Sanyal and Jones on Trade in Middle Products

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Abstract

This paper reports for the first time empirical estimates of one of the most ingenious models of trade in intermediate products, the model of Sanyal and Jones. We show how the restrictions implied by the Sanyal and Jones production structure can be imposed and tested with the help of aggregate data even though information about the allocation of inputs and outputs between tiers and industries is not available. A new theoretical concept, that of "disjoint production" is introduced. We also propose a new functional form that is a generalization of the Symmetric Normalized Quadratic restricted profit function.

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1 Introduction

The main purpose of this paper is to put to the test of the data one of the most elegant models of trade in intermediate products, the model of Sanyal and Jones (1982). Models of trade in intermediate goods have received much attention in the literature since it is widely recognized that most international trade is in raw materials and nonfinished goods, and that even most so-called "finished" products must still go through a number of transformations at home (imports) or abroad (exports) before being ready to meet final demand. The model of Sanyal and Jones (S&J) allows for this feature and it is particularly intriguing in that it postulates a very detailed and rather complex production structure.

Of course, the idea that trade takes place almost exclusively in intermediate goods is not new. Thus, much of the early empirical work on import determination was consistent with the view that imports are an input to the technology,[1] and Burgess (1974) has formally introduced what is now called the production theory approach to import determination. This work has been extended by Kohli (1978, 1991) and Woodland (1982) into the GNP/GDP function approach to modeling the demand for imports and the supply of exports. This approach treats all imports as inputs to the technology. They are used together with domestic factors to produce goods intended for domestic use, as well as goods destined for foreign markets. These exports, in turn, are used as inputs by the foreign technology.

Most of the previous work on the GDP function approach has allowed for joint production, and it has placed very few restrictions on the form of the technology, except for constant returns to scale, convexity, and free disposals. Assuming two domestic factors (labor, L, and capital, K), two middle products (imports, M, and exports, X), and two final goods (nontradables, Y and Z), the black box of technology is depicted in figure 1. Nothing filters out about what takes place inside the box.

S&J, on the other hand, make some very specific assumptions about the way inputs are combined into outputs. Thus, they assume that there are two production tiers. The two middle products are produced in the "input tier", and the two final goods are produced in the "output tier". Production is assumed to be nonjoint in input quantities at each level of production. There are thus four distinct sectors. Labor is mobile between sectors and tiers, whereas capital is only used in the input tier, and, moreover, it is sector specific. While labor and capital are used in the input tier, the production of final goods involves labor and one middle product. Trade takes place exclusively in middle products. Net exports of each middle product are equal to the difference between domestic production and domestic utilization. All these

[1] See Leamer and Stern (1970).

assumptions are captured by figure 2 which enables us to have a peep inside the box.[2]

Given the specificity of the S&J model, it seems worthwhile to confront it to the data, and to attempt to quantify some of its predictions. Furthermore, given the wide gap that exists between theory and empirical work in the international trade literature, we believe that it is important to attempt to estimate and test existing trade models.[3] Actually, there is an additional challenge here for the empirical economist since the data that the model requires — specific capital in the input tier, production of middle products in the input tier, utilization of middle products in the output tier, allocation of labor between the two tiers and within the two tiers — is generally not available.[4] However, as we shall see, there are ways to get around these difficulties if one is willing to make just one additional assumption. This paper thus illustrates how one can model some rather fancy production structures with aggregate data if one selects the appropriate representation of the technology as a starting point.

While the first purpose of this paper is to show how the restrictions implied by the S&J model can be modeled and tested, we have two further objectives. Thus, we develop a new theoretical concept, the notion of "disjoint production" which makes it possible to break up a technology into a number of subtechnologies. Moreover, we introduce a new functional form which is a generalization of the Symmetric Normalized Quadratic restricted profit function proposed by Kohli (1993b), and which is well suited to impose and test various forms of nonjointness and disjointness.

2 General Description of the Aggregate Technology

We begin with a general description of the aggregate technology. We assume that there are M inputs and outputs. We denote the vector of input and output quantities by $y \equiv [y_m]$; an element of y is positive if it relates to an output, and negative if it refers to an input. The vector of input and output prices is denoted by $p \equiv [p_m]$; all elements of p are assumed to be strictly positive. Let T be the production possibilities

[2] The rounded cones represent one-output two-input neoclassical production functions allowing for the substitution between inputs; the continuous lines indicate mandatory directions, whereas the dotted lines show alternative paths which may be followed. See Section 4 for a complete definition of the notation. This is actually the original version of the S&J model. In what follows, a generalized version will also be considered. See Appendix 2 for a formal description of the model.

[3] One might argue that this is somewhat unfair since S&J did probably not intend their model to provide a basis for empirical work. However, since the comparative statics results they obtain largely reflect the restrictions they place onto the technology, it seems appropriate to test the empirical validity of those restrictions. If one were to assume a different — perhaps more realistic — production structure, the comparative statics results would most likely be significantly altered as well.

[4] While data on *direct* factor requirements in particular industries are widely available, what is needed here are data on *total* (i.e. direct and *indirect*) factor requirements. That is, the factor requirements of intermediate products — which are netted out at the aggregate level — used by any industry would have to be known too, and allocated to that industry.

set, that is, the set of all feasible input and output combinations. We assume that T is a closed, non-empty, convex cone; that it is bounded from above, and that it allows for the free disposals of inputs.

Let us partition the set of inputs and outputs into two subsets, A and B, that contain, respectively, I and $J \equiv M - I$ elements. We denote the vector of the I input and output quantities in A by $y_A \equiv [y_i]$, and the vector of the $J \equiv M - I$ input and output quantities in B by $y_B \equiv [y_j]$; the corresponding price vectors are denoted by $p_A \equiv [p_i]$ and $p_B \equiv [p_j]$. As we shall see later on, the partitioning of the inputs and outputs may be dictated by economic theory, by statistical considerations, or by technical requirements. Without loss of generality (by appropriately reordering the inputs and outputs), we may assume that A contains the first I input and outputs, and that B contains the last J ones: thus $y \equiv (y_A', y_B')'$ and $p \equiv (p_A', p_B')'$.

The aggregate technology can now be described by a restricted profit function that is defined as follows:[5]

$$\pi(p_A, y_B) \equiv \max_{y_A}\{p_A' y_A : (y_A, y_B) \in T\}. \tag{1}$$

The restricted profit function treats the first I quantities as variable and the last J quantities as fixed. The restricted profit function is identical to the variable profit function, except that, although one generally allows for A to contain outputs as well as (variable) inputs, one usually assumes that all elements of B are inputs;[6] this is not the case here. Indeed, in what follows, both A and B will generally be assumed to contain inputs as well as outputs.

It is well known that the profit maximizing supply of variable outputs and demand for variable inputs can be obtained by differentiation:

$$y_A(p_A, y_B) = \nabla_{p_A}\pi(p_A, y_B), \tag{2}$$

where $\nabla_{p_A}\pi(\cdot)$ is the vector of the partial derivatives of $\pi(\cdot)$ with respect to p_A: $\nabla_{p_A} \equiv [\partial\pi(p_A, y_B)/\partial p_i], i = 1, \ldots, I$. Under competitive conditions, the *inverse* output (and input) supply (demand) functions can be similarly obtained:

$$p_B(p_A, y_B) = -\nabla_{y_B}\pi(p_A, y_B), \tag{3}$$

where $\nabla_{y_B}\pi(\cdot)$ is the vector of the partial derivatives of $\pi(\cdot)$ with respect to y_B: $\nabla_{y_B}\pi(\cdot) \equiv [\partial\pi(p_A, y_B)/\partial y_j], j = I + 1, \ldots, M$. Note that it follows from (2)-(3) and

[5] Given the assumptions made on T, the function $\pi(\cdot)$ is well defined for all quantity vectors y_B and for all strictly positive price vectors p_A. It is linearly homogeneous and convex in the elements of p_A; it is increasing (decreasing) in p_i if the corresponding element is an output (input); it is linearly homogeneous, concave, and decreasing in the elements of y_B.

