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Labour Productivity vs. Total Factor Productivity

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Abstract

In this paper, we examine the relationship between two commonly used measures of productivity, namely labour productivity and total factor productivity. We show that total factor productivity is an essential component of labour productivity. Labour productivity is further influenced by capital intensity, and, in the open economy context, by the terms of trade and the real exchange rate. Complete multiplicative decompositions of productivity are given for Switzerland for the period 1980 to 2002. Our analysis rests on a tight theoretical framework being based on the GDP function approach to modelling the production sector of an open economy.

Keywords: labour productivity, total factor productivity, index numbers, technological change, capital deepening, terms of trade, real exchange rate

JEL classification: D24, O47, E25, F43

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Labour Productivity vs. Total Factor Productivity

0. Introduction

Most headline productivity measures refer to the average product of labour, with productivity growth being typically explained by capital deepening and technological progress. Many economists, however, are more interested in total factor productivity (TFP). Although this is a less intuitive concept, total factor productivity, as indicated by its name, is more general in that it encompasses all factors of production, rather than just one of them. It turns out that TFP is an essential component of the productivity of labour. A contribution of this paper is to document this relationship. A multiplicative decomposition of Swiss labour productivity, 1980–2002, is provided as an illustration.

A second contribution of the paper is to move beyond the usual and rather restrictive two-input, one-output production function setting. Thus, we expand the model by adopting the GDP-function framework that allows for many inputs and outputs, including imports and exports. This makes it possible to show that labour productivity is influenced by additional forces, namely changes in the terms of trade and in the real exchange rate. A complete decomposition of Swiss productivity growth is provided for this case as well.

1. Labour productivity in the production function context

Let the aggregate technology be represented by the following two-input, one-output production function:

(1) $$y_t = f(v_{L,t}, v_{K,t}, t) \ ,$$

where y_t measures the quantity of output, $v_{L,t}$ denotes the input of labour services, and $v_{K,t}$ is the input of capital services, all three quantities being measured at time *t*. The production function itself is allowed to shift over time to account for technological change. We assume that the production function is linearly homogeneous, increasing, and concave with respect to the two input quantities. In what follows, we will also assume competitive behaviour and profit maximization.

The average product of labour ($a_{L,t}$), is defined as:

(2) $$a_{L,t} \equiv \frac{y_t}{v_{L,t}} .$$

In terms of production function (1) we can also write:

(3) $$a_{L,t} = a_L(v_{L,t}, v_{K,t}, t) \equiv \frac{f(v_{L,t}, v_{K,t}, t)}{v_{L,t}} .$$

The labour productivity index ($A_{t,t-1}$) can be expressed as one plus the rate of increase in the average product of labour between period *t-1* and period *t*:[1]

(4) $$A_{t,t-1} \equiv \frac{a_L(v_{L,t}, v_{K,t}, t)}{a_L(v_{L,t-1}, v_{K,t-1}, t-1)} .$$

Note that it follows from the linear homogeneity of the production function that $a_L(\cdot)$ is homogeneous of degree zero in $v_{L,t}$ and $v_{K,t}$. The same is therefore true for $A_{t,t-1}$, which thus depends on changes in *relative* factor endowments and on the passage of time only.

2. Accounting for labour productivity

We next turn to the task of accounting for the changes over time in labour productivity. Using (4) as a starting point, we can define the following index that isolates the effect of changes in factor endowments over consecutive periods of time:

(5) $$A^L_{V,t,t-1} \equiv \frac{a_L\left(v_{L,t}, v_{K,t}, t-1\right)}{a_L\left(v_{L,t-1}, v_{K,t-1}, t-1\right)} .$$

When defining $A^L_{V,t,t-1}$ we have held the technology constant at its initial (period *t-1*) state. $A^L_{V,t,t-1}$ has thus the Laspeyres form, so to speak. Alternatively, we can adopt the technology of period *t* as a reference. We then get the following Paasche-like index:

[1] Throughout this paper we will use the term labour productivity to designate the *average* productivity of labour. For a discussion of the relationship between *average* and *marginal* productivity, see Kohli (2004c).

(6) $$A_{V,t,t-1}^{P} \equiv \frac{a_L(v_{L,t}, v_{K,t}, t)}{a_L(v_{L,t-1}, v_{K,t-1}, t)} \ .$$

Since there is no reason *a priori* to prefer one measure over the other, we can follow Diewert and Morrison's (1986) lead and take the geometric mean of the two indexes just defined. We thus get:

(7) $$A_{V,t,t-1} \equiv \sqrt{\frac{a_L(v_{L,t}, v_{K,t}, t-1)}{a_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{a_L(v_{L,t}, v_{K,t}, t)}{a_L(v_{L,t-1}, v_{K,t-1}, t)}} \ .$$

Note that if capital deepening takes place, both $A_{V,t,t-1}^{L}$ and $A_{V,t,t-1}^{P}$ are greater than one, in which case $A_{V,t,t-1}$ must exceed one as well.[2]

