“ETH” in Quantum Mechanics – *attempts towards understanding what QM means*

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Dedicated to the memory of *Res Jost, Edward Nelson,* and *Ernst Specker*

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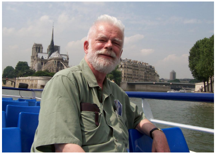

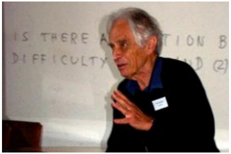

1918-1990 1932-2014 1920-2011

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II. Kochen-Specker & Bell

III. Physical systems, nature of time evolution in QM, isolated systems, preparation of states

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V. Indirect/Kraus observations of events and facts

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Introduction

Abstract

After a general introduction to Quantum Mechanics (QM) and to some of the problems surrounding it, I discuss results concerning the non-existence of hidden-variables theories, (Kochen-Specker theorem, Bell-type inequalities). Subsequently, I introduce a notion of "isolated (but open) physical system" suitable for the purposes of QM. Remarks on the preparation of states in QM follow. I then turn to proposing a theory of events and of direct/projective/von Neumann measurements. My approach is based a novel principle of "Loss of (Access to) Information". It gives rise to a novel picture of the time evolution of states in QM, called "ETH approach" - for "Events, Trees and Histories." A new type of quantum branching process is proposed and discussed. I conclude with an outline of the theory of indirect/Kraus measurements and with the discussion of examples illustrating the general theory.

1. Introduction to QM

- ABC in QM – what are the problems surrounding QM?
- Fundamentally, physical theories are never fully predictive
- Fundamental constants of nature – new theories as deformations of precursor theories
- The example of Matrix Mechanics
- Atomistic theories of matter as deformations of cont. theories
- What is a physical system – "realistic" ths. vs. quantum ths.
- Non-$\exists$ of hidden variables in QM – Kochen-Specker & Bell
- Quantum marginal problem ($\nearrow$ Klyachko, Christandl&Walter)
- No-signaling lemma (FFS) $\Rightarrow$ stochastic time evolution of states of systems featuring events
- Dictionary between classical theory and quantum theory

Part of this material is deferred to exercise sessions.

"If someone tells you they understand quantum mechanics then all you've learned is that you've met a liar."

(R.P. Feynman)

"Anyone who is not shocked by quantum theory has not understood it."

(N. Bohr)

"We have to ask what it means!"

(K. G. Wilson)

1. Is Quantum Probability Th. = Class. Probability Theory?

And – if not – how does it differ?

Class. (topol.) dynamical syst.:

M: (cp. topol.) state space; σ-alg., Σ, of Borel sets.

$\mathcal{A} := C(M)$

$\{\tau_{t,s}\}_{t,s\in\mathbb{R}} \subset Aut(\mathcal{A})$: time evol.

$\overset{1-1}{\longleftrightarrow}$ homeos $\{\phi_{t,s}\}_{t,s\in\mathbb{R}}$ of M

$\omega, \rho, \cdots$: States = prob. meas. on (M,Σ).

π: meas. class; $\mathcal{A}^{\pi} := L^{\infty}(M,\pi)$

$\Pi^{(i)} := \chi_{\Omega_i}(\cdot)$, $\Omega_i \in \Sigma$ ($\neq$ null set)

$\omega \in \pi$: state

$$Prob_\omega\{\Pi^{(0)}_{t_0}, \cdots, \Pi^{(n)}_{t_n}\} := \int_M d\omega(\xi) \prod_{i=0}^{n} \chi_{\Omega_i}(\phi_{t_i,t_*}(\xi)) \quad (1)$$

ω pure $\Leftrightarrow$ $\omega = \delta_{\xi_*}$, $\xi_* \in M$

$\rightarrow$ 0–1 laws

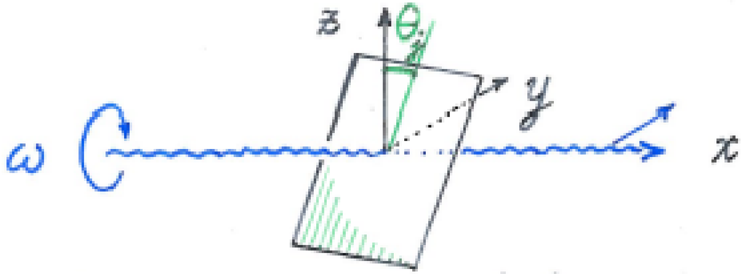
etc.