[6] See Diewert (1974) for an analysis of the properties of the variable profit function. Note also that the restricted profit function defined by (1) is equal to minus a joint cost function, except that we allow for fixed inputs as well as variable ones, and for variable outputs as well as fixed ones; see Hall (1973) and McFadden (1978) for a discussion of the properties of the joint cost function.

the linear homogeneity of $\pi(\cdot)$ that:

$$\pi(\cdot) = \sum_{i=1}^{I} p_i y_i = - \sum_{j=I+1}^{I+J} p_j y_j .$$

The substitution possibilities allowed for by the technology can be described by a set of price and quantity elasticities. Let $H \equiv [\eta_{mn}]$ be defined as:

$$H \equiv \begin{bmatrix} H_{pp} & H_{py} \\ \\ H_{yp} & H_{yy} \end{bmatrix} \equiv \begin{bmatrix} \partial \ln |y_i| / \partial \ln p_h & \partial \ln |y_i| / \partial \ln |y_j| \\ \\ \partial \ln p_j / \partial \ln p_i & \partial \ln p_j / \partial \ln |y_k| \end{bmatrix} . \tag{4}$$

Thus, the elements of $H_{pp} \equiv [\eta_{ih}]$ are the price elasticities of the supply of variable outputs and of the demand for variable inputs. The components of $H_{yy} \equiv [\eta_{jk}]$ are the quantity elasticities of the inverse supply (demand) functions of (for) the fixed outputs (inputs). We could also call them the inverse price elasticities of fixed output supply (input demand); they indicate the effect of a change in the composition of y_B on the prices of the fixed components. The elasticities in $H_{py} \equiv [\eta_{ij}]$ indicate the effect of a change in the fixed quantities (y_B) on the variable quantities (y_A), while the elasticities in $H_{yp} \equiv [\eta_{ji}]$ indicate the effects of changes in the prices of the variable elements (p_A) on the prices of the fixed elements (p_B). Note that all these elasticities generally are functions of p_A as well as of y_B. It can easily be seen that the elements of H can be calculated as:

$$\eta_{mi} = \frac{\pi_{mi}}{\pi_m} p_i, \qquad \eta_{mj} = \frac{\pi_{mj}}{\pi_m} y_j, \tag{5}$$

for $m = 1, \ldots, M; i = 1, \ldots, I; j = I+1, \ldots, M$, where π_m and π_{mn} are the first-order and second-order partial derivatives of $\pi(\cdot)$ with respect to the corresponding price or quantity.

3 Disjoint Production

The description of the technology reviewed in the previous section is very general, and it can be used to describe a wide range of multiple-input, multiple-output technologies. In particular, it is consistent with joint production, as well as with nonjoint production. Under joint production, the choice of the elements of y_A and y_B is relatively unimportant, and it may be determined by convenience or by statistical considerations. Under less general production structures, those involving some form of nonjointness for instance, this choice may be crucial. Thus, in many cases, particular production structures imply additional restrictions on the form of the restricted profit function, but these can only be imposed for particular partitions of the inputs and outputs. In this section, we look at one particular type of restriction on the form of the technology, one which is more general than nonjointness, and which seems to have gone unnoticed in the literature. We call it "disjointness". We first need some additional definitions and preliminary results.

Definition 1: The B-restricted production possibilities set, $Y^B(y_B)$, is the set of all quantity vectors y_A that are feasible for a given quantity vector y_B. Formally:

$$Y^B(y_B) \equiv \{y_A : (y_A, y_B) \in T\}.$$

If all elements of y_B were negative, and if all elements of y_A were positive, $Y^B(y_B)$ would define the output production possibilities set for input vector $-y_B$. If, on the other hand, all elements of y_B were positive, and if all elements of y_A were negative, $Y^B(y_B)$ would define the input requirements set of output vector y_B. Definition 1 is quite general, however, and it applies to arbitrary partitions of inputs and outputs. It follows from the properties of T that $Y^B(y_B)$ is a convex cone.

Consider now a partition of the J elements of B into K subsets, $K \leq J$. Let the corresponding quantity and price vectors be given by $y_B \equiv (y'_{B_1}, \ldots, y'_{B_k}, \ldots, y'_{B_K})'$ and $p_B \equiv (p'_{B_1}, \ldots, p'_{B_k}, \ldots, p'_{B_K})'$. Note that the y_{B_k}'s and p_{B_k}'s may well be vectors.

Definition 2: The $B(k)$-restricted production possibilities subset, $Y^{B_k}(y_{B_k})$, is the set of all quantity vectors y_A that are feasible for a quantity vector $(0, \ldots, y_{B_k}, \ldots, 0)$:

$$Y^{B_k}(y_{B_k}) \equiv \{y_A : (y_A, 0, \ldots, y_{B_k}, \ldots, 0) \in T\}.$$

Note that it follows from the fact that $Y^B(y_B)$ is a convex cone that $Y^{B_k}(y_{B_k})$ is a convex cone as well.

Definition 3: The technology is B-disjoint if there exists a partition of y_B such that the B-restricted production possibilities set can be written as:

$$Y^B(y_B) = \sum Y^{B_k}(y_{B_k}),$$

where all sets $Y^{B_k}(y_{B_k})$ are convex cones.

We now turn to the main result of this section:[7]

Theorem 1: The technology is B-disjoint if and only if the restricted profit function can be written as:

$$\pi(p_A, y_B) = \sum \pi^k(p_A, y_{B_k}),$$

where the $\pi^i(\cdot)$'s are linearly homogeneous and convex in prices; increasing in the prices of outputs and decreasing in the prices of inputs; linearly homogeneous, concave, and decreasing in the components of y_B.

Note that Theorem 1 is more general than the result discussed by Kohli (1983, Theorem 2); this is because disjointness is less restrictive than nonjointness. While

[7]See Appendix 1 for a proof of all theorems and lemmas.

all forms of nonjointness that have been analyzed in the literature assume that each production process involves either one output and several inputs, or one input and several outputs, disjointness implies that the technology can be separated into blocks, tiers, or industries, [8] but each block may well contain several inputs and outputs, and, moreover, it may involve joint production. That is, nonjointness is not necessary for disjointness. Nonjointness is a special case of disjointness that arises if either y_A or each one of y_{B_k} contain one component only. Note also that each element of y_B is exclusive – or specific – to one block.[9]

The concept of disjointness may seem farfetched. Yet, it is not difficult to come up with examples. Think of a multi-country model, where each country's production possibilities set is given by a convex cone. Some products may be traded, others may be nontraded. Factors may be mobile internationally or not. Goods as well as factors may be country-specific, or they may be perfect substitutes for their foreign counterparts. Whether production is joint, disjoint, or nonjoint as the national level, the *world* technology is clearly disjoint since it can be broken up into *national* production possibilities sets, and perhaps into smaller components still. Like the pieces of a puzzle, the various elements describing the national technologies fit together perfectly to yield the world production possibilities set.

4 The Sanyal & Jones Structure of Production

We now turn to the specific production structure implied by S&J, and investigate the nature of the restrictions that their model imposes on the form of the aggregate technology.[10] Here we are faced with two difficulties. First, there is the question of the sector specific capital in the input tier. Sector specific capital is generally not observable; this is particularly true here since not even the industries producing the middle products are observable. However, as shown by Kohli (1993a), if we are willing to assume that the relative supplies of the two sector-specific types of capital do not change much over time, we may aggregate them into a composite factor by invoking Leontief aggregation. This composite capital good then has all the properties of a public input which enters all industries (of the input tier) at the same time. The return to this composite factor is the sum of the returns of the two types of sector-specific forms of capital.