In the same vein, we can define an index that isolates the impact of technological change. That is, we compute the index of labour productivity, allowing for the passage of time, but holding factor endowments fixed, first at their level of period *t-1*, and then at their level of period *t*:

(8) $$A_{T,t,t-1}^{L} \equiv \frac{a_L(v_{L,t-1}, v_{K,t-1}, t)}{a_L(v_{L,t-1}, v_{K,t-1}, t-1)}$$

(9) $$A_{T,t,t-1}^{P} \equiv \frac{a_L(v_{L,t}, v_{K,t}, t)}{a_L(v_{L,t}, v_{K,t}, t-1)} \ .$$

Taking the geometric mean of these two indexes, we get:

(10) $$A_{T,t,t-1} \equiv \sqrt{\frac{a_L(v_{L,t-1}, v_{K,t-1}, t)}{a_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{a_L(v_{L,t}, v_{K,t}, t)}{a_L(v_{L,t}, v_{K,t}, t-1)}} \ .$$

Comparing (7) and (10) with (4), it can then easily be seen that $A_{V,t,t-1}$ and $A_{T,t,t-1}$ together yield a complete decomposition of the labour productivity index:

(11) $$A_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1} \ .$$

3. Total factor productivity

While labour productivity remains the concept of choice when it comes to the public

[2] This follows directly from the slope and linear homogeneity properties of the production function.

debate, most economists prefer to think in terms of TFP. The measure of TFP treats all inputs symmetrically. In the production function context, it can be defined as the increase in output that is not explained by increases in input quantities. Put differently, it is the increase in output made possible by technological change, holding all inputs constant. One state-of-the art definition of TFP, $Y_{T,t,t-1}$, is drawn from the work of Diewert and Morrison (1986). It too can be thought of as the geometric average of Laspeyres-like and Paasche-like measures:

$$(12) \qquad Y_{T,t,t-1} \equiv \sqrt{\frac{f(v_{L,t-1}, v_{K,t-1}, t)}{f(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{f(v_{L,t}, v_{K,t}, t)}{f(v_{L,t}, v_{K,t}, t-1)}} \ .$$

In view of (3), it is immediately clear that $Y_{T,t,t-1}$ as given by (12) is in fact identical to $A_{T,t,t-1}$ as defined by (10). That is, TFP in this model is equal to the contribution of technological change when explaining the productivity of labour. Labour productivity will exceed TFP to the extent that capital deepening occurs ($A_{V,t,t-1} > 1$).

4. Measurement

To make the decomposition (11) operational one needs to specify a functional form for the production function (1). One functional form well suited for this purpose is the Translog. In the production function context, and under linear homogeneity, it is as follows:[3]

$$(13) \qquad \begin{aligned} \ln y_t = \alpha_0 + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 + \\ \phi_{KT}(\ln v_{K,t} - \ln v_{L,t})t + \beta_T t + \frac{1}{2}\phi_{TT} t^2 \end{aligned}$$

One option, at this stage, would be to estimate function (13) econometrically, and then to use the resulting parameter estimates to calculate (7) and (10) in order to get the full decomposition of labour productivity.[4] It turns out, however, that as long as the true production function is indeed given by (13), it is not necessary to have estimates of its parameters to be able to proceed. Thus, Diewert and Morrison (1986) have

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[3] See Christensen, Jorgenson, and Lau (1973).

[4] See Kohli (1990, 1991, 2004c) for such an econometric approach.

shown that in this case TFP ($Y_{T,t,t-1}$) as defined by (12) can be calculated from knowledge of the data alone in the following way:

$$(14) \qquad Y_{T,t,t-1}\left(= A_{T,t,t-1}\right) = \frac{Y_{t,t-1}}{V_{t,t-1}} \ ,$$

where $Y_{t,t-1}$ is the index of real GDP:

$$(15) \qquad Y_{t,t-1} \equiv \frac{y_t}{y_{t-1}} \ ,$$

and $V_{t,t-1}$ is a Törnqvist index of input quantities:[5]

$$(16) \qquad V_{t,t-1} \equiv \exp\left[\sum_{j\in\{L,K\}} \frac{1}{2}(s_{j,t} + s_{j,t-1})\ln\frac{v_{j,t}}{v_{j,t-1}}\right] \ ;$$

$s_{j,t}$ in (16) is the income share of factor *j*:

$$(17) \qquad s_{j,t} \equiv \frac{w_{j,t}v_{j,t}}{p_t y_t} \ , \quad j \in \{L,K\} \ ,$$

where $w_{j,t}$ is the rental prices of factor *j*, and p_t is the price of output. Note that the GDP identity implies that $s_{L,t} + s_{K,t} = 1$.

