Practical output feedback tracking control for a class of stochastic system

Tahar Khalifa. Issat Sousse. Tunisia. tkhalifa@laposte.net

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Abstract

This study investigates the global adaptive practical tracking for a class of nonlinear stochastic systems with dynamic uncertainties and unmeasurable states via dynamic output feedback control. We show that we can control the error in (1) for stochastic system and generalize the work in [2]. An output feedback controller is constructed to guarantee that the closed-loop system is globally practically stable in probability and the output can be regulated to the all fixed ball almost surely.

Notations and preliminary results

Consider the following stochastic nonlinear system

\[ dx = f(x)dt + g(x)dw \]  
(1)

Where \( x \in \mathbb{R}^n \) is the system state, \( x \) is an \( n \)-dimensional standard Wiener process defined on the complete probability space \( (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P}) \). The Borel measurable functions \( f : \mathbb{R}^n \to \mathbb{R}^n \) and \( g : \mathbb{R}^n \to \mathbb{R}^{n \times m} \). For any given function \( V(x) \in C^2(\mathbb{R}^n) \), associated with system (1), the differential operator \( \mathcal{L} \) is defined as

\[ \mathcal{L}V = \frac{\partial V}{\partial x}(f(x) + g(x)\mathbb{E}w) = \frac{\partial V}{\partial x}f(x) + \frac{1}{2} \sum_{i,j} \frac{\partial^2 V}{\partial x_i \partial x_j}g_{ij}(x)g_{ij}(x) \]

Definition 1. [3] The solution process \( (x(t), t \geq 0) \) of stochastic differential system (1) is said to be bounded in probability, if

\[ \lim_{t \to \infty} \sup_{x \in \mathbb{R}^n} P\{x(t) \in R \} \leq \epsilon \]

Theorem 1. [6] Consider the system (1) and assume that \( f \) and \( g \) are \( C^2 \). if there exists a function \( C^2 \) function \( V(x) \), class \( \mathcal{K}_{\infty} \) functions \( \beta_1 \) and \( \beta_2 \) a constant \( \epsilon > 0 \), and a nonnegative function \( W(x) \) such that

\[ \begin{align*}
\beta_1(|x|) & \leq V(x) \leq \beta_2(|x|) \\
L \mathcal{V}(x) & \leq -W(x) + \epsilon 
\end{align*} \]

Then, there exists a unique almost sure solution on \( [0, \infty) \).

0.4 Controller

Let \( \rho > 0 \), we introduce the controller via the full-order observer

\[ u_i = -\sum_{l=1}^{n} (LM)^{l-1}k_i^l_d \]

(7)

\[ d_i = l_i + (LM)^{l-1}(y_i - z_i) dt \]

(8)

where \( z = [z_1, \ldots, z_n]^T \) with the initial value \( z(t_0) = 0 \).

Proof

The error dynamics and the closed-loop system

Let \( \eta_i = z_i - y_i \) and define the following scaling state \( \zeta_i = (\zeta_{1,i}, \ldots, \zeta_{n,i})^T \) and estimation error \( \hat{\zeta}_i \) as follows:

\[ \hat{\zeta}_i = \frac{c_i}{(LM)^{i-1}} + \zeta_i \]

(12)

Where \( \beta > 0 \) is constant. Now, using (12), the closed-loop systems (6) and (7) can be expressed compactly as

\[ d\eta = ((LM)\Delta \eta - \frac{1}{2} \frac{\partial^2 V}{\partial x^2} \eta + D \hat{\zeta} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} \eta + G \eta + d(t, x, u, \eta) dw(t) \]

(13)

\[ d\hat{\zeta} = ((LM)^{n-1} \Delta \hat{\zeta} - \frac{1}{2} \frac{\partial^2 V}{\partial x^2} \hat{\zeta} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} \hat{\zeta} + \frac{1}{2} \frac{\partial^2 V}{\partial x^2} \eta + D \eta + G \eta + d(t, x, u, \eta) dw(t) \]

(14)

\[ F \frac{\partial \zeta}{\partial x} \frac{\partial \eta}{\partial x} + \frac{\partial G}{\partial x} \frac{\partial \eta}{\partial x} \]

(15)

1.1 Lyapunov analysis

Consider the Lyapunov function defined by

\[ V = \alpha_1 \eta + \alpha_2 \hat{\zeta} \]

(16)

\[ V_1 = \frac{1}{\alpha_1^2} \hat{\zeta}^2 + \eta \]

The remainder of the proof is omitted on grounds of space

References

[3] Shu-Jun Li, Ji-Feng Zhang, Zong-Ping Jiang,Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems.Automatica 2007, volume 43, pp 258 251