Second, there is the assumption by S&J, reflected by figure 2, that each middle product is used as an input by only one output-tier industry. In our empirical work,

[8] We purposely avoid the term separable, since separability has a very definite meaning in production theory.

[9] While there may be many industries, each one possibly using and producing many of the same inputs and outputs, each industry has something unique about it, in that some of its inputs (e.g. specific factors) or outputs (final sales) appear in no other industry; this production structure can be viewed as a special case of the structure examined by Woodland (1977).

[10] See Appendix 2 for a formal description of the S&J model.

we will consider two types of middle products (importables and exportables), and two final goods (investment and consumption goods),[11] and it would be arbitrary to attribute either middle product to one or the other final good. In their paper, however, S&J provide a generalization of their model, one where both middle products are used by both output-tier industries. In what follows, we will mostly consider this generalized structure. Nevertheless, in the last part of our paper, we will briefly investigate the original S&J structure by considering in turns the cases where importables and exportables are attributed exclusively to one or the other final good industry.

In what follows, we therefore consider six inputs and outputs: labor (L), capital (K), imports (M), exports (X), investment goods (I), and consumption goods (C). The first three elements are viewed as inputs to the technology ($y_L \leq 0, y_K \leq 0, y_M \leq 0$), while the last three elements are outputs ($y_X \geq 0, y_I \geq 0, y_C \geq 0$).

For empirical implementation, we will treat the quantities of the final goods and of the composite factor as given, and the quantities of labor and traded goods as variable; that is, $y_A \equiv (y_M, y_X, y_L)'$ and $y_B = (y_I, y_C, y_K)'$. The motivation behind this particular partition will become clear later on. We can now describe the aggregate technology by the following restricted profit function:

$$\pi(p_M, p_X, p_L, y_I, y_C, y_K) \equiv \max\{p_M y_M + p_X y_X + p_L y_L : (y_A, y_B) \in T\}. \quad (6)$$

Function $\pi(\cdot)$ has all the properties of the restricted profit function as described in Section 2. Note that the value of $\pi(\cdot)$ is the balance of trade minus the labor wage bill. In absolute value, this is also equal to the value of final output (domestic sales) minus the total return to the composite capital good. The fact that $\pi(\cdot)$ treats p_L as given and y_L as variable, rather than the reverse, does not match the usual assumption in international trade theory that the endowments of all factors are given. However, this treatment of labor as a variable input is necessary at this stage to enable us to impose the S&J production structure onto the aggregate technology; we will revert to the alternative treatment of labor later on when assessing our results.

Theorem 2: The S&J production structure is disjoint.

[11] The disaggregation of final output between consumption goods and investment goods seems a natural choice from a national accounts, a macroeconomic or a growth theory perspective. Note that the distinction between tradables (middle products) and nontradables (end products) is already accounted for. Another possibility would be to disaggregate final output between goods and services, but there seems to be no compelling reason to do this, and it would raise additional data problems. Some people might argue that our disaggregation is somewhat arbitrary, since a particular good (e.g. a motor vehicle) may end up being classified as an export, a consumption good, or an investment good based on the accident of who purchases it. However, this misses the point: goods which are exported, consumed or purchased by business, in spite of the appearance, are fundamentally different products. A car that is exported does not go through the same channels as a car that ends up in a dealer's showroom: it is still a middle product at this stage (it may end up in a showroom in a foreign country) and as a result it typically contains less domestic value added than a car purchased by a domestic household. Similarly, a car rental company replenishes its fleet by buying its vehicles wholesale rather than retail, which again suggests that these contain less value added than cars purchased by households.

Theorem 3: Under the S&J production structure, restricted profit function (6) has the following additive form:

$$\pi(p_M, p_X, p_L, y_I, y_C, y_K) = \pi^1(p_M, p_X, p_L, y_I, y_C) + \pi^2(p_M, p_X, p_L, y_K),$$

where $\pi^1(\cdot)$ and $\pi^2(\cdot)$ are linearly homogeneous and convex in prices; increasing in output prices and decreasing in input prices; and linearly homogeneous, concave, and decreasing in quantities.

This theorem demonstrates that the restricted profit function of the aggregate technology can be written as the sum of the restricted profit functions of the output and input tiers. These are defined as follows:

$$\begin{aligned}\pi^1(p_M, p_X, p_L, y_I, y_C) &\equiv \max\{p_M y_M^1 + p_X y_X^1 + p_L y_L^1 : \\ &\quad (y_M^1, y_X^1, y_L^1) \in Y^{B_1}(y_I, y_C, 0)\} \qquad (7)\\ \pi^2(p_M, p_X, p_L, y_K) &\equiv \max\{p_M y_M^2 + p_X y_X^2 + p_L y_L^2 : \\ &\quad (y_M^2, y_X^2, y_L^2) \in Y^{B_2}(0, 0, y_K)\}. \qquad (8)\end{aligned}$$

These restricted profit functions have all the same properties as $\pi(\cdot)$ above. Note that the value of $\pi^1(\cdot)$ is minus the cost of producing y_I and y_C; that is, the value of importables, exportables, and labor used in the output tier. This is also equal to minus the value of domestic sales. The value of $\pi^2(\cdot)$, on the other hand, is equal to the variable profits drawn from the input tier; that is, the value of the production of importables and exportables, minus the wage bill of labor employed in the input tier. This is also equal to the revenue of the composite capital stock. The total value of $\pi^1(\cdot)$ and $\pi^2(\cdot)$, therefore, equal to the negative of the total labor wage bill and the trade account; as noted earlier, this is also equal to the total return of the composite capital stock minus domestic sales.

Theorem 3 is not the end of the story, though. The existence of well defined industries in both the output and the input tiers allows us to state the following two lemmas.

Lemma 1: Under the (generalized) S&J production structure, restricted profit function (7) has the following additive form:

$$\pi^1(p_M, p_X, p_L, y_I, y_C) = \pi^{1_I}(p_M, p_X, p_L, y_I) + \pi^{1_C}(p_M, p_X, p_L, y_C),$$

where $\pi^{1_i}(\cdot)(i = I, C)$ is linearly homogeneous, decreasing, and convex in prices, and linearly homogeneous in y_i.

Note that the $\pi^{1_i}(\cdot)$'s can be interpreted as the restricted profit (or revenue) functions of the two industries in the output tier. They are defined as follows:[12]

$$\pi^{1_i}(p_M, p_X, p_L, y_i) \equiv \max\{p_M y_{M_i}^1 + p_X y_{X_i}^1 + p_L y_{L_i}^1 : y_i = f^i(y_{M_i}^1, y_{X_i}^1, y_{L_i}^1)\}, \qquad (9)$$

[12] $f^i(\cdot)$ denotes the production function of industry i; see Appendix 2 for details.

$$i = I, C.$$

These revenue functions can also be interpreted as the negative of the industry cost functions.

Lemma 2: Under the S&J production structure, restricted profit function (8) has the following additive form:

$$\pi^2(p_M, p_X, p_L, y_K) = \pi^{2_M}(p_M, p_L, y_K) + \pi^{2_X}(p_X, p_L, y_K),$$

where $\pi^{2_h}(\cdot)(h = M, X)$ is linearly homogeneous and convex in prices, increasing in p_h and decreasing in p_L, and linearly homogeneous in y_K.

