Supergiant variability: theoretical pulsation periods and comparison with observations

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Received November 23, accepted December 13, 1983

Summary. The pulsation periods of stars evolving with mass loss covering the upper part of the HRD ($M \geq 15 \, M_{\odot}$) are determined for the fundamental radial mode and for the first and second overtones. Theoretical P-L-C (period-luminosity-colour) relations are given. It is found that the pulsation $Q$-terms present a minimum near $T_{\text{eff}} = 20,000$ K, and that the $Q$-terms increase both for hot main sequence stars as well as for cool and luminous red supergiants. The physical reasons for these changes are discussed.

For O-stars at the same location in the HR diagram, but having undergone a different mass loss, the pulsation period may be decreased by up to about 20% for the smaller remaining stellar mass. There is a large period difference between supergiants on the first redward tracks and those at a post-red supergiant (RSG) stage (but at the same location in the HR diagram): the post-RSG stars have periods longer by a factor of typically 1.7 or even more according to the remaining mass. This is an interesting signature of post-RSG stars, which also exhibit enrichments in the products of the CNO-cycles.

The comparison of models and observations show that about 40% of the supergiants have periods which agree well with the fundamental mode of radial pulsation. However, there is a large group of supergiants exhibiting periods by a factor of 1.5 to 4 longer than the corresponding periods of radial pulsation. Non-radial oscillation in $g$-mode connected with convection as well as post-RSG evolution may be responsible for these too long periods. We also give the period ratios of overtones and the velocity to light amplitude ratios.

Key words: stellar pulsation – supergiants – mass loss

1. Introduction

The variability in radial velocity, luminosity and colour of most of the supergiants is now well established (see for example Abt, 1957; Maeder and Rulener, 1972). However, many uncertainties remain as to the exact nature of the pulsation modes and the origin of the instabilities of supergiants. For valuable recent reviews on this subject the reader may refer to de Jager (1980) and to A. N. Cox (1983). Cox also gives an interesting survey of some recent theoretical developments about supergiant pulsations and makes the suggestion that non-radial $g$-modes may promote some mixing of the hydrogen envelope into the intermediate convective shell. The argument that non-radial $g$-modes are excited in supergiants is based mainly on the fact that many of the observed quasi-periods appear to be longer than those of the fundamental radial mode (cf. Maeder, 1980). However, the theoretical periods (Takeuti, 1979) used for comparison were not based on recent evolutionary models with mass loss. The process of mass loss modifies substantially the structure, the size and sometimes the composition of the outer envelopes, which all influence the pulsation periods. Moreover, at the same place in the HR diagram stars can be found on bluewards or redwards evolutionary tracks with obviously great differences in their structure. The present paper has the rather simple and limited purpose of improving the comparison basis by providing the calibrations of pulsation periods for a grid of evolutionary models with mass loss. The periods of the fundamental mode, the first and second overtones of radial pulsation will be given at various evolutionary stages.

Section 2 gives some specifications on the analysis and models, and also contains the main results of pulsation periods and pulsation constants. The effects of mass loss on pulsation are examined in Sect. 3 and the comparison between the observed and theoretical periods is performed in Sect. 4.

2. Method of analysis and results

Determinations of periods were made using the classical linear adiabatic theory. The justifications for this choice are quite straightforward: linearity due to the small amplitudes of supergiant variability (a few percents in the light curves, cf. Table 1 in Maeder, 1980), and adiabaticity because of the quasi-adiabatic displacements within the stars except for the very outermost layers of the envelope. The linear adiabatic theory, although not fully realistic, has the major advantage of being easy to implement on computer without numerical ambiguity, and it runs quite fast. Furthermore, comparisons with more detailed calculations show that this theory gives reliable values of pulsation periods and relative amplitudes within the stars (cf. Cox, 1980). The models used were chosen in the grids computed and described by Maeder (1981). They belong to the 15, 30, and 60 solar mass evolutionary tracks with medium mass-loss rates (case B); some models were also taken from cases A and C to allow us to account for the effects of the adopted mass-loss rates. The effective temperature of these models runs from $\log T_{\text{eff}} = 4.6$ to $\log T_{\text{eff}} = 3.6$, and the luminosity from $\log L/L_{\odot} = 4.5$ to $\log L/L_{\odot} = 6.0$; these models are thus covering the upper part of the HR diagram (Table 1).
Table 1. Parameters of supergiant models

