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RESEARCH STATEMENT: FOAMS AND CATEGORIFICATION

A. Main Project: Link homologies

A.1. **Previous Work.** The (colored) $\mathfrak{sl}_N$-homology is a categorification of the $\mathfrak{sl}_N$-invariant of framed oriented links colored by integers between 0 and N (uncolored means that only the color 1 is allowed). This homology has been widely studied and defined using many different techniques. The story started with the categorification of the Jones polynomial by Khovanov [Kho00] and its combinatorial description by Bar-Natan [BN02]. This corresponds to the case $N = 2$. While the $\mathfrak{sl}_3$-homology has a nice combinatorial model due to Khovanov [Kho04], such a model is still missing for $N \geqslant 4$, especially in the equivariant[1] setting.

Let me mention a few constructions known so far:

- The first definition of the $\mathfrak{sl}_N$-homology is due to Khovanov and Rozansky [KR08a, KR08b], where they use matrix factorizations. This has been extended to the colored case by Yonezawa [Yon11] and Wu [Wu14] and to the uncolored equivariant case by Krasner [Kra10].
- An approach using the derived categories of coherent sheaves on some smooth projective varieties is due to Cautis and Kamnitzer [CK08a, CK08b].
- Mazorchuk and Stroppel [MS09] exploited the BGG category $\mathcal{O}$ to define the $\mathfrak{sl}_N$-homology in the uncolored case. This has been generalized to the colored case by Sussan [Sus07].
- Webster [Web13] studies 2-representations of 2-quantum groups to generalize this approach to every simple Lie algebra.
- Mackaay, Stosic and Vaz [MSV09] used the Kapustin–Li formula [KL03] to define a combinatorial model for the uncolored $\mathfrak{sl}_N$-homology.
- Finally, Queffelec and Rose [QR14] have a combinatorial construction using the skew Howe duality.

Emmanuel Wagner and I [RW17a] give a new approach which somehow sticks to the original works of Khovanov and Bar-Natan. We define a TQFT-like functor $\mathcal{F}_N$ from the category of webs and foams to the category of graded $\mathbb{Z}[X_1, \dots, X_N]^{\mathfrak{S}_N}$-modules. Webs are plane trivalent graphs whose edges are labeled by non-negative integers with a flow preserving conditions on every edges. Foams are natural cobordisms between webs.

From this trivalent TQFT, we obtain a definition of the equivariant $\mathfrak{sl}_N$-homology thanks to the hypercube of resolution based on the Rickard complexes associated with every crossings. The definition of the functor $\mathcal{F}_N$ relies on two ingredients:

- A $\mathbb{Z}[X_1, \dots, X_N]$-valued invariant $\langle \bullet \rangle_N$ of closed foams. This is the core of our joint work and this solves a problem which was open for more than 10 years. It is defined as follows:

$$\langle F \rangle_N := \sum_{c \text{ coloring of } F} (-1)^{s(F,c)} \frac{\prod_{f \text{ facet of } F} P_f(c(f))}{\prod_{1 \leqslant i < j \leqslant N} (X_i - X_j)^{\frac{\chi(F_{ij}(c))}{2}}}. \tag{1}$$

  In this formula, a coloring of a foam F is a map from the set of facets of F to $\mathcal{P}(\{1, \dots, N\})$. The symbols appearing on the right-hand side of the identity only depends on the topological and combinatorial property of the foam F endowed with the coloring c. Note that the fact that $\langle F \rangle_N$ is a polynomial is actually not obvious.
- The so-called *universal construction* [BHMV95], which allows in favorable circumstances to derive a TQFT from an numerical invariant of closed cobordisms.

The *MOY calculus* [MOY98, Rob15] associates with every graph Γ a Laurent polynomial in q with non-negative coefficients.