It follows from the definition of $A_{t,t-1}$ that:

$$(18) \qquad A_{t,t-1} = \frac{Y_{t,t-1}}{v_{L,t}/v_{L,t-1}} \ .$$

Making use of (11), (14), (16), and (18), we thus find that:

$$(19) \qquad A_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)\right] \ .$$

Table 1 reports estimates of the decomposition (11) for Switzerland over the period 1981 to 2002.[6] Cumulated effects and geometric averages for the entire period are shown at the bottom of the table. One can see that labour productivity has increased by about 31% over the entire period; this amounts to about 1.2% *per annum*

[5] The Törnqvist index is a superlative index in the sense of Diewert (1976).
[6] See the Appendix for a description of the data.

on average. TFP, on the other hand, has increased by a much more modest 8.8% (about 0.4% per year). The bulk of the increase in labour productivity is due to capital deepening, which added about 20% (0.8% on average annually) to labour productivity over the sample period. One also observes some fairly large annual variations. Thus, labour productivity increased by as much as 3.4% (in 1992), and it actually fell on a couple of occasions (in 1987 and 2001). The contribution of capital deepening, although larger than that of TFP, is much steadier. This is illustrated in Figure 1 that shows the contributions of both components. It is visible that the fluctuations of labour productivity largely reflect those of TFP.

The fact that TFP is found to be rather small, and perhaps less than one would have expected, might be thought to reflect poorly on Switzerland and its capacity to innovate. One must remember, however, that TFP is essentially measured as a Solow *residual*; see expression (14). TFP is the growth in output that cannot be explained by the model, given the input data on hand. TFP, too some degree, is a measure of our ignorance. The more precisely inputs and outputs are measured, the smaller one should probably expect TFP to be. In this study, we have used a superlative index to aggregate outputs and inputs. Furthermore, we have used refined measures of hours worked and of the capital stock. This might explain why the residual is found to be rather modest.[7]

5. Domestic real value added

The model of the production function is rather limiting since it imposes the number of outputs to be one.[8] Moreover, the production function approach does not make it possible to take into account imports and exports. In what follows, we therefore opt for the description of the aggregate technology by a real value added (or real income) function, such as the one proposed by Kohli (2004a). It is based on the GDP function approach to modelling the production sector of an open economy.[9] We assume that the technology counts two outputs, domestic (nontraded) goods (D) and exports (X), as well as three inputs, labour (L), capital (K), and imports (M); imports are treated as

[7] Further progress could probably be made by weighting work hours by their marginal productivity, rather than simply adding them up; see Greenwood and Kohli (2003) for an analysis along these lines.
[8] Alternatively, one must assume that outputs are globally separable from domestic inputs.
[9] See Kohli (1978), Woodland (1982).

a variable input, i.e. as a negative output. This treatment recognises the fact that most foreign trade is in middle products, and that even most so-called finished goods that are imported must still transit through the production sector where they are combined with domestic value added before meeting final demand. We denote output (including import) quantities by y_i and their prices by p_i, $i \in \{D, X, M\}$. Furthermore, we denote the inverse of the terms of trade by q ($q \equiv p_M / p_X$) and the relative price of tradables vs. nontradables by e ($e \equiv p_X / p_D$). Note that for given terms of trade, a change in e can be interpreted as a change in the real exchange rate, an increase in e being equivalent to a real depreciation of the home currency. Let π_t be nominal GDP:

(20) $$\pi_t \equiv p_{D,t} y_{D,t} + p_{X,t} y_{X,t} - p_{M,t} y_{M,t} = p_t y_t \ .$$

Domestic real value added (z_t) – or real domestic income – is defined as nominal GDP deflated by the price of domestic output:

(21) $$z_t \equiv \frac{\pi_t}{p_{D,t}} = y_{D,t} + e_t y_{X,t} - e_t q_t y_{M,t} \ .$$

The difference between real GDP (y_t) and real value added (z_t) lies in the price index that is being used to deflate nominal GDP. In the case of real GDP, an index of the prices of all output components (i.e. including imports and exports) is used (p_t), whereas in the case of real value added (or real income), only the prices of the domestic components are retained ($p_{D,t}$).[10] The effect of this difference in treatment becomes apparent if one considers a change in the terms of trade or in the real exchange rate. An improvement in the terms of trade, for instance, mathematically has little or no impact on real GDP, but it results in an increase in real value added; see Kohli (2004a). For unchanged factor endowments, the productivity of labour is thereby enhanced. Over the past quarter century, it turns out that Switzerland has experienced a significant improvement in its terms of trade and a real appreciation in its currency. This is documented by Figure 2 that shows the terms of trade (measured by $p_{X,t} / p_{M,t}$) and the real exchange rate (measured by $p_{X,t} / p_{D,t}$). The trends in the two series suggest differing paths for real GDP and real value added. This impression is confirmed by Figure 3, which shows the normalized path of the

[10] Törnqvist indices are used in both cases.

two indices (y_t and z_t). Clearly, real value added has increased more rapidly than real GDP on average.