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Mass ((M_\odot))</th>
<th>Age ((10^6 \text{ yr}))</th>
<th>(\log L/L_\odot)</th>
<th>(\log T_e)</th>
<th>(M_{\text{bol}})</th>
<th>(\log R/R_\odot)</th>
<th>(\vartheta_e)</th>
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<tr>
<td>150</td>
<td>15.0</td>
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<td>4.28</td>
<td>4.48</td>
<td>-5.94</td>
<td>0.715</td>
<td>3.45 \times 10^4</td>
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<td>151</td>
<td>13.7</td>
<td>11.55</td>
<td>4.55</td>
<td>4.41</td>
<td>-6.61</td>
<td>0.981</td>
<td>1.61 \times 10^3</td>
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<tr>
<td>152</td>
<td>13.7</td>
<td>11.59</td>
<td>4.50</td>
<td>4.03</td>
<td>-6.50</td>
<td>1.729</td>
<td>1.32 \times 10^7</td>
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<tr>
<td>153</td>
<td>13.6</td>
<td>11.61</td>
<td>4.66</td>
<td>3.58</td>
<td>-6.87</td>
<td>2.701</td>
<td>1.06 \times 10^10</td>
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<tr>
<td>154</td>
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<td>12.68</td>
<td>4.58</td>
<td>4.02</td>
<td>-6.69</td>
<td>1.786</td>
<td>3.63 \times 10^7</td>
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<tr>
<td>155</td>
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<td>12.92</td>
<td>4.76</td>
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<td>4.44</td>
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<td>1.304</td>
<td>1.71 \times 10^3</td>
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<td>5.41</td>
<td>3.90</td>
<td>-8.76</td>
<td>2.439</td>
<td>3.73 \times 10^8</td>
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<td>4.35</td>
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<td>1.792</td>
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<td>5.82 \times 10^6</td>
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<tr>
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<td>60.0</td>
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<td>5.97</td>
<td>4.50</td>
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<td>1.538</td>
<td>1.56 \times 10^3</td>
</tr>
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<td>621</td>
<td>39.0</td>
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<td>5.82</td>
<td>4.50</td>
<td>-9.78</td>
<td>1.432</td>
<td>1.40 \times 10^3</td>
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Table 2. Computed periods of pulsation

<table>
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<tr>
<th>Model No.</th>
<th>Fundamental mode</th>
<th>1st overtone</th>
<th>2nd overtone</th>
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<tbody>
<tr>
<td></td>
<td>(P(d))</td>
<td>(\log Q)</td>
<td>(P)</td>
</tr>
<tr>
<td>150</td>
<td>0.128</td>
<td>-1.37</td>
<td>0.0916</td>
</tr>
<tr>
<td>151</td>
<td>0.289</td>
<td>-1.44</td>
<td>0.216</td>
</tr>
<tr>
<td>152</td>
<td>3.86</td>
<td>-1.44</td>
<td>2.93</td>
</tr>
<tr>
<td>153</td>
<td>219.2</td>
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<td>116.5</td>
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<tr>
<td>154</td>
<td>5.08</td>
<td>-1.42</td>
<td>3.79</td>
</tr>
<tr>
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<td>313.4</td>
<td>-1.13</td>
<td>162.2</td>
</tr>
<tr>
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<td>0.187</td>
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<td>0.119</td>
</tr>
<tr>
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<td>0.671</td>
<td>-1.43</td>
<td>0.504</td>
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<tr>
<td>302</td>
<td>42.8</td>
<td>-1.33</td>
<td>28.3</td>
</tr>
<tr>
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<td>904.0</td>
<td>-0.95</td>
<td>276.9</td>
</tr>
<tr>
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<td>0.277</td>
<td>-1.23</td>
<td>0.152</td>
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<td>1.52</td>
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<td>2.71</td>
<td>-1.42</td>
<td>2.13</td>
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<td>45.8</td>
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<td>27.4</td>
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<td>7.00</td>
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<td>4.58</td>
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<tr>
<td>621</td>
<td>0.827</td>
<td>-1.43</td>
<td>0.645</td>
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</table>

\[ a) \text{ Pulsation periods} \]

The parameters of the models for which the pulsations have been studied, the computed periods and the \(\log Q\)-values for the fundamental radial mode and the first two overtones are given in Tables 1 and 2. The isoperiodic lines for the fundamental mode are plotted in the HR diagram given in Fig. 1. It was found necessary to split the area under investigation into two zones, one for O-type stars and early B-type supergiants and the other for late B- to M-types. We shall discuss below the reason for this splitting. The following period-luminosity-colour relation is obtained from the models hotter than \(\log T_{\text{eff}} = 4.3\) in Tables 1 and 2:

\[
\log P_0 = -0.232 M_{\text{bol}} - 2.850 \log T_{\text{eff}} + 9.294. \tag{1}
\]

