Bell inequality. 2 spin-½ particles

$$A_{\vec{e}_1} := \vec{\sigma}\cdot\vec{e}_1 \otimes 1$$

$$B_{\vec{e}_2} := 1 \otimes \vec{\sigma}\cdot\vec{e}_2$$

$$[A_{\vec{e}_1}, B_{\vec{e}_2}] = 0$$

Initial state:

$$\Psi := \frac{1}{\sqrt{2}}\left(|\uparrow\rangle\otimes|\downarrow\rangle - |\downarrow\rangle\otimes|\uparrow\rangle\right)$$

(total spin $= 0$!)

Then

$$\langle \Psi, A_{\vec{e}_1}\cdot B_{\vec{e}_2}\,\Psi\rangle = -\vec{e}_1\cdot\vec{e}_2 \qquad (*)$$

(Choose $\vec{e}_1 = \begin{pmatrix}0\\0\\1\end{pmatrix}$, $\vec{e}_2 = \vec{e} = \begin{pmatrix}\sin\vartheta\sin\varphi\\ \sin\vartheta\cos\varphi\\ \cos\vartheta\end{pmatrix}$,

with $\varphi = 0$. Then $\vec{e}_1\cdot\vec{e}_2 = \cos\vartheta$!)

If local hidden variables existed

$$E(\vec{e}_1, \vec{e}_2) \equiv \langle \Psi, A_{\vec{e}_1} \cdot B_{\vec{e}_2} \Psi \rangle$$

$$= \int_{\Omega} a_{\vec{e}_1}(\omega)\, b_{\vec{e}_2}(\omega)\, d\rho_{[\Psi]}(\omega) \quad (xvi)$$

Since $A_{\vec{e}_1}^2 = 1$, $B_{\vec{e}_2}^2 = 1$,

$$|a_{\vec{e}_1}(\omega)| = 1, \quad |b_{\vec{e}_2}(\omega)| = 1.$$

Consider

$$F(\vec{e}_1, \vec{e}_2, \vec{e}_1', \vec{e}_2') := E(\vec{e}_1, \vec{e}_2) + E(\vec{e}_1, \vec{e}_2') + E(\vec{e}_1', \vec{e}_2) - E(e_1', \vec{e}_2').$$

Since

$$-2 \leq xy + xy' + x'y - x'y' \leq 2$$

for arb. x, y, x' and y' in $[-1, 1]$,

it follows from (xvi) that

$$-2 \leq F(\vec{e}_1, \vec{e}_2, \vec{e}_1', \vec{e}_2') \leq 2. \quad (BI)$$

$$xy + xy' + x'y - x'y' = x(y+y') + x'(y-y')$$

$|x| \leq 1$, $|y+y'| + |y-y'| \leq 2 \longrightarrow$ Done!

But choosing

$$\vec{e}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \vec{e}_2 = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ -\frac{1}{\sqrt{2}} \end{pmatrix},$$

$$\vec{e}_2\,' = \begin{pmatrix} 0 \\ -1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}, \quad \vec{e}_1\,' = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$

$$\left(\quad \vec{e}_1 \cdot \vec{e}_2 = -\frac{1}{\sqrt{2}}, \quad \vec{e}_1 \cdot \vec{e}_2\,' = -\frac{1}{\sqrt{2}} \right.$$

$$\vec{e}_1\,' \cdot \vec{e}_2 = -\frac{1}{\sqrt{2}}, \quad \vec{e}_1\,' \cdot \vec{e}_2\,' = \frac{1}{\sqrt{2}}$$

By $(*)$, page 26,

$$\left(\quad F(\vec{e}_1, \vec{e}_2, \vec{e}_1\,', \vec{e}_2\,') = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right.$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2} \quad \Big\}$$

$\longrightarrow$ This violated (BI)!

<u>Exercise:</u> Details of Bell + <u>CHSH.</u>

The no-signaling lemma, according to Faupin-Fröhlich-Schubnel.

See slides & FFS paper

Exercise: Details of estimates

Corollary: Time evolution in Q.M cannot be given exclusively by unitary propagation.

The Maassen experiment in quantum optics: See slides!

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4. A Theory of Events and Their Direct/Projective Observation

—

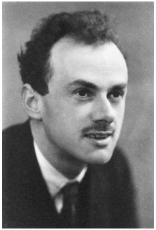
von Neumann Measurements

"I leave to several futures (not to all) my garden of forking paths"–
J. L. Borges

Les Diablerets, January 2017

1. Some basic questions and claims

Much confusion and disorientation surround the Foundations of Quantum Mechanics – so much thereof that most mathematicians do not want to touch this subject. There are many prejudices that are wrong or, to say the least, inaccurate and confusing. To mention but one example: We tend to teach to our students that the time-evolution of states of a system is described, in quantum mechanics, by the Schrödinger equation, and that the Schrödinger picture and the Heisenberg picture are equivalent. Well, nothing could be farther from the truth when considering systems accessible to observations! – Etc. Not having to make a career, anymore, I consider it to be my duty to attempt to alleviate some of this confusion – I believe I have made a little progress.



