1, 1, 1, 1, 4, 4, 5)	17	5	7	1	1	-1
15	$\langle 7321, 74 \rangle$	(1, 1, 1, 3, 4, 4, 7)	21	5	7	4	1	-1
16	$\langle 731, 74 \rangle$	(1, 2, 2, 3, 4, 4, 9)	25	5	7	5	7	-1
17	$\langle 7431 \rangle$	(0, 1, 1, 1, 3, 3, 4)	13	5	8	2	2	0
18	$\langle 7421 \rangle$	(0, 0, 1, 1, 2, 2, 3)	9	5	8	3	0	0
19	$\langle 7321, 741 \rangle$	(0, 1, 1, 2, 3, 3, 5)	15	5	8	4	2	0
20	$\langle 741 \rangle$	(0, 1, 1, 1, 2, 2, 4)	11	5	8	5	8	0

21	$\langle 732, 74 \rangle$	$(1, 1, 1, 2, 3, 3, 6)$	17	5	8	5	9	0
22	$\langle 7421, 743 \rangle$	$(1, 1, 2, 2, 5, 5, 7)$	23	5	9	3	4	1
23	$\langle 7321, 742 \rangle$	$(1, 1, 2, 3, 5, 5, 8)$	25	5	9	4	4	1
24	$\langle 732, 741 \rangle$	$(1, 2, 2, 3, 5, 5, 9)$	27	5	9	5	10	1
25	$\langle 7321, 743 \rangle$	$(1, 1, 1, 2, 4, 4, 6)$	19	5	10	4	8	2
26	$\langle 742 \rangle$	$(1, 1, 2, 2, 4, 4, 7)$	21	5	10	5	12	2
27	$\langle 743 \rangle$	$(1, 1, 1, 1, 3, 3, 5)$	15	5	11	5	16	3
28	$\langle 75 \rangle$	$(1, 1, 1, 1, 1, 2, 4)$	11	6	6	6	6	-4
29	$\langle 721, 75 \rangle$	$(1, 1, 2, 2, 2, 3, 6)$	17	6	7	6	7	-3
30	$\langle 7432, 75 \rangle$	$(1, 1, 1, 1, 3, 4, 4)$	15	6	8	2	2	-2
31	$\langle 7321, 75 \rangle$	$(1, 1, 1, 3, 3, 4, 6)$	19	6	8	5	2	-2
32	$\langle 731, 75 \rangle$	$(1, 2, 2, 3, 3, 4, 8)$	23	6	8	6	8	-2
33	$\langle 7431, 75 \rangle$	$(1, 2, 2, 2, 5, 6, 7)$	25	6	9	3	3	-1
34	$\langle 7421, 75 \rangle$	$(1, 1, 3, 3, 5, 6, 8)$	27	6	9	4	1	-1
35	$\langle 7321, 741, 75 \rangle$	$(1, 2, 2, 4, 5, 6, 9)$	29	6	9	5	3	-1
36	$\langle 732, 75 \rangle$	$(1, 1, 1, 2, 2, 3, 5)$	15	6	9	6	10	-1
37	$\langle 741, 75 \rangle$	$(1, 2, 2, 2, 3, 4, 7)$	21	6	9	6	9	-1
38	$\langle 7521 \rangle$	$(0, 0, 1, 1, 1, 2, 2)$	7	6	10	3	0	0
39	$\langle 7431, 751 \rangle$	$(0, 1, 1, 1, 2, 3, 3)$	11	6	10	3	4	0
40	$\langle 7421, 751 \rangle$	$(0, 1, 2, 2, 3, 4, 5)$	17	6	10	4	2	0
41	$\langle 7421, 743, 75 \rangle$	$(1, 1, 2, 2, 4, 5, 6)$	21	6	10	4	5	0
42	$\langle 7321, 751 \rangle$	$(0, 1, 1, 2, 2, 3, 4)$	13	6	10	5	4	0
43	$\langle 7321, 742, 75 \rangle$	$(1, 1, 2, 3, 4, 5, 7)$	23	6	10	5	5	0
44	$\langle 751 \rangle$	$(0, 1, 1, 1, 1, 2, 3)$	9	6	10	6	10	0
45	$\langle 732, 741, 75 \rangle$	$(1, 2, 2, 3, 4, 5, 8)$	25	6	10	6	11	0