Note that the $\pi^{2_h}(\cdot)$'s can be interpreted as the restricted profit functions of the two industries in the input tier. They yield the return to the specific capital, and they are defined as follows:

$$\pi^{2_h}(p_h, p_L, y_K) \equiv \max\{p_h y_h^2 + p_L y_{L_h}^2 : y_h^2 = f^h(y_{L_h}^2, y_K)\} \quad h = M, X. \tag{10}$$

Summing up, we get to the main result of this section:

Theorem 4: Under the (generalized) S&J production structure, restricted profit function (6) has the following additive form:

$$\begin{aligned}\pi(p_M, p_X, p_L, y_I, y_C, y_K) &= \pi^{1_I}(p_M, p_X, p_L, y_I) + \pi^{1_C}(p_M, p_X, p_L, y_C) + \\ &\quad \pi^{2_M}(p_M, p_L, y_K) + \pi^{2_X}(p_X, p_L, y_K),\end{aligned}$$

where $\pi^{1_i}(\cdot)(i = I, C)$ is linearly homogeneous, decreasing and convex in prices, and linearly homogeneous in y_i, and where $\pi^{2_h}(\cdot)(h = M, X)$ is linearly homogeneous and convex in prices, increasing in p_h and decreasing in p_L, and linearly homogeneous in y_K.

Compared to the general specification given by (6), the S&J production structure implies a number of important restrictions. Indeed, it is visible from Theorem 4 that:

$$\frac{\partial^2 \pi(\cdot)}{\partial y_i^2} = 0, \; \frac{\partial^2 \pi(\cdot)}{\partial y_I \partial y_C} = 0, \; \frac{\partial^2 \pi(\cdot)}{\partial y_i \partial y_K} = 0, \; \frac{\partial^3 \pi(\cdot)}{\partial p_M \partial p_X \partial y_K} = 0, \quad i = I, C.$$

We now turn to the task of investigating how these restrictions can be imposed and/or tested in empirical work.

5 Empirical Implementation

5.1 Functional Form

We require a functional form that is flexible enough to represent an arbitrary technology; that is, a functional form which does not restrict *a priori* the size or the sign of the various elasticities of substitution. Moreover, we require a functional form that allows for the restrictions implied by the S&J production structure to be imposed. Finally, given that curvature conditions often tend to be violated in empirical work based on flexible functional forms, we require a function that allows curvature conditions to be imposed globally without interfering with flexibility. To the best of our knowledge, there is no existing functional form for a restricted profit function that satisfies all these requirements. We therefore introduce the following new functional form for a restricted profit function:[13]

$$\pi = \frac{1}{2}\frac{\sum\sum\sum a_{ihj}p_i p_h |y_j|}{\sum a_i p_i} + \frac{1}{2}\frac{\sum\sum\sum b_{jki}|y_j||y_k|p_i}{\sum b_j |y_j|} + \sum\sum c_{ij}p_i|y_j|, \tag{11}$$

$$i, h = 1, \ldots, I;\; j, k = I+1, \ldots, I+J,$$

where $a_{ihj} = a_{hij}$, $\sum_i a_{ihj} = 0$, $b_{jki} = b_{kji}$, $\sum_j b_{jki} = 0$, and where the a_i's and b_j's can be set subject to the conditions $a_i \geq 0$, $b_j \geq 0$, $\sum a_i = 1$, and $\sum b_j = 1$.[14]

This function is linearly homogeneous in prices and in quantities. Moreover, it is convex in prices as long as matrices $A^j \equiv [a_{ihj}]$ are positive semi-definite, and it is concave in quantities as long as matrices $B^i \equiv [b_{jki}]$ are negative semi-definite. If needed, these restrictions can be imposed.

Functional form (11) is a member of the family of Normalized Quadratic flexible functional forms introduced by Diewert and Wales (1987, 1988a). Two other examples are the Normalized Quadratic (NQ) variable profit function introduced by Diewert and Ostensoe (1988), and the Symmetric Normalized Quadratic (SNQ) variable profit function proposed by Kohli (1993b). However, (11) is more general than either one of these two functions. Indeed, it contains them both as special cases. Thus, (11) is identical to the SNQ if $\beta_j A^j = A$ and $\alpha_i B^i = B$, where $\beta_j \geq 0$, $\alpha_i \geq 0$. It is identical to the NQ if, moreover, $\beta_j = 0$ ($j = I+1, \ldots, I+J-1$) and $\alpha_i = 0$ ($i = 1, \ldots, I-1$).

Functional form (11) has $IJ(I+J)/2$ parameters, which is more than what is required by a flexible functional form. This larger number of parameters adds somewhat to the complexity of this function, and it may lead to problems of multicollinearity when the model is estimated. However, this function will be extremely useful for our purposes since, as we shall see, it allows for nonjointness and disjointness to be imposed and/or tested.

Compared to other functional forms available in the literature, (11) is quite unique in that it allows for nonjointness restrictions to be incorporated and for global curvature conditions to be imposed globally without interfering with flexibility. Neither the NQ, nor the SNQ allow for imposition of nonjointness and disjointness, whereas other functional forms such as the Generalized-Linear Generalized-Leontief (GLGL) introduced by Hall (1973) cannot be forced to satisfy curvature conditions globally without

[13] Naturally, this functional form is also well suited for a joint cost function.

[14] As long as all fixed quantities and all prices are nonzero, the numerators in (11) are necessarily strictly positive.

wrecking its flexibility if the number of either fixed or variable components exceeds two. The same is true for the Mean-of-Order-Two Mean-of-Order-Zero (M2MØ) functional form proposed by Diewert (1973) and applied by Kohli (1991). In the case of the Translog variable profit function (Diewert, 1974), finally, convexity can only be imposed locally, whereas concavity can be imposed locally or globally, but, in the latter case, only at the cost of destroying the flexibility of the function; in any case, the Translog is not well suited to model the restrictions implied by nonjointness and disjointness.

The derived variable output supply and input demand equations are obtained by differentiation of (11) with respect to the prices of the variable components:

$$\begin{aligned} y_i &= \frac{\sum\sum a_{ihj}p_h|y_j|}{\sum a_i p_i} - \frac{1}{2}\frac{a_i\sum\sum\sum a_{ihj}p_i p_h|y_j|}{(\sum a_i p_i)^2} + \\ & \frac{1}{2}\frac{\sum\sum b_{jki}|y_j||y_k|}{\sum b_j|y_j|} + \sum c_{ij}|y_j|, \end{aligned} \tag{12}$$

$$i = 1, \ldots, I.$$

Similarly, the fixed component inverse demand and supply functions are as follows:

$$\begin{aligned} \pm p_j &= \frac{1}{2}\frac{\sum\sum\sum a_{ihj}p_i p_h}{\sum a_i p_i} + \frac{\sum\sum b_{jki}|y_k|p_i}{\sum b_j|y_j|} - \\ & \frac{1}{2}\frac{b_j\sum\sum\sum b_{jki}|y_j||y_k|}{(\sum b_j|y_j|)^2} + \sum c_{ij}p_i, \end{aligned} \tag{13}$$

$$j = I+1, \ldots, I+J,$$

the minus sign holding if fixed component j is an output.