This relation applies to the fundamental mode and represents the model data with a root mean square of 0.021 in \(\log P_0\). (The
relation obtained from models cooler than $\log T_{\text{eff}} = 4.1$ is

$$\log P_0 = -0.275 M_{\text{bol}} - 3.918 \log T_{\text{eff}} + 14.543$$

(2)

which represents the model data with a root mean square of 0.053 in $\log P_0$. The isoperiod lines drawn in Fig. 1 are based on relations (1) and (2); average values are taken for a period of 2 d. It has to be specified that these relations apply to case B-star models on their first (redwards) crossing of the HR diagram. The difference in pulsation periods for stars with other mass loss rates or at more advanced evolutionary stages will be discussed in Sect. 3.

As a comparison the observational calibration from Maeder and Rufener (1972) based on late B- to G-type supergiants is

$$\log P = -0.346 M_{\text{bol}} - 3.01 \log T_{\text{eff}} + 10.60$$

(3)

This calibration is also plotted in Fig. 1 with dashed lines. The observed periods are generally too long with respect to the theoretical ones, especially at higher temperatures; discrepancies will be further discussed in Sect. 4.

b) Pulsation constants

The pulsation constants $Q_i$, defined by

$$Q_i = P_i \left( \frac{\bar{\varrho}}{\varrho_0} \right)^{1/2} = P_i \left( \frac{M/M_0}{(R/R_0)^3} \right)$$

(4)

are given in Table 2 for the various pulsation modes $i$ considered. For the fundamental pulsation mode the pulsation constant $Q_0$ verifies the following relation in the hot region:

$$\log Q_0 = -0.003 M_{\text{bol}} + 0.688 \log T_{\text{eff}} - 4.480$$

(5)

which is essentially independent of $M_{\text{bol}}$. In the cooler region, the relation is

$$\log Q_0 = -0.054 M_{\text{bol}} - 0.864 \log T_{\text{eff}} + 1.635$$

(6)

with some moderate dependence on $M_{\text{bol}}$. The rms for respectively relations (5) and (6) are the same as for relations (1) and (2). It is worth underlining that the sign of the coefficient of $\log T_{\text{eff}}$ is different for the two $\log Q_0$ relations. This is actually due to a minimum in the $\log Q_0$ values at $\log T_{\text{eff}} = 4.2$.

The variations of $P$ and $Q$ with $M_{\text{bol}}$ and $T_{\text{eff}}$ can be easily understood on the basis of two well known properties of stellar pulsations. First, all other things being equal, $Q$ (which is proportional to the reciprocal of the dimensionless eigenvalue) decreases with the increasing central condensation. Second, all other things being equal, $Q$ increases with decreasing adiabatic coefficient $\Gamma_1$. It is well known that for massive ZAMS stars the decrease of $\Gamma_1$ with increasing mass leads to an increase of $Q$ which is not compensated by the slight increase of $\bar{\varrho}/\varrho_0$. As $\bar{\varrho}$ decreases with increasing mass, $P$ gets higher values.

As the star evolves from the ZAMS, $\Gamma_1$ does not change much at first, while $\varrho_0/\bar{\varrho}$ increases rapidly leading to a decrease of $Q$. However, $\bar{\varrho}$ decreases quickly enough to allow $P$ to increase.

However, as the effective temperature decreases, the extent of the hydrogen and helium ionization regions, where $\Gamma_1$ is small, grows to finally cover a large fraction of the radius, when the star is
in the red giant stage. At the same time, an external convective envelope develops (the line where models have a convective zone extending over 5% of the total radius is given in Fig. 2). Moreover as \( \varphi / \bar{\varphi} \) becomes very large, the pulsation amplitudes are large only in the outer layers which make the properties of the pulsation very sensitive to the structure of these outer layers and make \( T_{\text{eff}} \) the main parameter influencing \( Q \). Thus, there will be a minimum in \( Q \) when the ionization zones are large enough. Afterwards, \( Q \) will increase as the star moves to low \( T_{\text{eff}} \). This explains the signs of the coefficient of \( \log T_{\text{eff}} \) in Eqs. (5) and (6).

