Titles and abstracts

SMS-DMV workshop

Spectra and L2-invariants

10 - 13 September 2018 Geneva, Switzerland

Invited talks

Laurent Bartholdi (University of Göttingen)

Amenability of associative algebras

Amenability of groups developed into a fundamental notion in part because of the sundry definitions it affords: via isoperimetry (Følner), L^2-spectrum (Kesten), Poisson boundary, etc. In the context of associative algebras, there is up to now only one definition, via isoperimetry; numerous natural questions arise, which I shall discuss, and sometimes answer by introducing a stronger version of amenability for modules over an associative algebra. The prime example of interest is that of a group ring.

Jean Bellissard (University of Münster, Georgia Institute of Technology)

Periodic Approximations to Aperiodic Hamiltonians

(joint work with Siegfried Beckus and Giuseppe De Nittis) This talk will provide a glimpse of the content of a series of articles already written or under writing, concerning the calculation of the spectrum of a self-adjoint operator by approximating the operator with a sequence or a family of self-adjoints operators. A special emphasis will be put on the case of Hamiltonians describing the quantum motion of a particle in an aperiodic medium, by approximating the medium by periodic ones as the periods goes to infinity.

Marc Burger (ETH Zürich)

On the real spectrum compactification of Teichmueller space

Using real semi algebraic geometry, and following an approach suggested by G. Brumfiel, we show how to compactify the Teichmueller space of all complete hyperbolic structures on a surface of finite topological type by a compact space, the real spectrum boundary, on which the mapping class group acts by homeomorphisms. The real spectrum boundary is functorial by restriction to subsurfaces, it dominates the Thurston boundary and it has the remarkable property that point stabilizers in the mapping class group are always abelian by finite. This is joint work with A. Iozzi, A. Parreau, and B. Pozzetti.

Corina Ciobotaru (University of Fribourg)

Central limit theorems on Grassmannians

For given $0 < r < n$, the real Grassmanian Gr(r,n) is a specific compact subset of the boundary at infinity of the symmetric space X of SL(n,R) and X is identified with the symmetric and positive definite n x n matrices of determinant one. Fix any Borel probability measure P on Gr(r,n) and let U_{kk} be random samples with values in Gr(r,n) that are i.i.d. accordingly to P. With P and the empirical probability measures P_{kk} on Gr(r,n) corresponding to the samples $U_1, \ldots, U_k$ we associate a unique matrix M_P and unique random matrices M_{kk} in X. Then a generalization of Law of Large Numbers and Central Limit Theorem hold true for M_{kk} and M_P. This is a recent joint work with Christian Mazza.

David Damanik (Rice University)

The Fibonacci Hamiltonian

The Fibonacci Hamiltonian is the central object in the study of Schrödinger operators with potentials displaying aperiodic order. In this talk we will explain how the spectral properties of this operator can be studied via a dynamical analysis of the trace map - a polynomial dynamical system that is induced by the self-similarity of the Fibonacci sequence. While initially a map of 3-space, the Fibonacci trace map may be restricted to invariant surfaces defined by the Fricke-Vogt invariant. On each of the spectrally relevant surfaces, the non-wandering set turns out to be a locally maximal hyperbolic set, which carries information about a variety of spectral quantities of interest. We will explain the resulting dynamical-spectral dictionary and the spectral results that can be obtained in this way. (This is joint work with Anton Gorodetski and William Yessen.)

Jean-Pierre Eckmann (University of Geneva)

Energy transport in Hamiltonian networks

I plan to give a review of networks of coupled systems of masses connected with springs. How does energy flow through such systems? Over the years, we have learned what matters and what are obstacles to a reasonable heat transport. This is work mostly done with Luc Rey-Bellet, Claude-Alain Pillet, Noe Cuneo, C. Eugene Wayne and Martin Hairer.

Alejandra Garrido (University of Düsseldorf)

Hausdorff spectra of free pro-p groups

Hausdorff dimension has become a standard tool for measuring the "fractalness" of a subset of a metric space. In the case of a finitely generated pro-p group, there are a range of natural filtrations that one could take (e.g. p-powers, dimension subgroups, iterated Frattini series), to define a metric and thus a Hausdorff dimension of subsets of the group. The Hausdorff spectrum of a pro-p group is the set of Hausdorff dimensions of all of its closed subgroups, and can be used as a measure of complexity of its subgroup structure. It was observed by Barnea and Shalev that if a pro-p group is a p-adic Lie group, then its Hausdorff spectrum is finite. They then asked whether this characterises p-adic Lie groups among pro-p groups, a question which has is still open after more than 20 years. I will introduce all the necessary notions and report on joint work with Garaialde-Ocaña and Klopsch on Hausdorff spectra of free pro-p groups.