Example of quantum syst.

Beam of (monochromatic) light

$=$ beam of photons

w. $n+1$ polarization filters



After passing j^{th} filter, photon pol. in dir. $\theta_j := \frac{j\pi}{2n}$

Initially, beam circ. pol.

$\Pi_+^{(j)}$: photon passes through j^{th} filter;

$\Pi_-^{(j)}$: photon absorbed in j^{th} filter.

$$\mathrm{Prob}_\omega\{\Pi_+^{(\theta)} | \Pi_+^{(\varphi)}\} = \cos^2(\theta - \varphi),$$

$$\mathrm{Prob}_\omega\{\Pi_-^{(\theta)} | \Pi_+^{(\varphi)}\} = \sin^2(\theta - \varphi),$$

$$\mathrm{Prob}_\omega\{\Pi_\pm^{(0)}\} = \frac{1}{2}.$$

Thus,

$$\mathrm{Prob}_\omega\{\Pi_+^{(0)}, \cdots, \Pi_+^{(n)}\} = \frac{1}{2}\left(\cos\left(\frac{\pi}{2n}\right)\right)^{2n} \xrightarrow[n\to\infty]{} \frac{1}{2}$$

$$\mathrm{Prob}_\omega\{\Pi_\pm^{(0)}, \Pi_+^{(n)}\} = 0$$

If syst. were class. dyn. syst.

$$\left(\tfrac{1}{2} =\right) \mathrm{Prob}_\omega\{\Pi_+^{(0)}, \cdots, \Pi_+^{(n)}\}$$

$$\leq \mathrm{Prob}_\omega\{\Pi_+^{(0)}, \Pi_+^{(n)}\} \; (= 0)$$

because $\Pi_+^{(j)} + \Pi_-^{(j)} = 1$, $\Pi_\pm^{(j)} \geq 0$.

Interference!

$\Rightarrow \mathrm{Prob}_\omega$ not given by (1)!

More sophisticated arguments

Kochen-Specker, Bell's <

K–S: Measure spin $s=1$.

$$S_x^2 + S_y^2 + S_z^2 = s(s+1) = 2,$$

$$(S_i^2)^2 = S_i^2,\ [S_i^2, S_j^2] = 0,\ \forall (\vec{e}_x, \vec{e}_y, \vec{e}_z)$$

If $\{S_i^2\}$ were class. rv's then 2 out of $\{S_x^2, S_y^2, S_z^2\}$ are $=1$, remaining one $=0$, $\forall(\vec{e}_x, \vec{e}_y, \vec{e}_z)$

$\longrightarrow$ impossible!

Bell: 2 indep. spins, $s=\frac{1}{2}$.

Class. rv:

$$|\langle \sigma_{\underline{1}} \cdot \sigma_{\underline{2}} \rangle + \langle \sigma_{\underline{1}} \cdot \sigma_{\underline{3}} \rangle + \langle \sigma_{\underline{4}} \cdot \sigma_{\underline{2}} \rangle - \langle \sigma_{\underline{4}} \cdot \sigma_{\underline{3}} \rangle| \leq 2$$

$\forall\ \underline{1}, \underline{2}, \underline{3}, \underline{4}$.

QM:

$$\max |\langle \sigma_{\underline{1}} \cdot \sigma_{\underline{2}} \rangle + \langle \sigma_{\underline{1}} \cdot \sigma_{\underline{3}} \rangle + \langle \sigma_{\underline{4}} \cdot \sigma_{\underline{2}} \rangle - \langle \sigma_{\underline{4}} \cdot \sigma_{\underline{3}} \rangle| = 2\sqrt{2}$$

2. What is a Quantum Dynamical System?

$\mathcal{A} = C(M) \longmapsto \mathcal{A}$: NC C^*-alg.

w. $\{\tau_{t,s}\}_{t,s\in\mathbb{R}} \subset {}^*\mathrm{Aut}(\mathcal{A})$,

$\tau_{t,s} \circ \tau_{s,u} = \tau_{t,u}$: time evolution

"autonomous": $\tau_{t,s} = \tau_{t-s}$.