46	$\langle 7421, 752 \rangle$	$(1, 1, 3, 3, 4, 6, 7)$	25	6	11	4	4	1
47	$\langle 7421, 743, 751 \rangle$	$(1, 2, 3, 3, 5, 7, 8)$	29	6	11	4	6	1
48	$\langle 7321, 742, 751 \rangle$	$(1, 2, 3, 4, 5, 7, 9)$	31	6	11	5	6	1
49	$\langle 7321, 743, 75 \rangle$	$(1, 1, 1, 2, 3, 4, 5)$	17	6	11	5	9	1
50	$\langle 732, 751 \rangle$	$(1, 2, 2, 3, 3, 5, 7)$	23	6	11	6	12	1
51	$\langle 742, 75 \rangle$	$(1, 1, 2, 2, 3, 4, 6)$	19	6	11	6	13	1
52	$\langle 7421, 743, 752 \rangle$	$(1, 1, 2, 2, 3, 5, 5)$	19	6	12	4	8	2
53	$\langle 7321, 743, 751 \rangle$	$(1, 2, 2, 3, 4, 6, 7)$	25	6	12	5	10	2

54	$\langle 7321, 752 \rangle$	$(1, 1, 2, 3, 3, 5, 6)$	21	6	12	5	8	2
55	$\langle 742, 751 \rangle$	$(1, 2, 3, 3, 4, 6, 8)$	27	6	12	6	14	2
56	$\langle 743, 75 \rangle$	$(1, 1, 1, 1, 2, 3, 4)$	13	6	12	6	17	2
57	$\langle 7321, 743, 752 \rangle$	$(2, 2, 3, 4, 5, 8, 9)$	33	6	13	5	12	3
58	$\langle 752 \rangle$	$(1, 1, 2, 2, 2, 4, 5)$	17	6	13	6	16	3
59	$\langle 743, 751 \rangle$	$(1, 2, 2, 2, 3, 5, 6)$	21	6	13	6	18	3
60	$\langle 7321, 753 \rangle$	$(1, 1, 1, 2, 2, 4, 4)$	15	6	14	5	16	4
61	$\langle 743, 752 \rangle$	$(2, 2, 3, 3, 4, 7, 8)$	29	6	14	6	20	4
62	$\langle 753 \rangle$	$(2, 2, 2, 3, 3, 6, 7)$	25	6	15	6	24	5
63	$\langle 754 \rangle$	$(1, 1, 1, 1, 1, 3, 3)$	11	6	16	6	32	6
64	$\langle 76 \rangle$	$(1, 1, 1, 1, 1, 1, 3)$	9	7	7	7	7	-5
65	$\langle 721, 76 \rangle$	$(1, 1, 2, 2, 2, 2, 5)$	15	7	8	7	8	-4
66	$\langle 7432, 76 \rangle$	$(1, 1, 1, 1, 3, 3, 3)$	13	7	9	3	3	-3
67	$\langle 7321, 76 \rangle$	$(1, 1, 1, 3, 3, 3, 5)$	17	7	9	6	3	-3
68	$\langle 731, 76 \rangle$	$(1, 2, 2, 3, 3, 3, 7)$	21	7	9	7	9	-3
69	$\langle 7431, 76 \rangle$	$(1, 2, 2, 2, 5, 5, 6)$	23	7	10	4	4	-2
70	$\langle 7421, 76 \rangle$	$(1, 1, 3, 3, 5, 5, 7)$	25	7	10	5	2	-2
71	$\langle 7321, 741, 76 \rangle$	$(1, 2, 2, 4, 5, 5, 8)$	27	7	10	6	4	-2
72	$\langle 741, 76 \rangle$	$(1, 2, 2, 2, 3, 3, 6)$	19	7	10	7	10	-2
73	$\langle 732, 76 \rangle$	$(1, 1, 1, 2, 2, 2, 4)$	13	7	10	7	11	-2
74	$\langle 7521, 76 \rangle$	$(1, 1, 3, 3, 3, 5, 5)$	21	7	11	4	1	-1
75	$\langle 7431, 751, 76 \rangle$	$(1, 2, 2, 2, 4, 5, 5)$	21	7	11	4	5	-1
76	$\langle 7421, 751, 76 \rangle$	$(1, 2, 4, 4, 6, 7, 9)$	33	7	11	5	3	-1