In order to derive the price and quantity elasticities, we first note that the elements of the Hessian of $\pi(\cdot)$ are as follows:

$$\pi_{ii} = \frac{\sum a_{iij}|y_j|}{\sum a_i p_i} - 2a_i\frac{\sum\sum a_{ihj}p_h|y_j|}{(\sum a_i p_i)^2} + a_i^2\frac{\sum\sum\sum a_{ihj}p_i p_h|y_j|}{(\sum a_i p_i)^3} \tag{14}$$

$$\begin{aligned} \pi_{ih} &= \frac{\sum a_{ihj}|y_j|}{\sum a_i p_i} - a_i\frac{\sum\sum a_{ihj}p_h|y_j|}{(\sum a_i p_i)^2} - a_h\frac{\sum\sum a_{ihj}p_i|y_j|}{(\sum a_i p_i)^2} + \\ & a_i a_h\frac{\sum\sum\sum a_{ihj}p_i p_h|y_j|}{(\sum a_i p_i)^3} \end{aligned} \tag{15}$$

$$\begin{aligned} \pi_{ij} &= \frac{\sum a_{ihj}p_h}{\sum a_i p_i} - \frac{1}{2}a_i\frac{\sum\sum a_{ihj}p_i p_h}{(\sum a_i p_i)^2} + \frac{\sum b_{jki}|y_k|}{\sum b_j|y_j|} - \\ & \frac{1}{2}b_j\frac{\sum\sum b_{jki}|y_j||y_k|}{(\sum b_j|y_j|)^2} + c_{ij} \end{aligned} \tag{16}$$

$$\pi_{jj} = \frac{\sum b_{jji}p_i}{\sum b_j|y_j|} - 2b_j\frac{\sum\sum b_{jki}|y_k|p_i}{(\sum b_j|y_j|)^2} +$$

$$b_j^2 \frac{\sum\sum\sum b_{jki}|y_j||y_k|p_i}{(\sum b_j|y_j|)^3} \tag{17}$$

$$\pi_{jk} = \frac{\sum b_{jki}p_i}{\sum b_j|y_j|} - b_j \frac{\sum\sum b_{jki}|y_k|p_i}{(\sum b_j|y_j|)^2} - b_k \frac{\sum\sum b_{jki}|y_j|p_i}{(\sum b_j|y_j|)^2} + b_j b_k \frac{\sum\sum\sum b_{jki}|y_j||y_k|p_i}{(\sum b_j|y_j|)^3}, \tag{18}$$

$$i, h = 1, \ldots, I; \; j = I+1, \ldots, I+J.$$

The elements of H can then easily be calculated with the help of (5).

5.2 Imposing the Sanyal & Jones Production Structure

In our empirical application, we will consider three variable components (M, X, L) and three fixed components (I, C, K). In order to be able to impose the S&J production structure, we begin by setting $a_M = a_X = 0$ and $a_L = 1$. Thus, we introduce here the same type of asymmetry as is present, for instance, in the NQ restricted profit function proposed by Diewert and Ostensoe (1988). While asymmetrical treatment of variable components is generally not desirable, it is necessary here if we want to replicate the S&J production structure given that their model does treat labor, imports and exports in a very asymmetrical way. To restore some symmetry, however, we apply the same treatment to the fixed components and thus set $b_I = b_C = 0$ and $b_K = 1$.

In view of Theorem 4 and of (15)–(18), it is then apparent that the generalized S&J production structure requires the following restrictions.

- disjointness of the input and output tiers:

$$b_{IKM} = b_{IKX} = b_{IKL} = b_{CKM} = b_{CKX} = b_{CKL} = 0; \tag{19}$$

- nonjointness in input quantities (output tier):

$$b_{ICM} = b_{ICX} = b_{ICL} = 0; \tag{20}$$

- almost nonjointness in input prices and quantities (input tier):

$$a_{MXK} = 0. \tag{21}$$

It then follows that all four of $\pi^{1_j}(\cdot)(j = I, C)$ and $\pi^{2_i}(\cdot)(i = M, X)$ are Normalized Quadratic, and thus they are themselves flexible.[15]

Furthermore, it is possible to impose the original S&J production structure that is somewhat more restrictive. According to the original model, each industry in

[15] See Diewert and Wales (1987).

the output tier uses only one traded input. However, there is no indication as to which one, of importables and exportables, is used in the investment good industry as opposed to the consumption good industry. We will therefore let the data speak out, and consider both hypotheses in turns.

- original S&J production structure, version $MI : XC$ (importables used in the investment good industry and exportables used in the consumption good industry):
$$a_{MMC} = a_{MXC} = a_{MXI} = a_{XXI} = c_{MC} = c_{XI} = 0; \tag{22}$$

- original S&J production structure, version $MC : XI$ (importables used in the consumption good industry and exportables used in the investment good industry):
$$a_{MMI} = a_{MXI} = a_{MXC} = a_{XXC} = c_{MI} = c_{XC} = 0. \tag{23}$$

5.3 Curvature Conditions

As anybody who has worked with flexible functional forms is well aware of, it is a common occurrence in empirical work that the required curvature conditions are not met. Yet, these curvature conditions are an important part of the theory, and it makes little economic sense to consider a model that does not satisfy them. As noted above, restricted profit function (11) is globally convex in prices if and only if matrices A^j are positive semi-definite, and it is globally concave in fixed quantities if and only if matrices B^i are negative semi-definite. To impose these restrictions, we use the technique of Wiley, Schmidt, and Bramble (1973). As shown by these authors, a sufficient condition for a matrix A^j to be positive semi-definite is that it can be written as:
$$A^j = T^j T^{j\,\prime},$$
where $T^j \equiv [\tau_{ihj}]$ is a lower triangular matrix. Diewert and Wales (1987), exploiting a result by Lau (1978), have shown that the above condition is also necessary for A to be positive semi-definite. Thus, with $I = 3$, and recalling that $\sum_i a_{ihj} = 0$, we define:

$$T^j \equiv \begin{bmatrix} \tau_{11j} & 0 \\ \tau_{21j} & \tau_{22j} \end{bmatrix}.$$

We then have:

$$A^j \equiv \begin{bmatrix} a_{11j} & a_{12j} \\ a_{21j} & a_{22j} \end{bmatrix} = \begin{bmatrix} \tau_{11j}^2 & \tau_{11j}\tau_{21j} \\ \tau_{11j}\tau_{21j} & \tau_{21j}^2 + \tau_{22j}^2 \end{bmatrix}.$$

Applying this reparameterization to all matrices $A^j (j = I+1, \ldots, I+J)$ ensures that the resulting restricted profit function is convex in output prices.

Similarly, a matrix B^i is negative semi-definite if it can be written:

$$B^i = -M^i M^{i\,\prime},$$

where $M^i \equiv [\mu_{jki}]$ is a lower triangular matrix. This reparameterization can be applied to all matrices $B^i (i = 1, \ldots, I)$ to ensure that (11) is globally concave in quantities.

5.4 Technological Change

It is well known that growth in inputs alone is not sufficient to explain all of output growth. Since our data is uncorrected for the effects of technological change, it is important to allow for the technology to shift over time. This is achieved by adding a time index (t) as an additional variable, and by letting this index interact with all prices and quantities. We therefore rewrite the variable profit function as follows:

$$\begin{aligned} \pi \quad = \quad & \frac{1}{2}\frac{\sum\sum\sum a_{ihj} p_i p_h |y_j|}{\sum a_i p_i} + \frac{1}{2}\frac{\sum\sum\sum b_{jki}|y_j||y_k| p_i}{\sum b_j |y_j|} + \\ & \sum\sum c_{ij} p_i |y_j| + \sum\sum d_{ij} p_i |y_j| t \end{aligned} \tag{24}$$

$$i, h = M, X, L; \; j = I, C, K.$$

This extension adds nine unknown parameters to the model, and it implies that the time index appears as an additional variable in each inverse output supply and direct input demand equation. Naturally (15)–(18) and restrictions (22)–(23) must be modified accordingly.

5.5 Data

We require price and quantity series for all six inputs and outputs. For comparison purposes, we use exactly the same data as Kohli (1993b). These are annual figures covering the period 1948–1988, and they are derived from the U.S. National Income and Product Accounts (NIPA).[16] All prices are normalized to unity for 1982. The quantities are expressed in billion 1982 dollars; t is defined as a time trend with unit annual increments and normalized to zero for 1982.