$\omega, \rho, \cdots$: states on $\mathcal{A}$ prescribed at time t_*.

Time evol. in Heisenberg picture: $a(t) := \tau_{t,t_*}(a)$, (2)

$a \in \mathcal{A}$.

$\mathcal{P}$ subset of $\mathcal{A}$ closed under *

$\langle \mathcal{P} \rangle :=$ C^*-algebra in $\mathcal{A}$ generated by $\mathcal{P}$.

$(\mathcal{A},\omega) \longrightarrow$ rep. π_ω of $\mathcal{A}$ on Hilbert space $\mathcal{H}_\omega$, $\Omega \in \mathcal{H}_\omega$: cyclic for $\pi_\omega(\mathcal{A})$ s.t.

(3) $\omega(a) = \langle \Omega, \pi_\omega(a)\Omega \rangle$, $\forall a \in \mathcal{A}$.

(GNS construction)

π: Rep. of $\mathcal{A}$ on some $\mathcal{H}$

$\mathcal{A}^\pi$: weak* closure of $\mathcal{A}$ in $B(\mathcal{H}$, a von Neumann alg.

$\mathcal{S}^\pi$: Normal states on $\mathcal{A}^\pi$

$(\mathcal{A},\pi,\mathcal{A}^\pi) \leftrightarrow$ measure class

$\mathcal{S}^\pi \leftrightarrow$ {probability measures}

Let $a = a^* \in \mathcal{A}$ have finite spec.:

$$\pi(a) = \sum_{i=1}^{k} \alpha_i \, \Pi_i^{(a)}, \qquad (4)$$

e.v.'s spec. proj. $\in \mathcal{A}^\pi$

$$\Pi_i^{(a)}\Pi_j^{(a)} = \delta_{ij}\Pi_i^{(a)}, \sum_{i=1}^{k} \Pi_i^{(a)} = \mathbb{1}. \qquad (5)$$

For $\mathcal{B}$ *subalg. of $\mathcal{A}$, define

$$\mathcal{B}' \cap \mathcal{A} := \{a \in \mathcal{A} \mid [a,b] = 0, \forall b \in \mathcal{B}\}$$

Back to Physics

Duality between "observables" and "indeterminates":

"Observables" or "potential props." $\leftrightarrow$ ops. in

$$:= \{a_i = a_i^* \in \mathcal{A} \mid i \in I\} \qquad (6)$$

with: $a \in \mathcal{O}$, f cont. $\mathbb{R}$-valued function on $\mathbb{R} \Rightarrow f(a) \in \mathcal{O}$.

Choose a rep. π of $\mathcal{A}$; let $(\mathcal{A}^\pi, \mathcal{S}^\pi)$ be as above.

Definition.

- $\mathcal{E}_{\geqslant t} := \langle \{ \prod_{i=1}^{n} a_i(t_i) \mid a_i \in \mathcal{O}, t_i \geqslant t \} \rangle^\pi$
  v.N. alg. of "possible events" at times $\geqslant t$.
- $\mathcal{E} := \left(\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geqslant t} \right)^{-\pi}$

"Indeterminates":

$$\mathcal{J} := \mathcal{E}' \cap \mathcal{A}^\pi \qquad (7)$$

Fundamental Duality Principle ($\rightarrow$ entanglement $\mathcal{E}$):

(8) $a(t) = \tau_{t,t_*}(a)$, $a \in \mathcal{O}$, $t \in \mathbb{R}$, may corresp. to meas./event at time t only if $\mathcal{J}$ non-trivial, containing (sub)alg. $\simeq \mathcal{E}$.

$$\Downarrow$$

$$\mathcal{A}^\pi \underset{(8)}{\supsetneqq} \mathcal{E} \supseteq \mathcal{E}_{\geqslant t} \supsetneqq \mathcal{E}_{\geqslant t'} \supseteq \{\mathbb{C}\mathbb{1}\} \qquad (9)$$

"Loss of information" ($\mathcal{L}o\mathcal{I}$)

As an "initial condition" choose a state $\omega \in \mathcal{S}^\pi$.