As for sufficiently low \( T_{\text{eff}} \), \( Q \) increases and \( \bar{\varphi} \) decreases. \( P \) can only increase when the star moves rightwards and it will increase faster near the M.S. in agreement with Eqs. (1) and (2).

c) Overtones, amplitude ratios

Various other characteristics of the pulsations have also been calculated and are now being discussed. Let us first examine the periods of the first two overtones. The period ratio \( P_1/P_0 \) of the first overtone to the fundamental mode is sometimes used, either for determining the pulsation constant \( Q_0 \), or for invoking non-radial pulsation by excluding radial modes (cf. for example Christy, 1966). The various periods \( P_i \) and pulsation constants are given in Table 2; for the first overtone, the following relation is verified throughout:

\[
\log P_1/P_0 = -0.88 \log Q_0 - 1.35
\]

and for the second overtone:

\[
\log P_2/P_0 = -0.92 \log Q_0 - 1.51.
\]

The eigenfunction amplitude varies within the star. We found that the surface to center relative amplitudes ratio is directly proportional to the concentration factor

\[
\frac{A_s}{A_c} = 0.178 \frac{\varphi_c}{\bar{\varphi}} \quad \text{with} \quad A = \delta r/r
\]

from \( \varphi_c/\bar{\varphi} \simeq 30 \) (ZAMS models) to \( \varphi_c/\bar{\varphi} = 3 \times 10^5 \). For more centrally condensed models, no simple relation holds any more. The above relation shows, as is normally expected, that the more centrally condensed a star is, the more important may be the envelope pulsations with respect to variations in the center.

The ratio \( A_{kv}/A_V \) of the amplitudes of the velocity to light ratio at the surface of the star is an even more interesting quantity, since it can be compared to observational data (\( A_{kv}/A_V \) is expressed in \( \text{km s}^{-1} \text{mag}^{-1} \)). This ratio, corrected for the effect of integration over the stellar surface, shows a rapid dependence on effective temperature, ranging from typically \( A_{kv}/A_V = 350 \) on the ZAMS, to 100 at the end of main sequence, and lowering to 3.5 at the reddest points at \( \log T_{\text{eff}} = 3.6 \). It is also known that for radial pulsation (Balona and Stobie, 1979) the function

\[
\frac{A_{kv}}{A_V} = \frac{P}{R_0}
\]

where \( P \) is the period of pulsation in days, and \( R_0 \) the equilibrium radius in solar units, is not varying too much. This expression still depends on effective temperature for hot stars (for example for RR Lyrae models, Christy, 1966), while it was found to be nearly constant (ranging from 2 to 4) for cool stars (\( \log T_{\text{eff}} < 4.0 \)).
3. Effects of stellar evolution with mass loss on the pulsation periods

Let us first investigate the effects of mass loss on the pulsation periods for stars near the main sequence. For this purpose we analyse three models, with initial $60 \, M_\odot$, at the end of the main sequence, but having undergone different mass loss rates during their main sequence lifetimes. These models are labelled 611, 601, and 621 in Tables 1 and 2. They correspond respectively to case A (constant mass), case B (intermediate mass loss rates, $47 \, M_\odot$ are left) and case C (high mass loss rates, $39 \, M_\odot$ are left) in the computed evolutionary tracks from Maeder (1981). Of course, these models are not exactly at the same place in the HR diagram and we have corrected this by using the calibrations [relations (1) and (5)] given above in order to enable a direct comparison. It is found that mass loss, which affects both the structure and the mean density, leads to a decrease of the pulsation periods and of the pulsation constants. The effects are rather small. In the above case of initial $60 \, M_\odot$ stars, where mass loss is large, the decrease of periods is 19% from case A to B and 22% from case A to C. For the $Q_\odot$-values the decreases respectively are 16% and 26% in the two examples above.

Let us now compare the periods of stars moving rightwards in the HR diagram with those of stars moving rightwards. At a given point in the HR diagram, stars after the post-red giant phase have a lower mass and a larger mass concentration than stars in the pre-red giant phase. Due to the combination of the properties recalled in Sect. 2b and Eq. (4) we get a larger $Q$ and a longer period for the older models. The increase in period will be all the more important as the mass loss during the red giant phase is large. This is verified by the data of Tables 1 and 2.