Naive Description of Systems in QM

In our courses, we tend to describe quantum-mechanical systems, S, in terms of

- a Hilbert space, $\mathcal{H}_S$ of pure state vectors – pure states are unit rays in $\mathcal{H}_S$, hence form a complex projective space $\mathbb{C}P^n$, where $n = \dim \mathcal{H}_S \leq \infty$
- *Pictures*: Configuration space picture/momentum space picture/energy picture $\cdots$ of vectors in $\mathcal{H}_S$
- a propagator, $(U(t, s))_{t,s \in \mathbb{R}}$, describing time evolution of states or observables.

Unfortunately, these data hardly encode any interesting (invariant) information about S that would enable one to draw conclusions about its physical properties, its dynamical evolution and about the events it may feature, and they give the erroneous impression that quantum theory might be a deterministic theory. (The Schrödinger Eq. for $U(t, s)$ is deterministic!)

→Fundamental questions and problems:

- What do we have to add to the usual formalism of quantum theory to arrive at a mathematical structure which – through interpretation – can be given physical meaning, *without the intervention of "observers" (at places where they obviously do not play any role)*?
- What is the origin of the *intrinsic randomness* of quantum theory, given the deterministic character of Schrödinger equation? Does it differ from classical randomness?
- What is the meaning of *states, "observables"* and *events* ($\nearrow$ R. Haag) in quantum mechanics? Do we understand the *time evolution* of states of quantum systems, and what does it have to do with solutions of the Schrödinger equation? (Very little!)
- What do we mean by an *isolated system* in quantum mechanics, and why is this an important notion*? How can one prepare a system in a prescribed state?
  *Because *only for isolated systems* a general description of the *Heisenberg time evolution of "observables"* is available!

Goal of Lecture

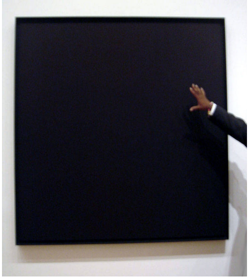
Sketch a theory of *events* and of direct and indirect *observations/ measurements* of events in QM based on two new ideas:

- *Loss of access to information, & entanglement with "lost" (inaccessible) degrees of freedom.*
- *Specification of a list of "instruments" serving to observe events.*

Some Claims

- The time evolution of qm states of a system that features observable events is *not* described by a linear Schrödinger equation; it is given by a (non-Markovian) stochastic branching process.
- There are thus no information- or unitarity paradoxes in quantum theory - even in the presence of black holes.
- The theory of operator algebras – including type-III_1 von Neumann algebras, etc. – of probability theory and stochastic processes, ... have been invented to be used in quantum theory, rather than to be ignored or ridiculed.

Metaphor for the "mysterious holistic aspects" of Quantum Mechanics



QM is QM-as-QM and everything else is everything else

"The one thing to say about art is that it is one thing. Art is art-as-art and everything else is everything else." (Ad Reinhardt)

2. Direct (projective, or von Neumann) Measurements

In classical Hamiltonian mechanics, observable physical quantities of an isolated system, S, are described by real, continuous functions on the phase space, Γ, of S. Their time evolution is governed by the usual *Hamiltonian equations of motion* formulated in terms of Poisson brackets.

Heisenberg's 1925 paper on quantum-theoretical *"Umdeutung"* contains revolutionary ideas, further elaborated upon by *Dirac*, of how to replace the basic concepts of Hamiltonian mechanics by new ones leading to a *quantum-mechanical description* of physical systems:

- Physical quantities of a system S are represented by *"symmetric matrices"*, $\widehat{F}$, (s.a. linear operators acting on a Hilbert space, $\mathcal{H}_S$)
- The Poisson bracket, $\{F, G\}$, of two phys. quantities, F and G, in a classical description of S is to be replaced, in QM, by

$$i\hbar^{-1}[\widehat{F}, \widehat{G}],$$

  where $\widehat{F}$ and $\widehat{G}$ are the s.a. operators representing the physical quantities corresponding (in classical mechanics) to F and G.