Let T_t be the production possibilities set at time *t*. We assume that T_t is a convex cone. The aggregate technology can be described by a real valued added function defined as follows:[11]

$$(22) \qquad z_t = g(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \max_{y_D, y_X, y_M} \left\{ \begin{array}{r} y_{D,t} + e_t y_{X,t} - e_t\, q_t y_{M,t} : \\ \left(y_{D,t}, y_{X,t}, y_{M,t}, v_{K,t}, v_{L,t}\right) \in T_t \end{array} \right\} .$$

In this context, the real value added per unit of labour ($b_{L,t}$) can be defined as follows:

$$(23) \qquad b_{L,t} \equiv \frac{z_t}{v_{L,t}} .$$

In terms of the real value-added function, we get:

$$(24) \qquad b_{L,t} = b_L(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \frac{g(q_t, e_t, v_{K,t}, v_{L,t}, t)}{v_{L,t}} .$$

The labour productivity index is now expressed as:

$$(25) \qquad B_{t,t-1} \equiv \frac{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} .$$

The Translog functional form is once again well suited for our purposes. In the context of the real value added function it can be written as:[12]

$$(26) \qquad \begin{aligned} \ln z_t &= \alpha_0 + \alpha_Q \ln q_t + \alpha_E \ln e_t + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} \\ &+ \frac{1}{2}\gamma_{QQ}(\ln q_t)^2 + \gamma_{QE} \ln q_t \ln e_t + \frac{1}{2}\gamma_{EE}(\ln e_t)^2 \\ &+ \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 + (\delta_{QK} \ln q_t + \delta_{EK} \ln e_t)(\ln v_{K,t} - \ln v_{L,t}) \\ &+ (\delta_{QT} \ln q_t + \delta_{ET} \ln e_t)\, t + \phi_{KT}(\ln v_{K,t} - \ln v_{L,t})\, t + \beta_T\, t + \frac{1}{2}\phi_{TT}\, t^2 \end{aligned}$$

Proceeding along the same lines as in Section 4, we can define the following index to capture the contribution of changes in the terms of trade to the productivity of labour:

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[11] See Kohli (2004a).
[12] See Diewert (1974).

(27) $$B_{Q,t,t-1} \equiv \sqrt{\frac{b_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{b_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)}} \ .$$

Similarly, we can identify the contribution of changes in the real exchange rate as:

(28) $$B_{E,t,t-1} \equiv \sqrt{\frac{b_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{b_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)}} \ ,$$

the contribution of changes in domestic factor endowments:

(29) $$B_{V,t,t-1} \equiv \sqrt{\frac{b_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{b_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)}} \ ,$$

and, finally, the contribution of technological progress:

(30) $$B_{T,t,t-1} \equiv \sqrt{\frac{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{b_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{b_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)}} \ .$$

Assuming that the real value added function is given by (26), it can be shown that these four effects together give a complete decomposition of the productivity of labour as defined by (25):[13]

(31) $$B_{t,t-1} = B_{Q,t,t-1} \cdot B_{E,t,t-1} \cdot B_{V,t,t-1} \cdot B_{T,t,t-1} \ .$$

The left-hand side of (31) can readily be computed in the following way:

(32) $$B_{t,t-1} \equiv \frac{z_t / v_{L,t}}{z_{t-1} / v_{L,t-1}} \ .$$

Under the hypothesis that the true real value added function is indeed Translog, the components on the right-hand side of (31) can be calculated on the basis of the data alone, that is without knowledge of the parameters of (26). One can thus show that:[14]

(33) $$B_{Q,t,t-1} = \exp\left[\frac{1}{2}(-s_{M,t} - s_{M,t-1}) \ln\frac{q_t}{q_{t-1}}\right]$$

(34) $$B_{E,t,t-1} = \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1}) \ln\frac{e_t}{e_{t-1}}\right]$$

[13] The demonstration is the same as in Kohli (2004a).
[14] See Kohli (2004a).

(35) $$B_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)\right]$$

(36) $$B_{T,t,t-1} \equiv \frac{Y_{t,t-1}}{V_{t,t-1}} \ ,$$

where s_M is the GDP share of imports ($s_M \equiv p_M y_M / \pi$), s_B is the trade balance relative to GDP ($s_B \equiv (p_X y_X - p_M y_M)/\pi$), and s_K is, as before, the GDP share of capital. It is noteworthy that $B_{V,t,t-1} = A_{V,t,t-1}$ and $B_{T,t,t-1} = A_{T,t,t-1}$. This implies:

(37) $$B_{t,t-1} = B_{Q,t,t-1} \cdot B_{E,t,t-1} \cdot A_{t,t-1} \ .$$

That is, the difference between the growth in real value added per unit of labour and that of real GDP per unit of labour is due to the terms-of-trade and the real exchange-rate effects. These are precisely the elements that account for the distinction between the two price indices (p_t vs. $p_{D,t}$) used to deflate nominal GDP to get either real GDP or real value added. Indeed, $p_{t,t-1}$, the change in domestic prices over consecutive periods and which is obtained as a Törnqvist index of the prices of domestic sales, exports and imports, can be written as follows:

