A lecture on quantisation

Lie Poisson bracket

$$\forall f, h \in \mathcal{F}(\mathfrak{g}^*) : \quad \{f, h\}(\Xi) = \Xi([df, dh])$$

Lie Algebra $\mathfrak{sl}_2(\mathbb{C})$

$$e_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad e_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[e_1, e_2] = e_3 \qquad [e_3, e_2] = -2e_2 \qquad [e_3, e_1] = 2e_1$$

$$[e_\alpha, e_\beta] = \mathcal{J}_{\alpha\beta}^{\gamma}\, e_\gamma$$

$$e_1^* = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad e_2^* = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad e_3^* = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$f_\alpha(A) := \langle e_\alpha, A \rangle \implies \{f_\alpha, f_\beta\} = \mathcal{J}_{\alpha\beta}^{\gamma} f_\gamma.$$

Hamiltonian system on $\mathfrak{sl}_2^* \times \mathfrak{sl}_2^* \times \dots \times \mathfrak{sl}_2^*$

$$H_j = \sum_{i \neq j} \frac{\operatorname{tr} A_i A_j}{u_j - u_i}, \qquad A_i \in \mathcal{O}_i^* \subset \mathfrak{sl}_2^*$$

$$\frac{\partial}{\partial u_j} A_i = \{A_i, H_j\}$$

Semenov-tian-Shansky r-matrix

$$r = \sum_\alpha e_\alpha^* \otimes e_\alpha$$