Theorem A.1 ([RW17a])**.** *The functor $\mathcal{F}_N$ categorifies the MOY calculus. This means that for any web Γ, $\mathcal{F}_N(\Gamma)$ is a finitely generated graded free $\mathbb{Z}[X_1, \dots, X_N]^{\mathfrak{S}_N}$-module whose graded rank is equal to $\langle \Gamma \rangle_N$. Moreover, using this functor in the hypercube obtained by replacing each crossing by its associated Rickard complex yields an equivariant colored $\mathfrak{sl}_N$-homology.*

[1] This means in particular that the ground ring is $\mathbb{Z}[X_1, \dots, X_N]$ with a grading given by $\deg(X_\bullet) = 2$, instead of $\mathbb{Z}$

Note that Ehrig, Tubbenhauer and Wedrich [ETW17] already used this functor to prove the functoriality of the colored $\mathfrak{sl}_N$-homology. They first use ideas of Bar-Natan [BN05] to established projective functoriality. Then the equivariant nature of the functor $\mathcal{F}_N$ enables them to work on a deformed version of the homology. It is then enough to check the consistency of some signs as Blanchet [Bla10] pointed out for the $\mathfrak{sl}_2$ case.

The integers labeling the links components are meant to represent *exterior* powers of V, the standard representation of $U_q(\mathfrak{sl}_N)$. A light modification of the MOY calculus enables to compute the Reshetikhin–Turaev invariants for links colored by symmetric powers of V. This is the *symmetric* MOY calculus. It associates with every web Γ a Laurent polynomial with non-negative coefficients. Symmetric powers are important since they appear in the definition of the Reshetikhin–Turaev invariants[2] for 3-manifolds.

Together with Emmanuel Wagner, we attempted to modify our construction in order to categorify the *symmetric* MOY calculus. We realized that it was not possible to have a construction as nice and general as in the exterior case. We need to narrow the category of webs and foams a little bit. We consider *vinyl* webs, they are the web analogues of braid closures. A similar restriction is considered for foams. We then manage to define a functor from the category of vinyl webs to the category of graded $\mathbb{Q}[X_1, \dots, X_N]^{\mathfrak{S}_N}$-modules. Surprisingly, the exterior evaluation formula (1) plays a key role in this construction.

Theorem A.2. [RW17b] *The functor $\mathcal{S}_N$ categorifies the symmetric MOY calculus. This means that for any web Γ, $\mathcal{S}_N(\Gamma)$ is a free graded $\mathbb{Q}[X_1, \dots, X_N]^{\mathfrak{S}_N}$-module whose (graded) rank is equal to $\langle\langle\Gamma\rangle\rangle_N$. Moreover, using this functor in the hypercube obtained by replacing each crossing by its associated Rickard complex yields a* symmetric *equivariant colored $\mathfrak{sl}_N$-homology.*

The definition of the functor $\mathcal{S}_N$ uses the family of functors $(\mathcal{F}_k)_{k\in\mathbb{N}}$ and a universal construction à la [BHMV95]. The proof of invariance is difficult: we need to compare our construction with the triply-graded homology ([Kho07], [Rou12]) which categorifies the HOMFLYPT polynomial. This homological theory is defined as the Hochschild homology of a complex of Soergel bimodules. From this detour, we obtain a foamy interpretation of singular Soergel bimodules of type A and of their 0th homology groups. Moreover, we exhibit a spectral sequence from the triply-graded homology to the symmetric homology.

A.2. **Upcoming developments.** There are many possible applications and generalizations to the construction described in the previous section. I am already working on some of them. The following list is by no mean exhaustive. I tried to sum up my future work into some *Projects*. Each of them is assigned some stars. One star means that the project is on-going and that the underlying mathematics are already understood. Two stars mean that the project has already started but some ideas are still missing. Finally, three stars mean that the project is both very ambitious and conjectural.

A.2.1. *Hochschild and Soergel bimodules.* The study of the categories of Soergel bimodules is a trending topic in representation theory. Indeed they are closely related to the positivity conjecture of Khazhdan–Lusztig (recently proved by Elias and Williamson [EW14]). Foams give an original point of view on these objects and can therefore be useful.