Definition of quantum-mechanical systems

- The Heisenberg time evolution of an operator $\widehat{F}$ representing a physical quantity of an isolated systems S is governed by

$$\frac{d}{dt}\widehat{F}(t) = i[\widehat{H}, \widehat{F}(t)],$$

where $t \in \mathbb{R}$ is time, and $\widehat{H}(= \widehat{H}(t))$ is the Hamilton operator of S. This determines a unitary propagator $U(t, s)$ with the property that

$$\widehat{F}(t) = U(s, t)\widehat{F}(s)U(t, s).$$

- General states of S are given as density matrices, P, acting on $\mathcal{H}_S$, where P is a positive, trace-class operator, with

$$\mathrm{tr}(P) = 1.$$

The original “Naive Copenhagen Interpretation of QM”

Suppose that – if S is prepared in a state P – a certain family of physical quantities, $A_1, A_2, A_3, \dots A_N$, are measured at times $t_1, t_2, t_3, \dots t_N$. Let

$$A_j(t_j) = \sum_n \alpha_j^{(n)} \Pi_j^{(n)}(t_j)$$

be the spectral decomposition of the operator $A_j(t_j)$, $j = 1, ..., N$. We would like to predict the probability of the *“history”* that the value $\alpha_j^{(n_j)}$ is measured at time $t_j, j = 1, ..., N$. According to *Born’s Rule*, as generalized by Lüders, Schwinger and Wigner, this probability is given by (see blackboard)

$$\mathrm{Prob}\{(\alpha_1^{(n_1)}, t_1), ..., (\alpha_N^{(n_N)}, t_N)\} = \mathrm{tr}\big(H_N(\underline{\alpha}, \underline{t})\, P\, H_N(\underline{\alpha}, \underline{t})^*\big), \quad (*)$$

where $$H_N(\underline{\alpha}, \underline{t}) := \prod_{j=1}^N \Pi_j^{(n_j)}(t_j).$$

LSW-formula

The Problems with the Copenhagen "Mumbo-Jumbo"

I Decoherence: Formula (*) only makes sense if the history $H_N(\underline{\alpha}, \underline{t})$ is *"consistent"* (i.e., decohers – see blackboard).

II Given that we know the propagator $\big(U(t,s)\big)_{t,s\in\mathbb{R}}$ of S and that S has been prepared in state P at some early time, *who* or *what* determines what physical quantities of S will be measured, and at which times? And: Do sharp measurement times make sense?
Alice, for example, might want to measure $A_1, A_2, \ldots$ at times $t_1, t_2, \ldots$, resp.;
Bob (who is unaware of Alice's measurements) wants to measure $B_1, B_2, \ldots$ at times $t_1', t_2', \ldots$, resp..
Both, Alice's and Bob's histories may be consistent! However, the operators $A_j(t_j)$ and $B_k(t_k')$ will, in general, not commute with each other, meaning that Alice's history and Bob's history are *incompatible* with each other, and that there does not exist a consistent refinement of the two histories. Now:

Will Nature obey Alice or Bob? *Ladies first?* And what happened before there were any Alices and Bobs around?

Conflicting Interpretations of QM

The Instrumentalist Approach

- Naive Copenhagen Interpretation
- "Consistent Histories", à la Griffiths & Gell-Mann - Hartle
- "Q-bism" (Mermin, ...)

The Realist Approach

- Everett's Many-Worlds Interpretation – *whatever it may mean*
- Bohmian Mechanics
- Collapse Mechanisms à la Ghirardi-Rimini-Weber, Penrose, ...

Much of this looks like utter nonsense to me – except for:

- The "ETH" Approach to QM (to be explained now!)

Some References

1. J. Fröhlich and B. Schubnel, "Do we understand quantum mechanics – finally?", in: Wolfgang Reiter et al. (eds.), *Erwin Schrödinger – 50 years after*, Zurich: European Math. Soc. Publ. 2013, pages 37 - 84.
2. J. Fröhlich and B. Schubnel, "Quantum Probability Theory and the Foundations of Quantum Mechanics", in: Philippe Blanchard and Jürg Fröhlich (eds.), *The Message of Quantum Science – Attempts Towards a Synthesis*, Lecture Notes in Physics vol. **899**, Berlin-Heidelberg: Springer-Verlag 2015, pages 131 - 193
3. M. Ballesteros, M. Fraas, J. Fröhlich and B. Schubnel, "Indirect retrieval of information and the emergence of facts in quantum mechanics", arXiv:1506.01213; and refs. given there.
4. J. Faupin, J. Fröhlich and B. Schubnel, "On the probabilistic nature of quantum mechanics and the notion of closed systems", to appear in Ann. Henri Poincar, 2015.

Please, take a look at some of the excellent papers by numerous colleagues that we have quoted in the works listed above.