77	$\langle 7421, 743, 76 \rangle$	$(1, 1, 2, 2, 4, 4, 5)$	19	7	11	5	6	-1
78	$\langle 7321, 751, 76 \rangle$	$(1, 2, 2, 4, 4, 5, 7)$	25	7	11	6	5	-1
79	$\langle 7321, 742, 76 \rangle$	$(1, 1, 2, 3, 4, 4, 6)$	21	7	11	6	6	-1
80	$\langle 751, 76 \rangle$	$(1, 2, 2, 2, 2, 3, 5)$	17	7	11	7	11	-1
81	$\langle 732, 741, 76 \rangle$	$(1, 2, 2, 3, 4, 4, 7)$	23	7	11	7	12	-1
82	$\langle 7621 \rangle$	$(0, 0, 1, 1, 1, 1, 1)$	5	7	12	3	0	0
83	$\langle 7521, 761 \rangle$	$(0, 1, 2, 2, 2, 3, 3)$	13	7	12	4	2	0
84	$\langle 7431, 761 \rangle$	$(0, 1, 1, 1, 2, 2, 2)$	9	7	12	4	6	0
85	$\langle 7421, 761 \rangle$	$(0, 1, 2, 2, 3, 3, 4)$	15	7	12	5	4	0
86	$\langle 7421, 752, 76 \rangle$	$(1, 1, 3, 3, 4, 5, 6)$	23	7	12	5	5	0

87	$\langle 7421, 743, 751, 76 \rangle$	$(1, 2, 3, 3, 5, 6, 7)$	27	7	12	5	7	0
88	$\langle 7321, 743, 76 \rangle$	$(1, 1, 1, 2, 3, 3, 4)$	15	7	12	6	10	0
89	$\langle 7321, 761 \rangle$	$(0, 1, 1, 2, 2, 2, 3)$	11	7	12	6	6	0
90	$\langle 7321, 742, 751, 76 \rangle$	$(1, 2, 3, 4, 5, 6, 8)$	29	7	12	6	7	0
91	$\langle 761 \rangle$	$(0, 1, 1, 1, 1, 1, 2)$	7	7	12	7	12	0
92	$\langle 732, 751, 76 \rangle$	$(1, 2, 2, 3, 3, 4, 6)$	21	7	12	7	13	0
93	$\langle 742, 76 \rangle$	$(1, 1, 2, 2, 3, 3, 5)$	17	7	12	7	14	0
94	$\langle 7521, 762 \rangle$	$(1, 1, 3, 3, 3, 4, 4)$	19	7	13	4	4	1
95	$\langle 7421, 752, 761 \rangle$	$(1, 2, 4, 4, 5, 6, 7)$	29	7	13	5	6	1
96	$\langle 7421, 743, 761 \rangle$	$(1, 2, 3, 3, 5, 5, 6)$	25	7	13	5	8	1
97	$\langle 7421, 743, 752, 76 \rangle$	$(1, 1, 2, 2, 3, 4, 4)$	17	7	13	5	9	1
98	$\langle 7321, 743, 751, 76 \rangle$	$(1, 2, 2, 3, 4, 5, 6)$	23	7	13	6	11	1
99	$\langle 7321, 742, 761 \rangle$	$(1, 2, 3, 4, 5, 5, 7)$	27	7	13	6	8	1
100	$\langle 7321, 752, 76 \rangle$	$(1, 1, 2, 3, 3, 4, 5)$	19	7	13	6	9	1
101	$\langle 732, 761 \rangle$	$(1, 2, 2, 3, 3, 3, 5)$	19	7	13	7	14	1
102	$\langle 742, 751, 76 \rangle$	$(1, 2, 3, 3, 4, 5, 7)$	25	7	13	7	15	1
103	$\langle 743, 76 \rangle$	$(1, 1, 1, 1, 2, 2, 3)$	11	7	13	7	18	1
104	$\langle 7421, 743, 752, 761 \rangle$	$(1, 2, 3, 3, 4, 5, 5)$	23	7	14	5	10	2
105	$\langle 7421, 762 \rangle$	$(1, 1, 3, 3, 4, 4, 5)$	21	7	14	5	8	2