In the case of an initial $60 \, M_\odot$ (cf. the case of models 602 and 605, which are not extreme cases), the period of the post RSG star is 1.70 times that of the redwards moving star (after correcting for location in the HR diagram; for $\log Q_\odot$, the corresponding ratio is 1.39). The longer periods are a striking signature of stars in a post-red supergiant stage. We emphasize that pulsation periods, as well as changes of surface abundance of CNO elements, appear as essential means to analyse the exact evolutionary status of blue and yellow supergiants and to know whether these stars have already evolved through the RSG stage.

In the case of less massive stars which do not evolve towards the WR stage but only make blue loops in the HR diagram, the increase of the period for the star on the blue loop is rather small. For models with initial mass of $15 \, M_\odot$, the cases of models 152 and 154 in Tables 1 and 2 show that the period increase amounts only to 7% after correcting for the locations in the HR diagram.

4. Comparison with observations

The theoretical pulsation periods obtained above and covering the whole upper HR diagram offer an improved comparison basis for observations. It is therefore interesting to examine whether significant deviations from the periods of radial pulsation exist in the observations. This fact would clearly have some consequences for our understanding of the nature of pulsations and of the evolutionary status of these stars.

The observational data used here are the values of $M_{bol}$, $\log T_{eff}$, $\log P$ for 32 supergiants with spectral types ranging from B1 to G0 collected by Burki (1977). Figure 3 shows the distribution of $\Delta \log P$, where $\Delta \log P=\log P_{obs} \ - \log P_\odot$, where $P_\odot$ stands for stars moving rightwards in the HR diagram.

From this sample the following conclusions can be drawn:

a) The two stars with $\Delta \log P < 0.1$ (HD 14956 and HD 99953, both B2Ia), are clearly "pathological" from our point of view, i.e. their actual periods being about 10 times the predicted ones indicate that something special occurs with these stars (this could be, for example, undetected binarity). Thus, these two stars deserve some further specific observations.

b) The 30 other stars exhibit a roughly bimodal distribution, the first mode being centered on $\Delta \log P=0$ with a half-width of 0.1, and the second centered near 0.4 with a half-width of about 0.2.

c) The first group of stars centered on $\Delta \log P=0$ has pulsation periods corresponding to the values of the fundamental radial mode. This suggests that some fraction (about 40%) of supergiants may be radially pulsating in the fundamental mode. A large part of the width in the distribution certainly comes from the difficulty in assigning precise values for the quasiperiods, as well as from uncertainties in luminosity and $T_{eff}$ determinations.

One might wonder whether a sizeable fraction of supergiants on the bluewards tracks really exists, since the models predict very short lifetimes on such tracks. However, there is evidence for a general excess of blue supergiants indicating a possible main sequence widening (cf. Meylan and Maeder, 1982). Of course the right models for these supergiants in excess (with mixing,
overshooting, etc.) are still to be found and some effect on the pulsation periods may also be expected to come from this side.

d) There is a larger group of stars with actual periods 1.5 to 4 times larger than the predicted ones. This large difference cannot be explained simply by errors, moreover there is no single star with a period much shorter than predicted. Thus, we conclude that the existence of variable supergiants with periods longer than the fundamental periods of radial pulsation is real (cf. Maeder, 1980).

e) Some effects which could influence the values of the period were studied in the last paragraph. Among others rotation may also be considered. Rotation in general was shown to have little effect upon pulsational period, unless the adiabatic exponent \( \Gamma_1 \) is very close to 4/3 (Simon, 1969; Stothers, 1981). In this case, e.g. for extremely rare supergiants, rotation may lead to a marked decrease of the period of pulsation. Some test computations, using the formalism of Stothers, show that these conclusions remain valid for supergiant stars. Therefore, we conclude that, owing to the sign and size of the effects, rotation cannot be responsible for the observed differences.

f) We may envisage that these too long periods may be explained either by non-radial pulsations (cf. Maeder, 1980; Cox, 1983), or following our discussion of Sect. 3 by stars at a farther evolutionary stage and with more mass loss than in the case of models on bluewards tracks or by these two phenomena together. These two causes could reasonably be expected, since on one side non-radial \( g \) modes are related to convective instability and the brightest supergiants, even in the early types, have large convective zones. On the other side, blue and yellow supergiants are also expected on the bluewards tracks or on the blue loops.

Consequently, the improved models used here not only confirm the trends previously found but also lead to a more precise view of the comparison of observed and theoretical periods. We think that the best way to discriminate between the two possibilities for explaining the too long periods is to make simultaneous observations and analysis the colour-magnitude diagram, variability and CNO surface abundances for well-selected supergiant stars in clusters and associations.

References

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