(38) $$\begin{aligned} p_{t,t-1} &= \exp\left[\sum_{i\in\{D,X,M\}} \pm\frac{1}{2}(s_{i,t} + s_{i,t-1}) \ln\frac{p_{i,t}}{p_{i,t-1}}\right] \\ &= \frac{p_{D,t}}{p_{D,t-1}} + \exp\left[\frac{1}{2}(-s_{M,t} - s_{M,t-1}) \ln\frac{q_t}{q_{t-1}}\right] + \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1}) \ln\frac{e_t}{e_{t-1}}\right] \end{aligned}$$

where we have taken into account the fact that $s_D + s_X - s_M = s_D + s_B = 1$. This expression demonstrates that p_t, the commonly used GDP price deflator, encompasses two components, the terms-of-trade effect and the real exchange-rate effect, that should best be viewed as real – rather than price – elements.

A decomposition of the productivity of labour according to (31) is reported in Table 2. One finds that labour productivity, in terms of real value added, has increased by close to 41% over the sample period. The impact of the real exchange rate was negligible,[15] but favourable movements in the terms of trade have lifted labour

[15] This is due to the fact that, in the eighties, when the real exchange rate was dropping rapidly, net exports were close to zero. Later, as a larger current account surplus developed, the real appreciation

productivity by 7.8% (about 0.3% per year). This is nearly as much as the contribution of technological change, as suggested by our estimate of TFP. Our results are further illustrated by Figure 4, which is based on (37). It shows the annual contributions of changes in the terms of trade and in the real exchange rate, together with the real-value-added and GDP labour productivity indices. It appears that at times the movement in the terms of trade has very much dominated the dynamics of real value added. In 1986, for instance, labour productivity increased by 3.6% as indicated by $B_{t,t-1}$, whereas, judging from the estimate of $A_{t,t-1}$, real GDP per unit of labour only increased by 0.6%. Except for a negligible real exchange-rate effect, the difference is due to the massive terms-of-trade effect (3.1%) that Switzerland experienced that year.

Figure 5 provides a final illustration of our results. It shows the cumulated effects of productivity gains ($A_t \equiv \prod_{h=1}^{t} A_{h,h-1}$, and so on). Thus, it indicates the paths of labour productivity (in terms of real value added and in terms of real GDP) and of TFP over the past quarter century. The vertical distance between $A_{T,t}$ and A_t results from capital deepening, whereas the vertical distance between A_t and B_t is accounted for by the foreign-trade (terms-of-trade and real exchange-rate) effects. One again, we see that capital deepening has dominated in the Swiss case, and that the role of trade effects has been about as important as that of TFP.

6. Conclusions

Productivity is an important, yet elusive concept. In this paper we have focused on the two measures of productivity most prevalent in the literature, labour productivity and TFP. Furthermore, we have identified and quantified the main forces at work: technological progress, capital deepening, terms-of-trade changes, and changes in the real exchange rate. We have shown that TFP is an essential component of labour productivity. However, we have found that in the Swiss case, capital deepening has played an even larger role in explaining the growth of labour productivity. Improvements in the terms of trade have played a very substantial role as well.

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slowed down markedly. Nonetheless, the real exchange-rate effect must be considered for things to add up, or more precisely, for the multiplicative decomposition to be complete.

Appendix: Description of the data

All data are annual for the period 1980 to 2002. We require the prices and quantities of all inputs and outputs. The data for GDP and its components, in nominal and in real terms, are taken from the *Office fédéral de la statistique (OFS)* website. Prices are then obtained by deflation. Data on labour compensation and on the operating surplus are also retrieved from the *OFS* website. The quantity of capital services is assumed to be proportional to the stock. The necessary figures, together with data on labour input (measured in hours worked) are *Swiss National Bank* estimates; see Fox and Zurlinden (2004). The rental price of labour and capital are then obtained by dividing labour and capital income by the corresponding quantity series. For the purpose of Section 4 output is expressed as an implicit Törnqvist index of real GDP; see Kohli (2004b) for details. In Section 5, the price of nontraded goods is computed as a Törnqvist price index of the deflators of consumption, investment and government purchases.