Thierry Giordano (University of Ottawa)

Cohomology of Cantor minimal systems and a model for $\mathbb{Z}^2$-actions

Joint work with Ian F. Putnam and Christian F. Skau In 1992, Herman, Putnam and Skau used ideas from operator algebras to present a complete model for minimal actions of the group Z on the Cantor set, i.e. a compact, totally disconnected, metrizable space with no isolated points. The data (a Bratteli diagram, with extra structure) is basically combinatorial and the two great features of the model were that it contained, in a reasonably accessible form, the orbit structure of the resulting dynamical system and also cohomological data provided either from the K-theory of the associated C*-algebra or more directly from the dynamics via group cohomology. This led to a complete classification of such systems up to orbit equivalence. This classification was extended to include minimal actions of $\mathbb{Z}^2$ and then to minimal actions of finitely generated abelian groups. However, what was not extended was the original model and this has handicapped the general understanding of these actions. In this talk I will indicate how we can associate to any dense subgroup H of $\mathbb{R}^2$ containing $\mathbb{Z}^2$ a minimal action of $\mathbb{Z}^2$ on the Cantor set, such that its first cohomology group is isomorphic to H.

Rostislav Grigorchuk (Texas A& M College Station)

On the question "Can one hear the shape of a group?" and Hulanicki type theorem for graphs

In my talk I will address the following question raised independently by Alain Valette and Koji Fujiwara: "Can one hear the shape of a group?" – a rephrasing of the famous question of Mark Kac (which in fact can be traced down to Lipman Bers). Few approaches to the negative answer will be suggested. In particular it will be explained how to get uncountably many non pairwise quasi-isometric isospectral groups. Part of the arguments will be based on the result about spectra of covering graphs which is a weak version for graphs of the famous result of Hulanickii characterizing amenable groups in terms of weak containment. The talk will be based on a joint work with Artem Dudko, and, partially, on a joint result with Tatiana Nagnibeda and Aitor Perez.

Harald Helfgott (University of Göttingen)

Growth in linear algebraic groups and permutation groups: towards a unified perspective

Given a finite group G and a set A of generators, the diameter $\mathrm{diam}(\Gamma(G, A))$ of the Cayley graph $\Gamma(G, A)$ is the smallest ℓ such that every element of G can be expressed as a word of length at most ℓ in $A \cup A^{-1}$. We are concerned with bounding $\mathrm{diam}(G) := \max_A \mathrm{diam}(\Gamma(G, A))$.

It has long been conjectured that the diameter of the symmetric group of degree n is polynomially bounded in n. In 2011, Helfgott and Seress gave a quasipolynomial bound ($\exp((\log n)^{(4+\epsilon)})$). We will discuss a recent, much simplified version of the proof, emphasising the links in commons with previous work on growth in linear algebraic groups.

Alessandra Iozzi (ETH Zürich)

Irreducible lattices and bounded cohomology

We show some of the similarities and some of the differences between irreducible lattices in product of semisimple Lie groups and their siblings in product of locally compact groups. In the case of product of trees, we give a concrete example with interesting properties, among which some in terms of bounded cohomology (ℓ^2-stability) and quasimorphisms.

Anders Karlsson (University of Geneva)

Spectral Zeta Functions

Spectral zeta functions is one type of generating function formed out of the spectrum of Laplace operators, which has proven to be a useful tool in number theory and physics. I will discuss these functions for some finite and infinite graphs, a topic that has not been much studied (in contrast to the Ihara zeta function, which is an entirely different function). In asymptotics for families of graphs, classical number theoretic zeta functions appear. In these ways certain all-important problems in analytic number theory, such as the Riemann hypothesis or real zeros of Dirichlet L-functions, get surprising reinterpretations.

Johannes Kellendonk (Université de Lyon)

Envelopping semigroup of tiling dynamical systems

The envelopping (or Ellis) semigroup E(X,G) of a group G of homeomorphisms on a compact space X is the closure of this group in the space of all maps from X to X equipped with the topology of pointwise convergence. The topological and algebraic properties of E(X,G) depend on the properties of the dynamical system and can be used to characterise the latter. We investigate the minimal ideal of E(X,G) for systems with finite coincidence rank and provide examples of calculation which are related to tilings.

Wolfgang Lück (University of Bonn)

Universal L^2-torsion, L^2-Euler characteristic, Thurston norm and polytopes

Given an L^2-acyclic connected finite CW-complex, we define its universal L^2-torsion in terms of the chain complex of its universal covering. It takes values in the weak Whitehead group $Wh^w(G)$. We study its main properties such as homotopy invariance, sum formula, product formula and Poincaré duality. Under certain assumptions, we can specify certain homomorphisms from the weak Whitehead group $Wh^w(G)$ to abelian groups such as the real numbers or the Grothendieck group of integral polytopes, and the image of the universal L^2-torsion can be identified with many invariants such as the L^2-torsion, the L^2-torsion function, twisted L^2-Euler characteristics and, in the case of a 3-manifold, the Thurston norm and the (dual) Thurston polytope.

