An earlier version of this paper was presented at the SSHRC International Conference on Index Number Theory and the Measurement of Prices and Productivity, Vancouver, B.C., June 30 - July 3, 2004

Labor Productivity: Average vs. Marginal

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Revised December 2004

Abstract

In this paper, we examine the relationship between the average productivity of labor and its marginal counterpart. As long as the income share of labor is fairly steady over time, which happens to be true for the United States, the two measures give very similar results. However, the apparent stability of the labor share is the outcome of several opposing forces. Thus, capital deepening tends to increase it due to the relatively large value of the Hicksian elasticity of complementarity. This is essentially offset by the fact that technological change is mostly labor augmenting. Furthermore, changes in the terms of trade and in the real exchange rate impact on the labor share as well, although these effects are quantitatively small in the case of the United States. Our analysis rests on a tight theoretical framework being based on the GDP function approach to modeling the production sector of an open economy. Full multiplicative decompositions of both measures of labor productivity are provided, and the link with total factor productivity is documented as well. Our estimates are based on both econometric and index-number approaches.

Keywords: labor productivity, total factor productivity, index numbers, technological change, capital deepening, terms of trade, real exchange rate

JEL classification: D24, O47, E25, F43

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Labor Productivity: Average vs. Marginal

0. Introduction

Most headline productivity measures refer to the *average* product of labor, with productivity growth being typically explained by capital deepening and technological progress. One might argue, however, that from an economic perspective a more relevant measure of the productivity of labor rests on its *marginal* product. This is certainly true if one is interested in the progression of real wages. It turns out, though, that as long as the income share of labor remains constant through time, the two productivity measures give identical results. It so happens that in the case of the United States, as shown in Figure 1, the share of labor has been fairly steady over the past thirty years.[1] Hence the distinction between the average and the marginal concepts might appear to be largely irrelevant. This impression is reinforced by Figure 2 that shows the path of both measures of labor productivity. The stability of the labor share also explains why the Cobb-Douglas production function – which restricts the Hicksian elasticity of complementarity between inputs to be unity and thus forces the input shares to be constant – appears to fit U.S. data reasonably well. Any increase in the relative endowment of capital or any technological change, independently of whether it is labor or capital augmenting, necessarily leaves factor shares unchanged, and thus impacts on the average and on the marginal products of labor to exactly the same extent. A more thorough look at the evidence, however, reveals that the apparent constancy of U.S. factor shares is the outcome of several opposing forces and that use of a functional form more flexible than the Cobb-Douglas does more justice to the data. Indeed, we find that over the past three decades the Hicksian elasticity of complementarity between labor and capital has been significantly greater than one. This means that capital deepening leads to an increase in the share of labor and thus raises its marginal product by relatively more than its average product. Technological change, on the other hand, has an offsetting effect: by being essentially labor augmenting, and in view of the large elasticity of complementarity, it tends to reduce the share of labor and thus to raise its average product relative to its marginal product. One contribution of this paper is to

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[1] See the Appendix for a description of the data.

disentangle these effects and to give a multiplicative decomposition of the average and marginal productivity of labor in the United States for the past three decades.

While much of the debate on productivity focuses on the productivity of labor, many economists are more interested in *total factor productivity*. Although this is a less intuitive concept, total factor productivity, as indicated by its name, is more general in that it encompasses all factors of production, rather than just one of them. It turns out that total factor productivity is an essential component of the average productivity of labor. A second contribution of this paper is to document this relationship. Moreover, we present estimates derived from two different approaches, an econometric approach and one based on index numbers.

A third contribution of the paper is to move beyond the rather restrictive two-input, one-output production-function setting. Thus, we expand the model by adopting the GDP-function framework that allows for many inputs and outputs, including imports and exports. This not only makes it possible to get a better estimate of the elasticity of complementarity between domestic primary inputs, but it also shows that there are additional forces at work, such as changes in the terms of trade and in the real exchange rate. Complete multiplicative decompositions of both measures of labor productivity and of total factor productivity are provided for this case as well.

1. The two-input, one-output case

Assume that the aggregate technology can be represented by the following two-input, one-output production function:

$$(1) \qquad y_t = y(v_{L,t}, v_{K,t}, t) \ ,$$

where y_t measures the quantity of output, $v_{L,t}$ denotes the input of labor services, and $v_{K,t}$ is the input of capital services, all three quantities being measured at time *t*. Note that the production function itself is allowed to shift over time to account for technological change. We assume that the production function is linearly homogeneous, increasing, and concave with respect to the two input quantities.