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Figure 1

Labour productivity, capital deepening, and total factor productivity, 1981-2002

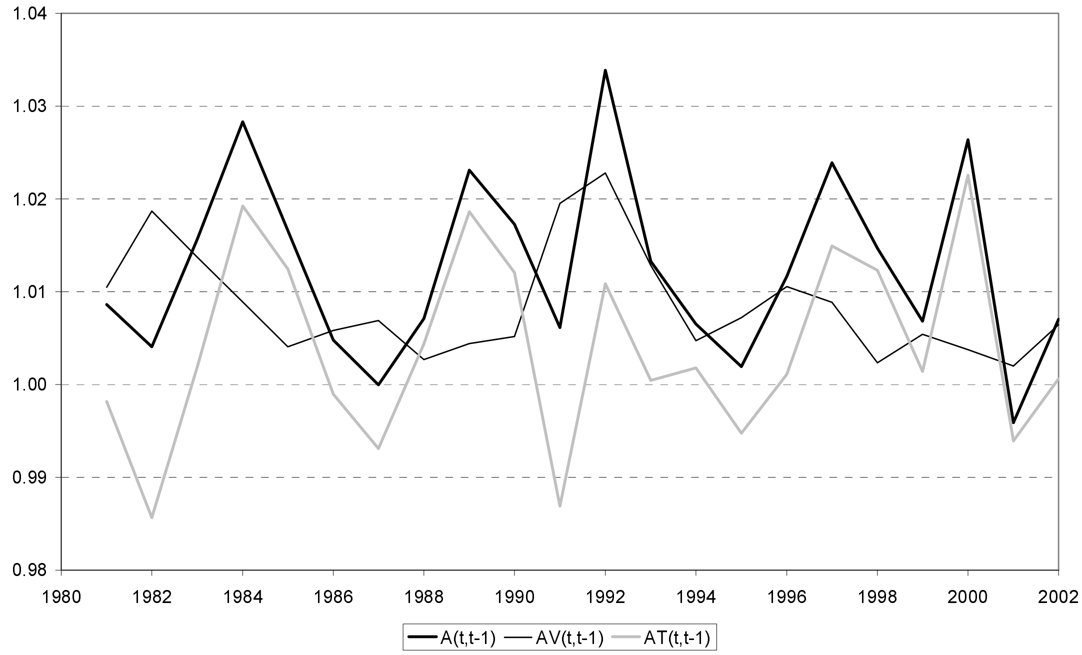

Figure 2

Real value-added and real GDP, 1981-2002

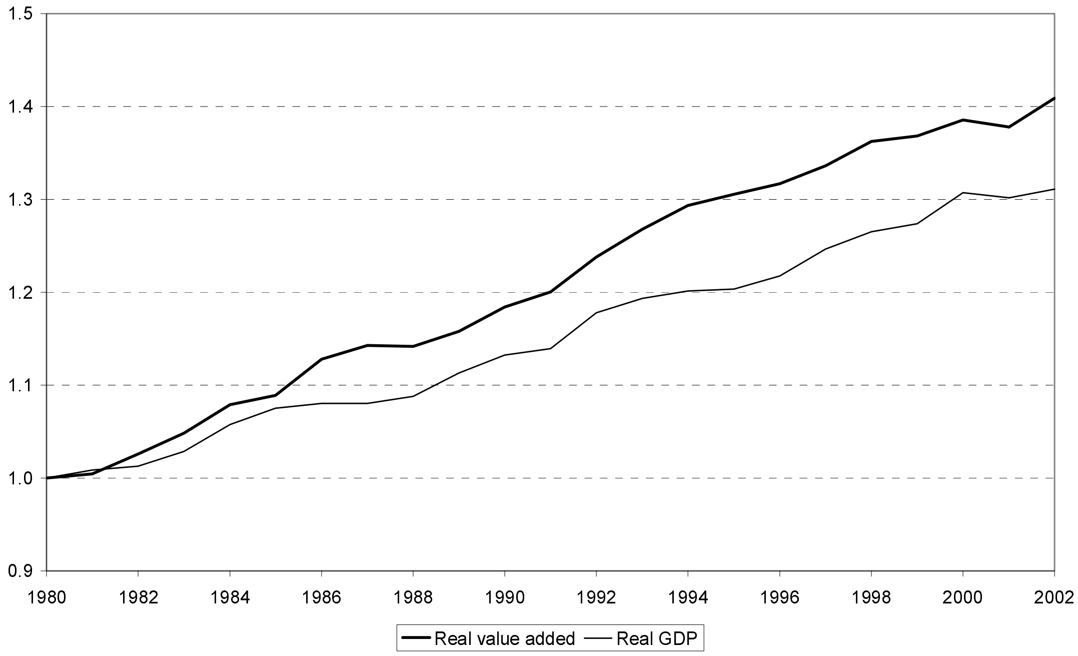

Figure 3

Terms of trade and real exchange rate, 1980-2002

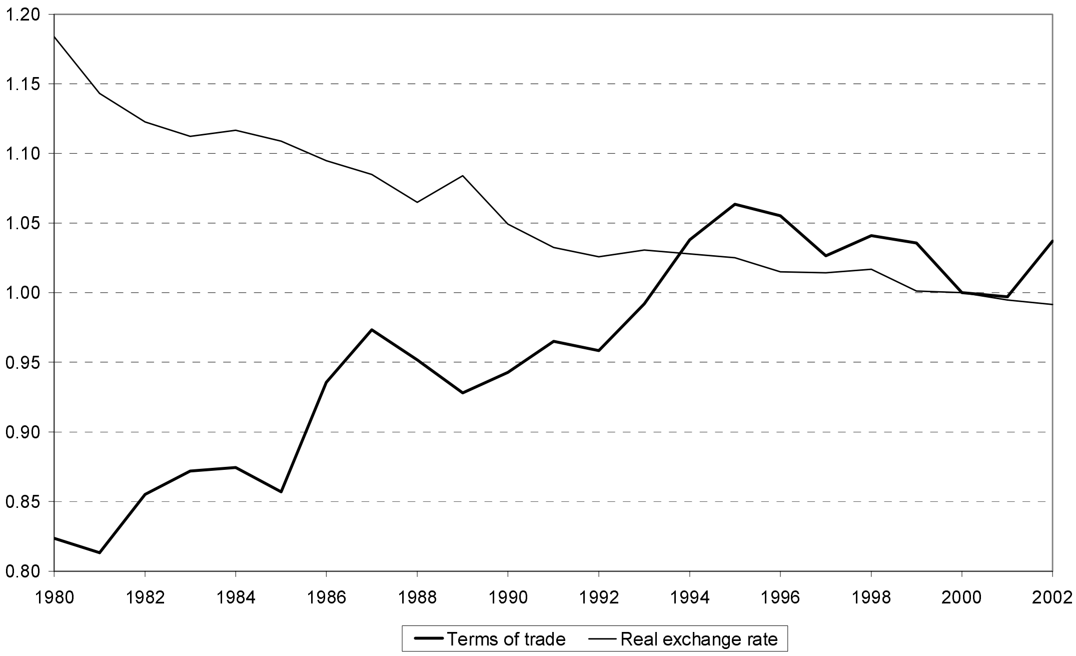

Figure 4

Labour productivity and the contribution of terms-of-trade and real exchange-rate changes, 1981-2002