Nicolas Matte Bon (ETH Zürich)

Rigidity for full groups of pseudogroups acting on the Cantor set

A pseudogroup is a semigroup of homeomorphism between open subsets of a space which is closed under certain operations. A pseudogroup naturally defines a group, called its (topological) full group. It is known that given two minimal pseudogroups acting on the Cantor set, every group isomorphism between their full groups extends to a continuous isomorphisms between the ambient pseudogroups. I will explain a necessary and sufficient condition under which an arbitrary homomorphism from a topological full group to another group of homeomorphism extends to a continuous morphism of pseudogroups. I will illustrate this criterion in some concrete examples such as groups of interval exchanges, Higman-Thompson's groups, topological full groups of minimal subshifts.

Felix Pogorzelski (Leipzig University)

Approximation of spectral measures via dynamical systems

This talk deals with the continuity of spectral measures with respect to convergence of group dynamical systems as advanced in recent work of Beckus/Bellissard/de Nittis. We outline how for uniquely ergodic limit systems, the convergence of topological dynamical systems in the Chabauty-Fell topology implies weak* convergence of spectral quantities such as the autocorrelation measure of Delone sets or the density of states measure for random bounded operators. As an outlook towards further directions, we might address the more subtle question of convergence of spectral measures in singleton sets.

Joint work with Siegfried Beckus.

Roman Sauer (Karlsruhe Institute of Technology)

Profinitess question for l^2-Betti numbers

While the first l^2-Betti number of a finitely presented residually finite group is determined by the profinite completion, we give examples of S-arithmetic groups showing that this is not true for any higher l^2-Betti number. However, we show profiniteness results for higher l^2-Betti numbers of S-arithmetic groups in a more restrictive setting. Further, we will see that the sign of the Euler characteristic is profinite among arithmetic groups but not in general. Joint work with H. Kammeyer, S. Kionke, J. Raimbault.

Alain Valette (Université de Neuchâtel)

Diameters in box spaces

If G is a finitely generated residually finite group, and $(N_i)_{i>0}$ is a decreasing sequence of finite index normal subgroups with trivial intersection, we study the diameter of G/N_i as a function of the order $|G/N_i|$. For $0 < \alpha \geq 1$, we say (after Breuillard and Tointon) that the box space $\square_(N_i)G$ has property D_α, if the diameter of G/N_i grows at least like $|G/N_i|^\alpha$. With Ana Khukhro, we proved in 2015 that, if G maps onto $\mathbb{Z}$, then for every $\alpha < 1$, the group G admits a box space with D_α. When G embeds as a subgroup of $SL_N(\mathbb{Z})$, we look at box spaces obtained by intersecting G with principal congruence subgroups of $SL_N(\mathbb{Z})$. Take $G = \mathbb{Z}^2 \rtimes_A \mathbb{Z}$, where A is an infinite order matrix in $SL_2(\mathbb{Z})$. In joint work with Etienne Grezet, we show that, if an arithmetic box space of G has D_α, then $\alpha \leq 1/3$ if A is parabolic, or if A is hyperbolic and the box space satisfies some extra arithmetic conditions. This is related to questions in number theory, some of which are still open.

Contributed talks

Amitay Kamber (Hebrew University of Jerusalem)

(Joint work with Konstantin Golubev (ETHZ), who is also coming to the conference.)

Density Theorems and Almost-Diameter of Locally Symmetric Spaces

Recently it as been observed by Sardari and Lubetzky-Peres that the almost-diameter of a $(q+1)$-regular Ramanujan graph with n vertices $\log_q(2)$, which is an optimal result. We generalize the results to arbitrary locally symmetric spaces (over R or a p-adic field) and show that similar results actually follow from a weaker density hypothesis, in the spirit of the work of Sarnak-Xue. The density hypothesis can be stated as a claim about multiplicity of representations, which is stronger than Benjamini-Schramm convergence but weaker than the "naive" Ramanujan conjecture. A recent conjecture of Sarnak says that they are expected to hold in great generality.

Steffen Kionke (Karlsruhe Institute of Technology)

p-adic limits of Betti numbers

We show that Betti numbers in pro-p towers converge to a limit in the field of p-adic numbers. The limiting "p-adic Betti numbers" provide new invariants which are analogues of L^2 Betti numbers. However, these Betti numbers behave quite differently. For instance, they can distinguish certain amenable groups, but they are equal for all free groups. We give an introduction to these invariants and mention open problems and applications.

Michael Schrödl (Karlsruhe Institute of Technology)

L^2-Betti numbers of random rooted simplicial complexes and Benjamini-Schramm convergence

We define unimodular measures on the space of rooted simplicial complexes and associate to each measure a chain complex and a trace function. As a consequence we can define L^2-Betti numbers of unimodular measures and show that they are continuous under Benjamini-Schramm convergence.