Under competitive conditions and profit maximization, the following first-order conditions must be met:

(2) $$y_L(v_{L,t}, v_{K,t}, t) \equiv \frac{\partial y(v_{L,t}, v_{K,t}, t)}{\partial v_{L,t}} = \frac{w_{L,t}}{p_t}$$

(3) $$y_K(v_{L,t}, v_{K,t}, t) \equiv \frac{\partial y(v_{L,t}, v_{K,t}, t)}{\partial v_{K,t}} = \frac{w_{K,t}}{p_t} \ ,$$

where $w_{L,t}$ and $w_{K,t}$ represent the rental prices of labor and capital, respectively, and p_t is the price of output. The partial derivative $y_L(\cdot)$ on the left-hand side of (2) is the *marginal* product of labor. The *average* product of labor ($g_{L,t}$) on the other hand, is simply defined as:

(4) $$g_{L,t} \equiv \frac{y_t}{v_{L,t}} \ .$$

In terms of production function (1) we can also write:

(5) $$g_{L,t} = g_L(v_{L,t}, v_{K,t}, t) \equiv \frac{y(v_{L,t}, v_{K,t}, t)}{v_{L,t}} \ .$$

The index of *average* labor productivity ($A_{t,t-1}$) can be expressed as one plus the rate of increase in the average product of labor between period *t-1* and period *t*:

(6) $$A_{t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)} \ .$$

Similarly, we can define the index of *marginal* labor productivity ($M_{t,t-1}$) as:

(7) $$M_{t,t-1} \equiv \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)} \ .$$

Note that it follows from the linear homogeneity of the production function that both $g_L(\cdot)$ and $y_L(\cdot)$ are homogeneous of degree zero in $v_{L,t}$ and $v_{K,t}$. The same is therefore true for the two measures of labor productivity, which thus depend on changes in *relative* factor endowments and on the passage of time only.

Next, we define $s_{L,t}$ as the share of labor in total revenues (i.e. GDP):

(8) $$s_{L,t} \equiv \frac{w_{L,t} v_{L,t}}{p_t y_t} \ .$$

It follows from (1), (2), (4) and (5) that:

$$(9) \qquad s_{L,t} = s_L(v_{L,t}, v_{K,t}, t) \equiv \frac{y_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t)} \ .$$

Using (9) in (6) and (7), we find:

$$(10) \qquad M_{t,t-1} = S_{t,t-1} \cdot A_{t,t-1} \ ,$$

where $S_{t,t-1}$ is the labor share index:

$$(11) \qquad S_{t,t-1} \equiv \frac{s_L(v_{L,t}, v_{K,t}, t)}{s_L(v_{L,t-1}, v_{K,t-1}, t-1)} \ .$$

This index is greater or smaller than one, depending on whether the share of labor has increased or fallen between period *t-1* and period *t*.

2. The role of the Hicksian elasticity of complementarity

According to (10), the marginal productivity of labor will be higher (lower) than its average productivity if technological progress and changes in relative factor endowments lead to an increase (decrease) in the share of labor over time. Using (9) as a stating point, we find:

$$\begin{aligned} \frac{\partial s_L(\cdot)}{\partial v_{K,t}} &= \frac{g_L(\cdot)\partial y_L(\cdot)/\partial v_{K,t} - y_L(\cdot)\partial g_L(\cdot)/\partial v_{K,t}}{g_L(\cdot)^2} \\ (12) \qquad &= \frac{v_{L,t}}{y(\cdot)^2}\left[y_{LK}(\cdot)y(\cdot) - y_L(\cdot)y_K(\cdot)\right] \\ &= \frac{s_L(\cdot)s_K(\cdot)}{v_{K,t}}(\psi_{LK} - 1) \end{aligned}$$

where $y_{LK}(\cdot) \equiv \partial^2 y(\cdot)/(\partial v_{L,t}\partial v_{K,t})$ and ψ_{LK} is the Hicksian elasticity of complementarity between labor and capital:[2]

$$(13) \qquad \psi_{LK} \equiv \frac{y_{LK}(\cdot)y(\cdot)}{y_L(\cdot)y_K(\cdot)} \ .$$

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[2] In the two-input case, ψ_{LK} is necessarily positive; that is, the two inputs are necessarily Hicksian complements for each other. Moreover, the Hicksian elasticity of complementarity is then equal to the inverse of the Allen-Uzawa elasticity of substitution.

Thus, capital deepening will lead to an increase (decrease) in the share of labor if and only if the elasticity of complementarity if greater (smaller) than one.

Next, to assess the impact of the passage of time, we take the partial derivative of s_L with respect to *t* to find:

$$\frac{\partial s_L(\cdot)}{\partial t} = \frac{g_L(\cdot)\partial y_L(\cdot)/\partial t - y_L(\cdot)\partial g_L(\cdot)/\partial t}{g_L(\cdot)^2}$$

$$(14) \qquad = \frac{v_{L,t}}{y(\cdot)^2}\left[y_{LT}(\cdot)y(\cdot) - y_L(\cdot)y_T(\cdot)\right]$$

$$= \frac{s_{L,t}(\cdot)y_T(\cdot)}{y(\cdot)}(\psi_{LT} - 1)$$

where $y_T(\cdot) \equiv \partial y(\cdot)/\partial t$, $y_{LT}(\cdot) \equiv \partial^2 y(\cdot)/(\partial v_{L,t}\partial t)$, and ψ_{LT} is defined as follows:

$$(15) \qquad \psi_{LT} \equiv \frac{y_{LT}(\cdot)y(\cdot)}{y_L(\cdot)y_T(\cdot)} \ .$$

The ratio $y_{LT}(\cdot)/y_L(\cdot)$ is the elasticity of the real wage rate with respect to time. The ratio $y(\cdot)/y_T(\cdot)$, on the other hand, is the inverse of the instantaneous rate of technological change (μ_t); ψ_{LT} will thus be greater than one if and only if technological change tends to favor labor relative to capital, in the sense that the wage rate increases by relatively more than the return to capital.[3] In that case the share of labor will increase with the passage of time.