Project A.1 ($\star$)**.** *Detail the correspondence between Soergel bimodules and foams. In particular, translate the relations satisfied by foams in an algebraic language.*

Project A.2 ($\star$, avec Emmanuel Wagner)**.** *Describe the full Hochschild homology of singular Soergel bimodules of type A using foams.*

A.2.2. *Super link Homologies.* A foamy description of the Hochschild homology of Soergel bimodules would immediately yields a new definition of the triply-graded homology. However, our motivation for this last project was not the triply-graded homology but the categorification of the Reshetikhin–Turaev invariant of links associated with the super quantum group $U_q(\mathfrak{gl}_{M|N})$. Indeed, we believe that the $\mathbb{Z}_2$-graduation of the representation of $\mathfrak{gl}_{M|N}$ should lift onto the $\mathbb{Z}$-graduation given by Hochschild homological degree.

Project A.3 ($\star\star$, with Hoel Queffelec, David Rose and Emmanuel Wagner)**.** *Categorifies the link invariants associated with $U_q(\mathfrak{gl}_{M|N})$.*

[2] For the 3-manifolds invariants, q is a root of unity (see Section A.2.3).

When $M = N = 1$ (and more generally when $M = N$), the quantum invariant (at the decategorified level) is the Alexander polynomial. It is very likely that this new approach relates to the *classical* categorification of the Alexander polynomial: the Heegaard–Floer homology for links [OS03]. More precisely, we plan to compare our construction with the recent combinatorial results of Ellis, Petkova and Vertesi [EPV15] or the ones of Manolescu, Ozsváth and Sarkar [MOS06]. This would give a new bridge between quantum topology and symplectic geometry. We think that the colored version of these homologies should categorify the ADO invariants.

Finally, the existence of a super homology is a new hint in favor of Gorsky–Gukov–Stošić's conjectures [GGS13] claiming the existence of a quadruply-graded and of various symmetries between the different link homologies.

A.2.3. *Web algebras.* One of the reason of the attention reserved to link homologies is their (conjectural) connection with 3+1 dimensional TQFT. Indeed a categorification of the Reshetikhin–Turaev invariants for 3-manifolds should be such a TQFT. The Reshetikhin–Turaev invariants for 3-manifolds are defined using symmetric powers (this is what we do in [RW17b]) and to set q to be a root of unity. This last point causes a few problems. In [Kho16], Khovanov observes the category of p-complexes over a field of characteristic p categorifies $\mathbb{Q}[\zeta_p]$ (for ζ a pth primitive root of unity).

In order to use this observation in the framework of link homologies, Khovanov suggests to look for some p-DG algebra structures (over a field of characteristic p). He shows with Qi, that the nilHecke algebra is naturally endowed with such a structure [KQ15]. For the symmetric homology, the algebra to look at should be an analogue of the web algebras in the exterior case (see [Kho02] for $N = 2$, [Rob13] for $N = 3$ and [Mac14] for the general case). The difficulty of such a construction arises because we can only (until now) define the symmetric homology for braids closure.

Project A.4 ($\star\star\star$, with Emmanuel Wagner)**.** *Define the web algebra for the symmetric homologies.*

Project A.5 ($\star\star\star$)**.** *Endow the symmetric web algebras with structures of p-DG-algebras in characteristic p.*

This last project is extremely ambitious. It is the next key step in the categorification program of quantum $\mathfrak{sl}_N$-invariants. It makes no doubt that many difficulties will occur. Let me just say that the pursue of 3+1 dimensional TQFT legitimates on its own the whole categorification program.

A.2.4. *Arbitrary colorings.* The link homologies defined in section A.1 categorify the Reshetikhin–Turaev invariants associated with the exterior and symmetric powers of the standard representation of $U_q(\mathfrak{sl}_N)$. One may want to extend these constructions to any finite dimensional representation of $U_q(\mathfrak{sl}_N)$. For doing so, one can use an idea of Khovanov to categorify the colored Jones polynomial. I already used it for the case $N = 3$ [Rob16]. The point is to construct a *tensor resolution* of any simple finite dimensional module.