5.6 Stochastic Specification and Estimation Technique

We assume that the demand and supply functions (12) and (13) are exact, except for errors in optimization. All demand and supply equations are converted into current

[16] The series are available from the author upon request; a detailed description of their construction can be found in Kohli (1991).

value terms by multiplying by the corresponding exogenous price or quantity. Furthermore, to avoid likely problems of heteroskedasticity, we divide both sides of all equations by p_L and $-y_K$.

In our theoretical model, in order to be able to impose the S&J production structure, we treat the prices of imports, exports and labor services, together with the capital stock and the quantities of investment goods and consumption goods as exogenous. Except for the capital stock, this is unlikely to be true in a statistical sense, however. Indeed, the rental price of labor depends on the supply of labor, the domestic output mix reflects demand conditions, and the United States can hardly be viewed as a small open economy, which means that the prices of middle products are likely to be endogenous as well. For these reasons we estimate the model using the algorithm of Berndt, Hall, Hall and Hausman (1974); this is essentially a nonlinear version of iterative three stage least squares. We use the following instruments: a constant; the time trend; the beginning-of-period capital stock; the official discount rate; the average sales and excise tax rate; the budget deficit, the government wage bill, and foreign capital inflows, each one as a ratio of GDP; consumer savings as a ratio of disposable income; and the U.S. population. Hypothesis testing is carried out using the Wald test statistic which is χ^2 distributed.

It follows from the GDP identity that (12)–(13), once that they are expressed in value terms, add up to zero. Hence, one equation must be omitted before proceeding with the estimation, but, the results do not depend on which equation is left out. All in all, we have 205 observations (five equations times 41 annual observations) to estimate up to 36 unknown parameters.

6 Empirical Results

We began by estimating the unrestricted version of the model. It turned out that several of the τ_{ihj} and μ_{jki} estimates were essentially zero. This is not unusual in a model of this size, and it suggests that a semiflexible functional form would have been sufficient.[17] We did purposely not restrict the flexibility of the function, however, since this might interfere with the testing for disjointness and nonjointness. Estimates of the remaining (nonzero) parameters, together with their asymptotic t-values,[18] are reported in Table 1. Curvature conditions were imposed and thus they are met globally. We also verified that all monotonicity conditions are satisfied for all observations.

Table 2 shows 1988 estimates of the various price and quantity elasticities contained in (4); these were obtained with the help of (5). When interpreting these elasticities, one must remember that they are defined for given prices of imports,

[17] See Diewert and Wales (1988b).

[18] These must be interpreted with care, however, since A and B do not have full rank; see Gourieroux, Holly, and Monfort (1982).

exports and labor services, for given outputs of investment goods and consumption goods, and for a given endowment of capital. It is the demand for imports and labor, the supply of exports, and the prices of investment goods, consumption goods and capital services which do the adjusting. This partition between exogenous and endogenous variables was dictated by the requirements of the S&J structure of production. Consider η_{MM}, for instance. Its 1988 value is -0.179, which indicates that a 10% increase in the price of imports would, *ceteris paribus*, lead to a 1.8 percent reduction in the demand for imports. The other things being held constant are not only the price of exports and the endowment of capital, but also the wage rate and the output of investment goods and consumption goods. The impact of the import price increase is rather small, but this is not surprising since the quantity of imports can only decrease to the extent that the demand for labor rises and/or the supply of exports falls.[19] This experiment might be relevant in a Keynesian context with wage rigidities and the output of nontraded goods being demand determined. In international trade theory, however, one is accustomed to treat the prices of goods and the endowments of factors as given. Fortunately, it is possible to express the comparative statistics of the model for changes in the quantities of goods and in the rental prices of domestic factors as functions of changes in goods prices and factor endowments. The resulting elasticities, which correspond to the traditional GDP setting,[20] are reported in Table 3. They tend to be somewhat larger than the ones reported by Kohli (1993b); this is due to the differences in functional forms and stochastic specification. There are, however, important similarities. Thus both studies show that an increase in the endowment of labor relative to capital favors imports and alters the output mix towards exports and investment goods, away from consumption goods. Consequently, an improvement in the terms of trade favors labor over capital, and so does an increase in the price of investment goods or a drop in the price of consumption goods.

We next turn to the test of the restrictions implied by the generalized S&J production structure. We reestimated the model, successively imposing restrictions (19), (20), and (21). The test results are reported in Table 4. Thus, we find that disjointness of the input and output tiers is decisively rejected by the data. Nonjointness in input quantities in the output tier, conditional on disjointness, on the other hand, cannot be rejected, but this restriction would have little meaning on its own. The same holds true for almost nonjointness in input price and quantities in the input tier, conditional on the previous two sets of restrictions, which is found to do little violence to the data. It thus appears that the S&J production structure is rejected by the data in the U.S. case, essentially because it does not seem that the technology can be broken up into two tiers. Nevertheless, we report in Table 5 the corresponding aggregate quantity and price elasticities. Note that, under the S&J production struc-

[19] The demand for labor is actually shown to decrease, which means that all of the adjustment rests on the supply of exports.

[20] See Kohli (1991).

ture, all quantity elasticities of the price equations ($\eta_{jk}, j, k = I, C, K$) are necessarily zero. We find that the remaining elasticities do not differ a great deal from the ones shown in Table 2.

While convexity has been imposed, and thus holds globally, the two profit (or revenue) functions in the output tiers — $\pi^{1_I}(\cdot)$ and $\pi^{1_C}(\cdot)$ — violate some of the monotonicity requirements. Even though the aggregate revenue function is increasing in the price of exports, $\pi^{1_I}(\cdot)$ and $\pi^{1_C}(\cdot)$ should actually be decreasing in this variable. This is because, while exports are a net output for the aggregate technology, exportables are an input in the production of investment and consumption goods. The net supply of exports is equal to the production of exportables in the input tier minus the utilization of exportables in the output tier. Although monotonicity is satisfied at the aggregate level, it is not met at the level of the individual sectors: the S&J model truly implies formidable restrictions on the structure of the technology.

Looking at the input tier, we find that our estimate of $\pi^2(\cdot)$ violates several of the monotonicity requirements: it is not increasing in the price of exports throughout the sample, and it fails to be decreasing in the rental price of labor. This last observation is actually quite telling. Under the S&J production structure, for given prices of middle products and fixed capital endowment, an increase in the price of labor services necessarily depresses the rental price of both types of specific capital. The total return to capital must therefore fall. This is not necessarily true in a more general model where production is not assumed to be disjoint. The increase in the price of labor may well be absorbed by increases in the prices of investment and consumption goods, so that the rental price of capital need not decrease. Indeed, judging from the positive sign of the unrestricted estimate of η_{KL} in Table 2, this is exactly what happens. The reader can verify from Table 5 that, even if one forces the S&J structure onto the model, this positive effect persists, albeit at the expense of monotonicity.

We finally test both versions of the original S&J specification. It is apparent from the last two lines of Table 4 that both restriction sets (22) and (23) are very severely rejected, although the assumption that importables are used in the investment good industry alone and exportables in the consumption good industry exclusively does somewhat better than the alternative.

At this juncture, one might be tempted to argue that the fact that the S&J model does not seem to be supported by the data is really not very surprising since it contains some rather restrictive simplifying assumptions. One could argue that to be fair to S&J, one should make the model more realistic and thus improve its chances of passing the test of the data. One way of achieving this would be to allow for capital — either mobile or sector-specific — to be utilized in the output tier also. Another possible avenue would be to allow for the production of exportables to involve the utilization of importables.[21] Although extensions along these lines might well be worth undertaking in future research, they are beyond the scope of this paper.