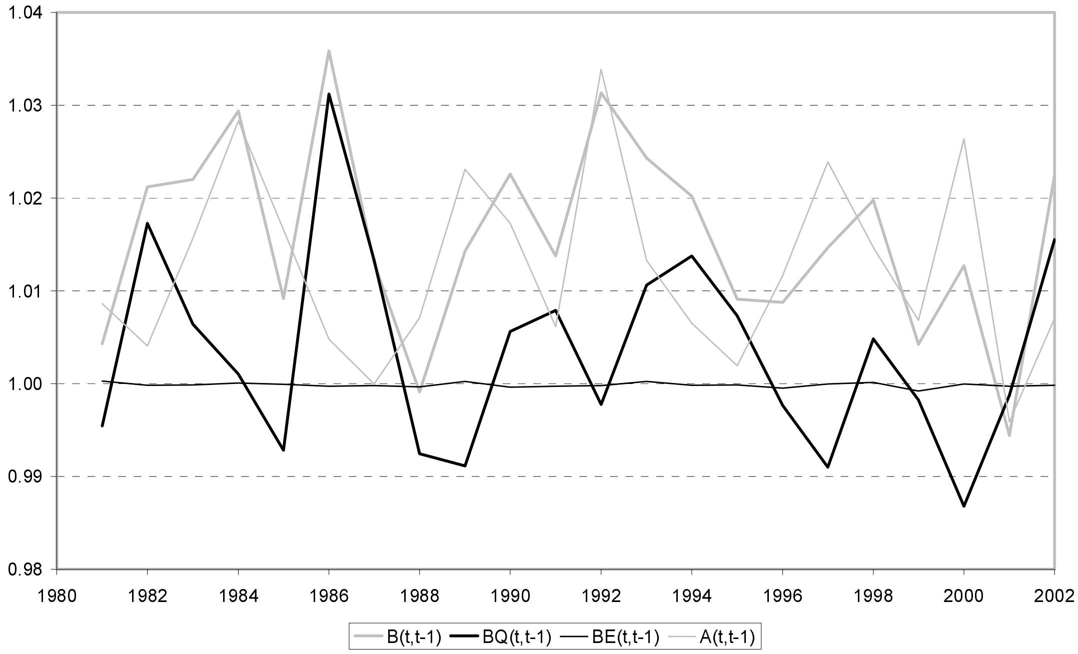

Figure 5

Labour productivity and totals factor productivity: Cumulated effects, 1980-2002

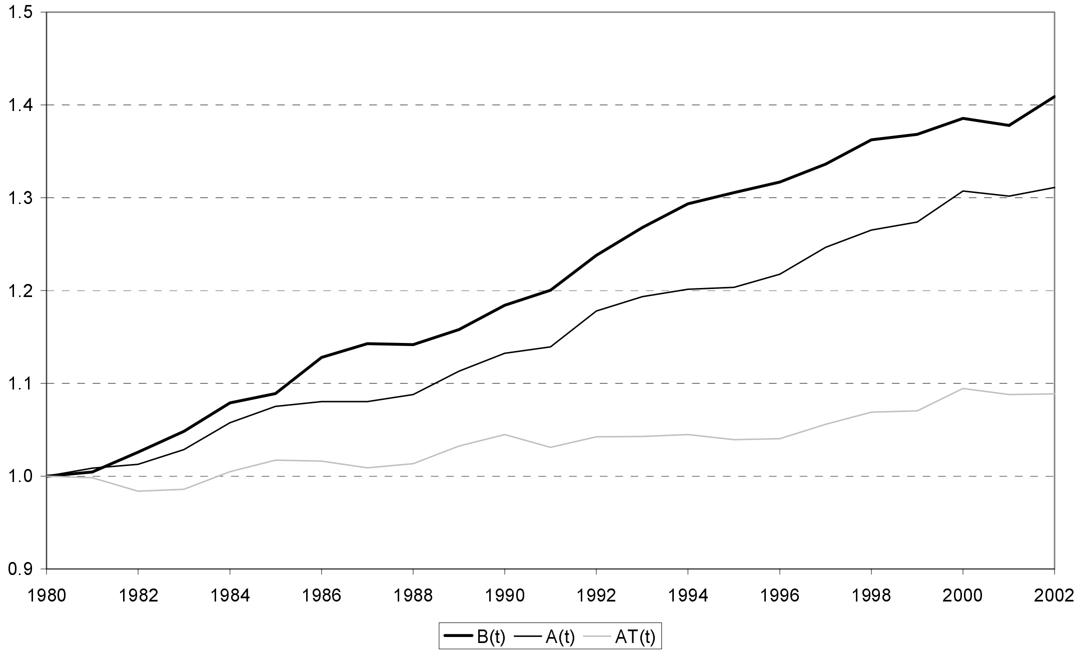

Table 1

**Decomposition of the productivity of labour:
2-input Translog production function**

	$A_{t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$
1981	1.00860	1.01047	0.99816
1982	1.00405	1.01868	0.98564
1983	1.01565	1.01373	1.00189
1984	1.02829	1.00888	1.01923
1985	1.01658	1.00407	1.01245
1986	1.00478	1.00583	0.99896
1987	0.99994	1.00689	0.99310
1988	1.00708	1.00268	1.00438
1989	1.02309	1.00440	1.01862
1990	1.01728	1.00516	1.01206
1991	1.00614	1.01950	0.98690
1992	1.03386	1.02276	1.01085
1993	1.01328	1.01283	1.00044
1994	1.00650	1.00472	1.00178
1995	1.00192	1.00720	0.99476
1996	1.01165	1.01055	1.00109
1997	1.02389	1.00884	1.01492
1998	1.01468	1.00234	1.01232
1999	1.00682	1.00541	1.00140
2000	1.02636	1.00375	1.02253
2001	0.99591	1.00199	0.99394
2002	1.00704	1.00643	1.00061
1981-2002	**1.31091**	**1.20436**	**1.08847**
mean	1.01238	1.00849	1.00386

Table 2

**Decomposition of the productivity of labour:
2-input, 3-output Translog real domestic value added function**