Project A.6 ($\star$, with Matthew Hogancamp)**.** *Categorify the Reshetikhin–Turaev invariants associated with arbitrary representation.*

A.2.5. *Relations with algebraic geometry.* The applications of the categorification of quantum $\mathfrak{sl}_N$ invariants are not bound to low dimensional topology. For example, we obtain in [RW17a] a formula for computing the Littlewood–Richardson coefficients; more generally we exhibit the multiplicative structure constants of the cohomology rings of partial flag varieties. Furthermore, Lobb and Zentner [LZ14] give a moduli space interpretation of the $\mathfrak{sl}_N$ web invariant: they associate with any web Γ a projective variety and prove that the Euler characteristic of this variety is equal to $\langle\Gamma\rangle_N$ evaluated in $q = 1$. Emmanuel Wagner and I conjecture that $\mathcal{F}_N(\Gamma)$ is isomorphic to the intersection cohomology of the variety. We have already proved this for some webs for which the associated varieties are partial flag varieties[3].

Project A.7 ($\star\star\star$, with Emmanuel Wagner)**.** *Modify the construction of Lobb and Zentner by the use of the intersection cohomology.*

This construction should have a symmetric analogue. We give a first hint [RW17b], but for now we cannot formulate any satisfactory conjecture.

[3]We then have an isomorphism as Frobenius algebra.

A.2.6. *Deformed homologies.* The equivariant flavor of our definition of the $\mathfrak{sl}_N$-homology encompasses all the deformations of the (non-equivariant) $\mathfrak{sl}_N$-homology. This deformation are the starting point for the definition of the Rasmussen invariant s_2 (case $N = 2$) [Ras10]. The invariant s_2 is the main ingredient which enabled to give a simple and combinatorial proof of the Minor conjecture for torus knots. The Rasmussen invariant for other values of N were first thought not to give any more information than s_2. This has been disproved by Lewark [Lob09, Lew13]. Moreover, we now know that different deformation (for a fixed N) give rise to different invariants. Rose and Wedrich [RW15] started to study these deformations. However, since the model they use is quite complicated, they could not keep track of the filtrations which are essential for the study of invariant of Rasmussen type.

Project A.8 ($\star\star$)**.** *Understand the filtration of the deformed $\mathfrak{sl}_N$ homologies.*

These deformed $\mathfrak{sl}_N$ homologies are very much connected with the branching rules[4] in the $(\mathfrak{sl}_N)_{n\in\mathbb{N}}$ sequence.

Project A.9 ($\star\star$)**.** *Investigate the branching rules of the $\mathfrak{sl}_N$-homologies thanks to foams.*

The symmetric homology that we define has the same equivariant nature. Hence, one can deform the theory, just like in the exterior case. For now, it is not clear that those deformed versions are of any interest. However, we think that if we manage to define Rasmussen-like invariants in this new framework, they should not relate to the slice genus, but to the braid index. We expect to get some Morton–Franks–Williams type inequalities.

A.2.7. *Other directions.* The Reshetikhin–Turaev invariants are defined for any simple Lie algebra $\mathfrak{g}$. So far the type A has receive almost all the attention. It makes sense to broaden the categorification program to other type. The type D seems the more accessible, since it has already been studied from a combinatorial point of view. (see for instance [KR07] or [VW14]).

Project A.10 ($\star\star\star$, with Pedro Vaz and Emmanuel Wagner)**.** *Categorifies the Reshetikhin–Turaev link invariants associated with $\mathfrak{so}_{2N}$.*

Foams (or at least some of them) can be seen as Poincaré duals to triangulations of 3-manifolds (then they are usual called *spines*).

Project A.11 ($\star\star\star$, with Rinat Kashaev and Emmanuel Wagner)**.** *Understand if we can define an invariants of 3-manifolds with formula (1).*

Everitt and Turner [ET14] have a very nice interpretation of the Khovanov homology as a derived functor of the inverse limit of a presheaf.

Project A.12 ($\star\star$, with Paul Turner)**.** *Adapt the construction of Everitt–Turner to the (exterior) $\mathfrak{sl}_N$-homology.*

Finally, I am very interested with the potential connections between foams I study (most probably the symmetric model) and the *spin foams* of physicists.