[21] This might be particularly important at a time when production becomes increasingly globalized and intermediate components often go through various stages of transformation in different countries.

Moreover, such an exercise would amount to formulating an essentially new theory of trade in middle products, complete with its own X-ray of the black box of technology. Such a new model, which would deviate from the S&J model in some key respects, would probably yield significantly different comparative statics results, so that some of the essence of the S&J model might be lost.

7 Conclusions

The purpose of this paper was to show how a very intricate production structure such as the one proposed by S&J could be implemented empirically and tested, even in the absence of information about the allocation of inputs and outputs between industries and tiers, simply by using aggregate production data. This also lead us to introduce a new theoretical concept, that of production disjointness, and to propose a new flexible functional form for restricted profit functions that is a generalization of the Symmetric Normalized Quadratic function.

While the S&J production structure does not seem to be supported by the data in the U.S. case, there are nevertheless several empirically relevant lessons which we can draw from this exercise. Thus, the idea that trade takes place almost exclusively in middle products remains intact: even a casual examination of trade data shows that this is indeed the case. Also, the hypothesis that capital is sector specific tends not to be rejected by the data. This confirms the findings of Kohli (1993a) in the context of a simple two-by-two specific-factors model. What seems to be rejected, on the other hand, is the idea of the existence of two separate tiers. Disjointness and nonjointness are powerful assumptions which, by imposing a strong structure on the form of the technology, can yield very remarkable results. It may be, however, that, in the aggregate, the way that inputs are combined into outputs is just too complex for the nonjointness and disjointness restrictions to pass the test of the data. In fact, the production technology of an entire country probably contains numerous tiers and many occurences of disjointness and nonjointness of all kinds and at various levels. It thus forms such a complex web that ultimately, we are inclined to think, the only hope there is to adequately approximate it with a parsimonious model is to allow for joint production. We may be able to say something about the shape of the black box of technology, but not about its detailed contents.

Appendix 1: Proofs of the Theorems and Lemmas

Proof of Theorem 1 The proof follows directly from application of McFadden's (1978) composition rule 4, "Composition rules for gauge and profit functions", Table 5. **Q.E.D.**

Proof of Theorem 2: Let us partition input and outputs in the following way: set A contains imports, exports, and labor services, while subset B contains investment goods, consumption goods, and the composite capital stock. The B-restricted production possibilities set is therefore defined as follows:

$$Y^B(y_I, y_C, y_K) \equiv \{(y_M, y_X, y_L) : (y_M, y_X, y_L, y_I, y_C, y_K) \in T\}.$$

Next partition set Y^B into two subsets, the first one containing investment goods and consumption goods, and the second one containing capital. The corresponding $B(k)$-restricted production possibilities sets are as follows:

$$\begin{aligned} Y^{B_1}(y_I, y_C, 0) &\equiv \{(y_M^1, y_X^1, y_L^1) : (y_M^1, y_X^1, y_L^1, y_I, y_C, 0) \in T\} \\ Y^{B_2}(0, 0, y_K) &\equiv \{(y_M^2, y_X^2, y_L^2) : (y_M^2, y_X^2, y_L^2, 0, 0, y_K) \in T\}. \end{aligned}$$

$Y^{B_1}(\cdot)$ thus is the B-restricted production possibilities set of the output tier, whereas $Y^{B_2}(\cdot)$ is the B-restricted production possibilities set of the input tier. Recalling that $y_M = y_M^1 + y_M^2$, $y_X = y_X^1 + y_X^2$, and $y_L = y_L^1 + y_L^2$, it immediately follows that:

$$Y^B(y_I, y_C, y_K) = Y^{B_1}(y_I, y_C, 0) + Y^{B_2}(0, 0, y_K),$$

i.e. the S&J technology is disjoint between production tiers. **Q.E.D.**

Proof of Theorem 3: The proof follows immediately from Theorems 1 and 2. **Q.E.D.**

Proof of Lemma 1: The existence of two separate industries in the output tier implies that the output-tier technology is nonjoint in input quantities. The proof then follows immediately by interpreting restricted profit function (7) as the negative of the joint cost function of the output tier and by making use of Hall's (1973) result. **Q.E.D.**

Proof of Lemma 2: The existence of two well defined industries at the input-tier level, and our assumption that the two sector-specific capital stocks can be aggregated into a composite factor which has the properties of a public input, implies that the input-tier technology is almost nonjoint in input prices and quantities; the proof then follows immediately from Kohli (1985). **Q.E.D.**

Proof of Theorem 4 The proof follows immediately from application of Theorem 3 and Lemmas 1 and 2. **Q.E.D.**

Appendix 2: The Sanyal & Jones Production structure

The (generalized) S&J production structure can be formally described as follows:

Tier 1 (output tier):

$$y_i = f^i(y^1_{M_i}, y^1_{X_i}, y^1_{L_i}), \quad y^1_{M_i}, y^1_{X_i}, y^1_{L_i} \leq 0; y_i \geq 0;\ i = I, C$$

Tier 2 (input tier):

$$y^2_h = f^h(y^2_{L_h}, y_K), \quad y^2_{L_h} \leq 0, y^2_h \geq 0; y_K \leq 0;\ h = M, X$$

Full employment of labor:

$$y_L = \sum_h y^2_{L_h} + \sum_i y^1_{L_i}.$$

Net exports:

$$y_h = y^2_h + y^1_{h_I} + y^1_{h_C}, \quad h = M, X.$$

Under profit maximization, the following relationships hold in the price space:

Rental price of the composite capital good:

$$p_K = -\sum_h p_h \frac{\partial f^h(\cdot)}{\partial y_K}$$

Wage rate:

$$p_L = -p_h \frac{\partial f^h(\cdot)}{\partial y^2_{Lh}}, \quad h = M, X$$

Output prices:

$$\begin{aligned}
\frac{p_h}{p_i} &= -\frac{\partial f^i(\cdot)}{\partial y^1_{hi}}, \quad i = I, C; h = M, X \\
\frac{p_L}{p_i} &= -\frac{\partial f^i(\cdot)}{\partial y^1_{Li}}, \quad i = I, C,
\end{aligned}$$

where the minus signs result from the convention that inputs are measured negatively. Following S&J, we assume that all production functions are well behaved, i.e. monotonically increasing, linearly homogeneous, and strictly quasi-concave.

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Table 1: Parameter Estimates
(asymptotic t-values in parentheses)

c_{MI}	-0.53055	(-2.71)	d_{MI}	-0.01093	(-1.62)
c_{MC}	-0.30489	(-1.75)	d_{MC}	-0.01375	(-2.81)
c_{MK}	0.65045	(2.07)	d_{MK}	0.02301	(2.21)
c_{XI}	0.59421	(2.35)	d_{XI}	0.01417	(2.43)
c_{XC}	0.49448	(0.90)	d_{XC}	0.02211	(5.16)
c_{XK}	-1.02820	(-1.05)	d_{XK}	-0.02997	(-3.27)
c_{LI}	-1.15674	(-5.43)	d_{LI}	0.01444	(3.21)
c_{LC}	-0.35819	(-0.72)	d_{LC}	0.00367	(0.64)
c_{LK}	0.36751	(0.42)	d_{LK}	-0.00607	(-0.57)
τ_{MMK}	-0.31520	(-1.97)	μ_{IIX}	0.37751	(2.47)
τ_{MXK}	0.89184	(4.11)	μ_{ICX}	0.52373	(1.73)
τ_{XXK}	-0.40651	(-1.42)	μ_{IIL}	0.34666	(1.22)
			μ_{ICL}	-0.69097	(-2.55)

