(iii) Why must a "realistic" interpretation of QM fail? The no-signaling lemma

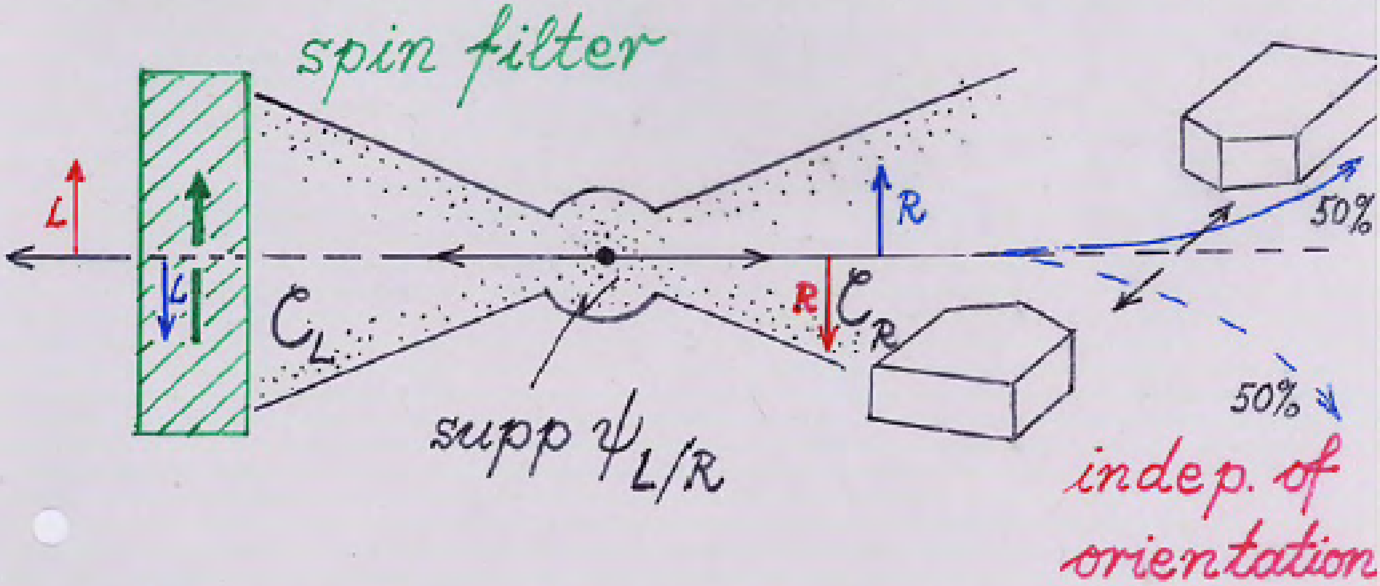
QM does sometimes predict facts (e.g., "relaxation to g.s., ⋯)

Yet, there remains an irred. element of chance whenever $\mathcal{A}_S$ is non-abelian $\Rightarrow \psi_t$ does not describe reality.

Example (n-s-ℓ/FPS)

2 electrons prepared in spin-singlet state:

$$\Psi_{\cdot}^{(2)} := (\psi_L \otimes \psi_R + \psi_R \otimes \psi_L) \otimes (|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$$



$_L\uparrow$ transm., $\downarrow_L$ absorbed

Experiment: If $_L\uparrow$ observed
then $_R\downarrow$ predicted; (1)
if $\downarrow_L$ obs. then $\uparrow_R$ predicted

$\psi_{L/R}$ propagates into $\mathcal{C}_{L/R}$, except for tiny tails.

Initial state of composed syst.:

$$\Phi_0 = \sum_{\alpha} \Psi_{\alpha}^{(2)} \otimes \chi_{filter,\alpha}$$

Dynamics: $H = H_0 + H_I$

H_0 : dyn. of uncoupled syst.

H_I : int. electrons-filter
localized around filter.

$$\Phi_t := e^{-itH}\Phi_0$$

<u>Lemma</u> ("no signaling")

Under "reasonable hyp." on H_I,

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_R} \Phi_t \rangle \simeq 0, \qquad (2)$$

for all t.

Consequ. of Cook arg. & "cluster props." – Suppose that (R)

$$\langle \Phi_t, \vec{S}_{\mathcal{C}_L} \Phi_t \rangle = (\hbar/2)\vec{e}_3 \qquad (L\uparrow)^*$$

(2) & $(L\uparrow)^*$, or $(\downarrow L)$, contradict (1)!

$\Rightarrow (\swarrow\uparrow)^*$, i.e. (R), *impossible!*

$\Rightarrow$ Frequ. of transm. of left el. through filter $\simeq 1/2$, …

Thus, Φ_t does not describe what happens in a det. way, but only what may happen. Einstein causality not invoked.

$\rightarrow$ J.F., P.P., C.S.

Dictionary

M: state space (a top. space) $\xleftrightarrow{\text{Gel'fand}}$	A C^*-algebra
$\mathcal{A} = C_0(M)$	$\mathcal{A}$ (e.g., $= M_n(\mathbb{C})$)
$\mathcal{O} = C_{\mathbb{R}}(M)$	$\mathcal{O}$ = list of s.a. ops $\in \mathcal{A}$
Homeos, flows on M $\leftrightarrow$ automs. of $C_0(M)$	*automorphisms of $\mathcal{A}$ ($\tau_{t,s}, \cdots$)

Vector bundles over M ($\leftarrow$ Swan-Serre $\rightarrow$)	(Finitely gen.) proj. modules over $\mathcal{A}$
Prob. measures $\mu, \omega, \cdots$	$\mathcal{S}$ – states $\mu, \omega, \cdots$ on $\mathcal{A}$

GNS construction

$$(\mathcal{A}, \omega) \longmapsto (\mathcal{H}_\omega, \pi_\omega, \Omega \in \mathcal{H}_\omega) \quad \text{with}$$

$$\omega(a) = \langle \Omega, \pi_\omega(a) \Omega \rangle_{\mathcal{H}_\omega}, \ \forall a \in \mathcal{A} \qquad (3)$$

$L^\infty(M, [\omega])$	$\mathcal{A}^\infty_{[\omega]} := \overline{\pi_\omega(\mathcal{A})}^{w} \subseteq B(\mathcal{H}_\omega)$
σ-alg., Σ (Borel sets)	(sub)alg. of "events"

Boltzmann-, K-S-entropy	von Neumann-, qm K-S entropy
Gibbs states, DLR	KMS states
	Tomita-Takesaki th.
cond. prob., cond. exp.; Radon-Nikod.	cond. exp. NC Radon-Nikodym
Stoch. processes on M	Semigroups of compl. positive maps (Lindbl.)
Martingales	decoherent histories