Improving Ash Cloud Forecasts via Bayesian learning algorithms

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Cordon Caulle volcano, Chile, 2011
Injection and transport of volcanic ash within the atmosphere

1. Ash mass is significant relative to air mass only at the eruption column source i.e. Costa et al, 2013

2. Transport in the atmosphere is passive, as mean settling time for mean size range in ash clouds is low (.036 km/hour @ 10 microns)

3. Simulations of ash clouds from eruption column source use wind solutions to obtain velocity and most models only concerned with conserving mass. This has been a successful strategy.

4. As the mass flux varies with altitude over time, so does the direction (azimuth) of the cloud streaming from the eruption column.

All that matters in simulating the position of ash clouds are the altitudes and times at which ash is injected into the atmosphere
As either wind direction or ash altitude changes, the direction in which the ash cloud moves changes...

*This information can be used to train a computer to track and forecast an ash cloud*
Method of training a computer to forecast ash clouds

1. **Define training criteria to track an ash cloud using satellite data**
   - Overlap = (Area with both model and satellite cloud)/Area model cloud
   - The Jacobians of posterior with model inputs \( w \) are the training data, with each \( w \) consisting of source mass, altitude, and time

2. **Determine the posterior distribution of model input data**
   - *Use Bayes Theorem*: For input parameters \( w \) to a transport model \( M \) given ash cloud observation data \( D \)
     
     \[
     P(w \mid D, M) = \frac{P(D \mid w, M) P(w \mid M)}{P(D \mid M)}
     \]

3. **Make and refine ash forecasts using posterior and training data**
   - *Posterior distributions weight each forecast to include uncertainty in a position*

These steps are accomplished within a single layer, supervised neural network
1. Define the training criteria

Overlap = Model Area with both satellite and model data / Model Area

Error in altitude from differences between model and satellite altitude

Error in mass from differences between model and satellite mass

Model is trained by adjusting model input parameters \( w \) so as to minimize an error function

\[
E_D = \sum_{n=1}^{N} \sum_i \frac{D(x_i, t^N) - M_w(x_i, t^n)^2}{\sigma_w^2}
\]

Each satellite observation \( D \) at position \( x_i \) time \( t^N \) is made up of ash emitted hours before.

Each previous time \( t^n \) has a point source altitude and source mass flux \( M_w \) consists of model output using input parameters \( w \) to a transport model \( M \)
2. **Define probability distributions of model input**

Treat $E_D$ as a negative log likelihood

$$P(D | w, M) = \exp(-E_D(w)) / Z_D$$

Defining a log prior probability over the parameters acts as a buffer against overfitting

$$P(w | M) = \exp(-\alpha E_w) / Z_w$$

Where $\alpha$ is the inverse in the variance of distribution in the differences between model output and prior values, $E_w$ is the sum of the square of those differences, and $Z_w$ is a normalizing constant.
3. **Determine posterior distributions for model input**

Then, using Bayes theorem, the posterior distribution is Gaussian

\[
P(w \mid D, M) = \frac{P(D \mid w, M)P(w \mid M)}{P(D \mid M)} = \frac{\exp(-Q(w))}{Z_Q}
\]

and the objective function \( Q(w) \) corresponds to the inference of input parameters \( w \) given the satellite data

A Taylor expansion of \( Q(w) \) about the maximum likelihood gives the cost functions used by Eckhardt et al. (2008), Stohl (2009; 2010), and many others to track either gas or ash clouds\(^1\)

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1. Denlinger, Pavolonis, and Sieglaff, 2010
4. **Make predictions of future ash clouds**

The probability that a future location $x(n+1)$ at time $t(n+1)$ will contain an ash cloud depends on the *overlap* of the model and satellite cloud (a fraction between 0 and 1) and the posterior distributions that produce that overlap.

$$P(\text{cload}(x^{n+1}, t^{n+1}) | D, w, M) = \int \text{Overlap}(w, t^{n+1})P(w | D, M)dw$$

All *nonzero* overlaps between model clouds and satellite clouds at time, $t^N$, weighted by the posterior distribution on $w$ determined at that time, are mapped into the *future*:

1. **Zero overlap gives zero probability**

2. **Nonzero overlap is weighted by how well model matches data**

3. **All fits, not just the best fit, are projected and added together**
Example: Track and forecast ash clouds from Eyjafjallajokull volcano from May 5-7\textsuperscript{th} 2010

1155 hours May 6, 2010

brown = ash

1240 hours May 7, 2010

Gudmundsson et al. 2012
Ash Cloud 1500h on May 6, 2010

Eyjafjallajökull volcano

Projected ash from different source elevations at 00h on May 6th, 2010

Satellite Cloud Load kg/km²

Data from Mike Pavolonis
Data reduction described in Pavolonis et al, 2013
Relative contribution of Model Input Parameters

*Posterior distributions of model input data*

Product of cloud area overlap and posterior distributions of model input
Ash Cloud 1500h on May 7, 2010
Eyjafjallajökull volcano

Projected ash from 00h to 600h on May 6th
Ash Cloud 1500h on May 7, 2010

Eyjafjallajökull volcano

Projected ash from 00h to 900h on May 6th
Ash Cloud 1500h on May 7, 2010
Eyjafjallajökull volcano

Projected ash from 00h to 1200h on May 6th
Ash Cloud 1500h on May 7, 2010

Eyjafjallajökull volcano

Projected ash from 00h to 1500h on May 6th
Conclusions

1. Wind direction varies strongly with altitude in the troposphere. Hence the path of volcanic ash injected into the atmosphere is very sensitive to the time(s) and altitude(s) at which the injection occurs.

2. This sensitivity may be exploited to forecast the future position of an ash cloud by matching the movement of the cloud to global forecast winds (GFS winds).

3. A supervised neural network provides a flexible and powerful framework by which a computer can be trained to automatically lock on to the position of an existing ash cloud, and then use the uncertainty in that position to characterize the uncertainty in forecasts of future ash clouds.

4. The use of Bayesian inference makes the forecast robust, and insulates it against overconfident predictions based solely on maximum likelihood.