Table 2: 1988 Price and Quantity Elasticities

$$\eta_{mn} \equiv \partial \ln z_m(p_M, p_X, p_L, y_I, y_C, y_K)/\partial \ln v_n,$$
$$z_n \in \{-y_M, y_X, -y_L, p_I, p_C, p_K\},\ v_m \in \{p_M, p_X, p_L, y_I, y_C, -y_K\}$$

	$n = M$	$n = X$	$n = L$	$n = I$	$n = C$	$n = K$
η_{Mn}	-0.179	0.508	-0.329	0.992	1.924	-1.915
η_{Xn}	-0.614	2.105	-1.491	0.994	2.217	-2.210
η_{Ln}	-0.076	0.284	-0.208	0.375	1.399	-0.774
η_{In}	0.552	-0.457	0.905	0.198	-0.217	0.019
η_{Cn}	0.312	-0.298	0.985	-0.063	1.538	-1.475
η_{Kn}	0.556	-0.530	0.974	-0.010	2.636	-2.626

Table 3: 1988 Price and Quantity Elasticities
GDP Function Setting (indirect estimates)

$$\varepsilon_{mn} \equiv \partial \ln z_m(p_M, p_X, p_I, p_C, y_L, y_K)/\partial \ln v_n,$$
$$z_m \in \{-y_M, y_X, y_I, y_C, p_L, p_K\},\ v_n \in \{p_M, p_X, p_I, p_C, -y_L, -y_K\}$$

	$n = M$	$n = X$	$n = I$	$n = C$	$n = L$	$n = K$
ε_{Mn}	-0.257	0.076	0.768	-0.587	2.139	-1.139
ε_{Xn}	-0.092	1.074	-0.093	-0.889	2.547	-1.547
ε_{In}	-0.427	-0.043	1.672	-1.202	1.581	-0.581
ε_{Cn}	0.095	-0.119	-0.351	0.375	0.248	0.752
ε_{Ln}	-0.493	0.486	0.655	0.353	-0.286	0.286
ε_{Kn}	0.331	-0.371	-0.303	1.343	0.360	-0.360

Table 4: Test results

restriction(s)	null hypothesis	df	$\chi^2_{.95}$	test statistic
1. *Disjointness of input and output tiers (DJIO)*				
(28)	unrestricted	6	12.59	23.65
2. *Nonjointness in input quantities, output tier (NJIQ)*				
(29)	DJIO	3	7.81	5.03
3. *Almost nonjointness in input prices and quantities, input tier (ANIPQ)*				
(30)	DJIO & NJIQ	1	3.84	0.03
4. *Original Sanyal & Jones structure (MI:XC)*				
(31)	Generalized S&J	8	15.51	125.60
5. *Original Sanyal & Jones structure (MC:XI)*				
(32)	Generalized S&J	8	15.51	415.13

Table 5: Generalized S&J Production Structure
1988 Price and Quantity Elasticities

$$\eta_{mn} \equiv \partial \ln z_m(p_M, p_X, p_L, y_I, y_C, y_K)/\partial \ln v_n,$$
$$z_n \in \{-y_M, y_X, -y_L, p_I, p_C, p_K\},\ v_m \in \{p_M, p_X, p_L, y_I, y_C, -y_K\}$$

	$n = M$	$n = X$	$n = L$	$n = I$	$n = C$	$n = K$
η_{Mn}	-0.139	0.447	-0.308	0.979	1.994	-1.973
η_{Xn}	-0.528	1.775	-1.247	0.384	4.655	-4.039
η_{Ln}	-0.072	0.245	-0.173	0.257	1.865	-1.122
η_{In}	0.556	-0.184	0.628	0	0	0
η_{Cn}	0.327	-0.647	1.319	0	0	0
η_{Kn}	0.582	-1.008	1.426	0	0	0

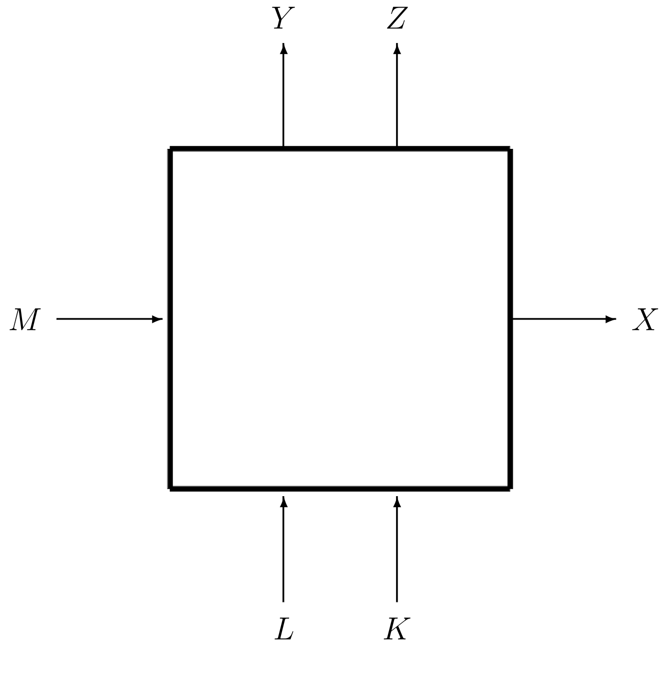

Figure 1: *The Black Box of Technology and Trade in Middle Products*

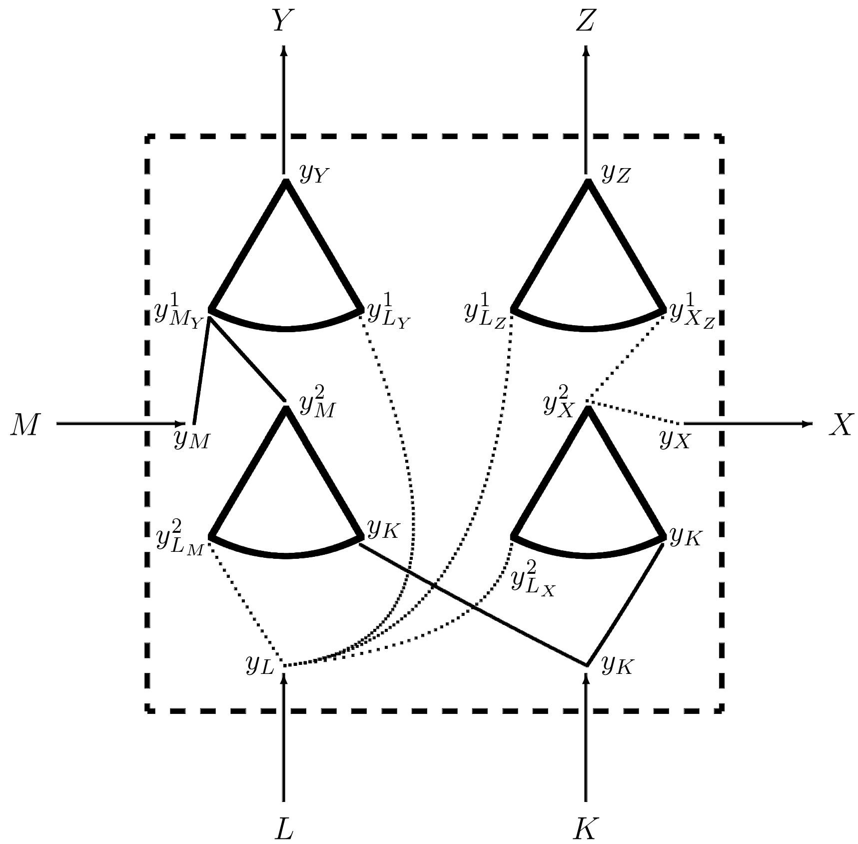

Figure 2: *The Sanyal & Jones Structure of Production: An X-Ray of the Black Box of Technology*