A.2.8. *Reverse engineering.* It is likely that our new combinatorial description sheds a new light on the tools used so far to define the $\mathfrak{sl}_N$-homology.

B. Side projects

B.1. **Signature invariant for webs.** Kauffman and Taylor [KT76] gave a beautiful 4-dimensional interpretation of the signature of a link. Let L be a link in $\mathbb{S}^3$ which is the boundary of B^4. We consider a properly embedded oriented surface Σ in B^4 whose boundary is equal to L. If W_Σ is the double branched cover of B^4 along Σ, $\sigma(L)$ is equal to the signature of W_Σ as an oriented 4-manifold. It is even possible to consider non-orientable surfaces, in this case, a correction term depending on the normal Euler number of the surface appears (see [GL78]).

Together with Catherine Gille, we define [GR] a signature invariant for web knotted in $\mathbb{S}^3$. We consider an embedded trivalent graph Γ together with a coloring: Every edge has a color a, b or c, such that at every vertex all colors are present. Let F be a colored foam properly embedded in B^4

[4]With *branching rules*, I mean the following: if M is an indecomposable $\mathfrak{sl}_N$-module, the inclusion of $\mathfrak{sl}_{N-1}$ in $\mathfrak{sl}_N$ yields a decomposition of M as $\mathfrak{sl}_{N-1}$-module. The branching rules makes this decomposition explicit.

such that $\partial F = \Gamma$. Interpreting a, b and c as the non trivial elements of $\mathbb{Z}_2 \oplus \mathbb{Z}_2$, one can construct W_F the so-called *Klein-covering* of B^4 along F. It turns out to be an orientable 4-manifold.

Theorem B.1 (Gille, R)**.** *The quantity $\sigma(\Gamma) := \sigma(W_F) + e(F)$ only depends on Γ, where $e(F)$ is analogue of the normal Euler number for foams.*

Project B.13 ($\star\star$, with Catherine Gille)**.** *Explain how to compute $\sigma(\Gamma)$ on a diagram of Γ and compute it on simplest knotted web. Understand how the signatures of a given knotted web for different colorings relate.*

Since we have managed to define a signature invariant for knotted webs, it is natural to hope that related invariants still make sense for embedded trivalent graphs.

Project B.14 ($\star\star\star$, with Catherine Gille)**.** *Define Tristram-Levine signature and Alexander modules for knotted webs.*

B.2. **Box totally dual integer polyhedra.** The solutions to a combinatorial problem of optimization can be considered as integral points in a certain vector space. The polyhedral approach aims to describe a polyhedron containing precisely these integral points by mean of linear inequalities. Beyond the structural properties that this point of view may reveal, its main interest is algorithmic and lies in the polynomial character of the optimization if the polyhedra is described by a polynomial number of inequalities.

A linear system is said to be *totally dual integer* (TDI) if its dual admits a integral optimal solution as soon as its optimum is finite. These systems play a key role in combinatorial optimization since they generate some min-max combinatorial relations. It is *box-TDI* if it remains TDI when the variables are bounded by some rational values.

Together with Patrick Chervet and Roland Grappe, we gave [CGR] a new characterization of box-TDI polyhedra. It is essentially different from the previous approaches and we can recover many results with very simple and short proofs.

A graph is *perfect* if the chromatic number of any induced sub-graph is equal to the size of a maximal clique of this sub-graph. Lovász [Lov72] proved that a graph G is perfect if and only if the system $A_G\mathbf{x} \leqslant \mathbf{1}, \mathbf{x} \geqslant \mathbf{0}$ is TDI where the matrix A_G is given by the cliques of G. A graph G is *box-perfect* if this system is box-TDI. In [DZZ16], Ding, Zang and Zhao conjecture a characterization of box-perfect graph and support their conjecture by large classes of examples.

Project B.15 ($\star\star$, with Patrick Chervet and Roland Grappe)**.** *Characterize box-perfect graphs.*

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