106	$\langle 7321, 752, 761 \rangle$	$(1, 2, 3, 4, 4, 5, 6)$	25	7	14	6	10	2
107	$\langle 7321, 743, 761 \rangle$	$(1, 2, 2, 3, 4, 4, 5)$	21	7	14	6	12	2
108	$\langle 7321, 743, 752, 76 \rangle$	$(2, 2, 3, 4, 5, 6, 7)$	29	7	14	6	13	2
109	$\langle 742, 761 \rangle$	$(1, 2, 3, 3, 4, 4, 6)$	23	7	14	7	16	2
110	$\langle 752, 76 \rangle$	$(1, 1, 2, 2, 2, 3, 4)$	15	7	14	7	17	2
111	$\langle 743, 751, 76 \rangle$	$(1, 2, 2, 2, 3, 4, 5)$	19	7	14	7	19	2
112	$\langle 7421, 743, 762 \rangle$	$(1, 1, 2, 2, 3, 3, 3)$	15	7	15	5	12	3
113	$\langle 7321, 762 \rangle$	$(1, 1, 2, 3, 3, 3, 4)$	17	7	15	6	12	3
114	$\langle 7321, 743, 752, 761 \rangle$	$(2, 3, 4, 5, 6, 7, 8)$	35	7	15	6	14	3
115	$\langle 7321, 753, 76 \rangle$	$(1, 1, 1, 2, 2, 3, 3)$	13	7	15	6	17	3
116	$\langle 752, 761 \rangle$	$(1, 2, 3, 3, 3, 4, 5)$	21	7	15	7	18	3
117	$\langle 743, 761 \rangle$	$(1, 2, 2, 2, 3, 3, 4)$	17	7	15	7	20	3
118	$\langle 743, 752, 76 \rangle$	$(2, 2, 3, 3, 4, 5, 6)$	25	7	15	7	21	3
119	$\langle 7321, 743, 762 \rangle$	$(2, 2, 3, 4, 5, 5, 6)$	27	7	16	6	16	4

120	$\langle 7321, 753, 761 \rangle$	$(1, 2, 2, 3, 3, 4, 4)$	19	7	16	6	18	4
121	$\langle 762 \rangle$	$(1, 1, 2, 2, 2, 2, 3)$	13	7	16	7	20	4
122	$\langle 743, 752, 761 \rangle$	$(2, 3, 4, 4, 5, 6, 7)$	31	7	16	7	22	4
123	$\langle 753, 76 \rangle$	$(2, 2, 2, 3, 3, 4, 5)$	21	7	16	7	25	4
124	$\langle 7321, 753, 762 \rangle$	$(2, 2, 3, 4, 4, 5, 5)$	25	7	17	6	20	5
125	$\langle 743, 762 \rangle$	$(2, 2, 3, 3, 4, 4, 5)$	23	7	17	7	24	5
126	$\langle 753, 761 \rangle$	$(2, 3, 3, 4, 4, 5, 6)$	27	7	17	7	26	5
127	$\langle 754, 76 \rangle$	$(1, 1, 1, 1, 1, 2, 2)$	9	7	17	7	33	5
128	$\langle 7321, 763 \rangle$	$(1, 1, 1, 2, 2, 2, 2)$	11	7	18	6	24	6
129	$\langle 753, 762 \rangle$	$(3, 3, 4, 5, 5, 6, 7)$	33	7	18	7	28	6
130	$\langle 754, 761 \rangle$	$(1, 2, 2, 2, 2, 3, 3)$	15	7	18	7	34	6
131	$\langle 763 \rangle$	$(2, 2, 2, 3, 3, 3, 4)$	19	7	19	7	32	7
132	$\langle 754, 762 \rangle$	$(2, 2, 3, 3, 3, 4, 4)$	21	7	19	7	36	7
133	$\langle 754, 763 \rangle$	$(3, 3, 3, 4, 4, 5, 5)$	27	7	20	7	40	8
134	$\langle 764 \rangle$	$(2, 2, 2, 2, 3, 3, 3)$	17	7	21	7	48	9
135	$\langle 765 \rangle$	$(1, 1, 1, 1, 1, 1, 1)$	7	7	22	7	64	10