"Everybody" agrees on following "Postulate": If $a \in \mathcal{O}$, as in (4),(5), is observed/

measured at time t then state right after meas. of a given by

$$\simeq \sum_{i=1}^{k} p_i \, \omega_i \qquad (10)$$

where $p_i = \omega\left(\Pi_i^{(a(t))}\right)$ (11)

(Born)

and meas. of a at time $t+0$ in state ω_i yields value α_i with certainty.

This "Copenhagen postulate" calls for an explanation!

$\mathfrak{M}$: von Neumann algebra;

ω : normal state on $\mathfrak{M}$.

Definition. $\{a,\omega]_{\mathfrak{M}}$, $a \in \mathfrak{M}$, is bd. linear functional on $\mathfrak{M}$:

(12) $\{a,\omega]_{\mathfrak{M}} := \omega([a,b])$, $b \in \mathfrak{M}$.

Lemma 1. Let $a = a^* \in \mathfrak{M}$ be as in (4), (5). Then

$$\|\{a,\omega]_{\mathfrak{M}}\| < \varepsilon \Leftrightarrow \omega(b) = \sum_{i=1}^{k} p_i \, \omega_i(b) + O(\varepsilon \|b\|)$$

where

$p_i = \omega\left(\Pi_i^{(a)}\right)$ and, for $p_i \neq 0$,

$$\omega_i(b) = p_i^{-1} \omega\left(\Pi_i^{(a)} b \, \Pi_i^{(a)}\right)$$

Definition.

$$\mathcal{E}_{\mathfrak{M}}^{\omega} := \{a \in \mathfrak{M} \mid \{a,\omega]_{\mathfrak{M}} = 0\}$$

"stabilizer" of ω

$\mathcal{Z}^{\omega}_{\mathfrak{M}}$: center of $\mathcal{C}^{\omega}_{\mathfrak{M}}$; (ab.)

Lemma 2.

(i) ω is a normalized trace on $\mathcal{C}^{\omega}_{\mathfrak{M}}$

(ii) $$\mathcal{C}^{\omega}_{\mathfrak{M}} = \int^{\oplus}_{\Lambda_\omega} \mathbb{M}_{n_\lambda}(\mathbb{C}) \oplus \text{"type } \mathrm{II}_1\text{'s"}$$

$$1 \leqslant n_\lambda < \infty, \quad \forall \lambda \in \Lambda_\omega .$$

($\nearrow$ classification of $\mathfrak{M}$'s)

(iii) (If, e.g., ω separating on $\mathfrak{M}$)

$\exists$ "conditional expectation"

$$\varepsilon : \mathfrak{M} \to (\mathcal{C}^{\omega}_{\mathfrak{M}} \to) \mathcal{Z}^{\omega}_{\mathfrak{M}}, \qquad (13)$$

satisfying

$$\varepsilon(x^* x) \geqslant \varepsilon(x)^* \varepsilon(x), \ \forall x \in \mathfrak{M},$$

$$\varepsilon(a x b) = a\, \varepsilon(x)\, b, \ \forall x \in \mathfrak{M}, \ \forall a, b \in \mathcal{Z}^{\omega}_{\mathfrak{M}} / \mathcal{C}^{\omega}_{\mathfrak{M}} .$$

Application.

$$\mathcal{C}^{\omega} := \mathcal{C}^{\omega}_{\mathcal{E}}, \ \mathcal{C}^{\omega}_{\geqslant t} := \mathcal{C}^{\omega}_{\mathcal{E}_{\geqslant t}}, \ \mathcal{Z}^{\omega}_{\geqslant t} := \mathcal{Z}^{\omega}_{\mathcal{E}_{\geqslant t}},$$

$$\varepsilon_{\geqslant t} : \mathcal{E}_{\geqslant t} \to \mathcal{Z}^{\omega}_{\geqslant t} .$$

Definition. (Variance of a)

For $a \in \mathcal{E}_{\geqslant t}$, define

$$\Delta^{\omega}_t a := \sqrt{\omega([a - \varepsilon_{\geqslant t}(a)]^2)}$$

Definition. ("Empirical props.")

Let $a \in \mathcal{O}$ satisfy (4), (5). We say that a is an "empirical property at time $\approx t$", given a resolution $\delta \geqslant 0$, iff

$$\Delta^{\omega}_t a(t) \leqslant \delta \qquad (14)$$