	$B_{t,t-1}$	$B_{Q,t,t-1}$	$B_{E,t,t-1}$	$B_{V,t,t-1}$	$B_{T,t,t-1}$
1981	1.00430	0.99545	1.00028	1.01047	0.99816
1982	1.02121	1.01728	0.99981	1.01868	0.98564
1983	1.02201	1.00639	0.99987	1.01373	1.00189
1984	1.02938	1.00102	1.00004	1.00888	1.01923
1985	1.00917	0.99281	0.99990	1.00407	1.01245
1986	1.03584	1.03118	0.99973	1.00583	0.99896
1987	1.01304	1.01329	0.99980	1.00689	0.99310
1988	0.99909	0.99242	0.99965	1.00268	1.00438
1989	1.01426	0.99112	1.00024	1.00440	1.01862
1990	1.02260	1.00561	0.99962	1.00516	1.01206
1991	1.01377	1.00788	0.99971	1.01950	0.98690
1992	1.03132	0.99776	0.99979	1.02276	1.01085
1993	1.02429	1.01062	1.00024	1.01283	1.00044
1994	1.02019	1.01376	0.99984	1.00472	1.00178
1995	1.00910	1.00731	0.99986	1.00720	0.99476
1996	1.00875	0.99765	0.99949	1.01055	1.00109
1997	1.01464	0.99100	0.99997	1.00884	1.01492
1998	1.01973	1.00484	1.00014	1.00234	1.01232
1999	1.00422	0.99823	0.99918	1.00541	1.00140
2000	1.01272	0.98677	0.99994	1.00375	1.02253
2001	0.99440	0.99874	0.99974	1.00199	0.99394
2002	1.02246	1.01551	0.99982	1.00643	1.00061
1981-2002	**1.40871**	**1.07823**	**0.99665**	**1.20436**	**1.08847**
mean	1.01570	1.00343	0.99985	1.00849	1.00386