3. Disembodied factor-augmenting technological change

To better track the impact of technological change on the share of labor, let us assume for a moment that technological change is disembodied, factor-augmenting, and that it takes place exponentially. We can then rewrite production function (1) as follows:

$$(16) \qquad y(v_{L,t}, v_{K,t}, t) = f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t}) = f(\tilde{v}_{L,t}, \tilde{v}_{K,t}) \ ,$$

where μ_L and μ_K are the rates of factor-augmenting technological change for labor and capital, respectively ($\mu_L \geq 0, \mu_K \geq 0$), and $\tilde{v}_{L,t}$ and $\tilde{v}_{K,t}$ are the quantities of labor

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[3] In that case, technological change is said to be pro-labor biased. See Kohli (1994) and Section 6 below.

and capital measured in terms of efficiency units: $\tilde{v}_{L,t} \equiv v_{L,t}e^{\mu_L t}$, $\tilde{v}_{K,t} \equiv v_{K,t}e^{\mu_K t}$. The marginal product of labor $y_L(\cdot)$ is then as follows:

$$(17) \qquad y_L(v_{L,t}, v_{K,t}, t) = \frac{\partial f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{\partial v_{L,t}} = e^{\mu_L t} f_L(\cdot) \ ,$$

where $f_L(\cdot) \equiv \partial f(\cdot)/\partial \tilde{v}_{L,t}$. The average product of labor, on the other hand, is simply equal to:

$$(18) \qquad g_L(v_{L,t}, v_{K,t}, t) = \frac{f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{v_{L,t}} \ ,$$

whereas the labor share can now be expressed as:

$$(19) \qquad s_L(v_{L,t}, v_{K,t}, t) = \frac{v_{L,t}e^{\mu_L t} f_L(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})} \ .$$

Differentiating this expression with respect to time, we get:

$$(20) \qquad \begin{aligned} \frac{\partial s_L(\cdot)}{\partial t} &= \frac{\mu_L \tilde{v}_{L,t} f_L(\cdot) + \tilde{v}_{L,t}(f_{LL}\mu_L \tilde{v}_{L,t} + f_{LK}\mu_K \tilde{v}_{K,t})}{f(\cdot)} \\ &\quad - \frac{\tilde{v}_{L,t} f_L(\cdot)(f_L \mu_L \tilde{v}_{L,t} + f_K \mu_K \tilde{v}_{K,t})}{f(\cdot)^2} \\ &= s_{L,t}(1 - s_{L,t})(\mu_K - \mu_L)(\psi_{LK} - 1) \end{aligned}$$

where we have taken into account the restrictions $f_{LL}\tilde{v}_{L,t} + f_{LK}\tilde{v}_{K,t} = 0$ and $f_L\tilde{v}_{L,t} + f_K\tilde{v}_{K,t} = f(\cdot)$ from the linear homogeneity of the production function. Thus, the labor share will increase with the passage of time if $\mu_K > \mu_L$ and $\psi_{LK} > 1$, or, alternatively, if $\mu_K < \mu_L$ and $\psi_{LK} < 1$. If technological change is Harrod-neutral, for instance ($\mu_L > 0$ and $\mu_K = 0$ in that case), and if labor and capital are relatively good complements, the share of labor will tend to fall over time. The increase in the available amount of labor measured in terms of efficiency units will tend to have a sufficiently large positive impact on the marginal product of capital for the share of capital to increase and the share of labor to fall.

4. The Cobb-Douglas functional form

Assume that production function (1) has the Cobb-Douglas form so that:

$$(21) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} v_{K,t}^{\beta_K} v_{L,t}^{1-\beta_K} e^{\mu t} \ ,$$

where $0 < \beta_K < 1$; μ is the rate of Hicks-neutral technological change. One would normally expect it to be positive. Note that the production function (21) could just as well have been written as:

$$(22) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} (v_{K,t} e^{\mu_K t})^{\beta_K} v_{L,t}^{1-\beta_K} \ ,$$

or as:

$$(23) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} v_{K,t}^{\beta_K} (v_{L,t} e^{\mu_L t})^{1-\beta_K} \ ,$$

where $\mu_K \equiv \mu / \beta_K$ and $\mu_L \equiv \mu/(1-\beta_K)$. That is, it is not possible, in the Cobb-Douglas case, to discriminate between the Hicks-neutral, the Solow-neutral, and the Harrod-neutral cases of technological change. In any case, it is well known that in the Cobb-Douglas case, the marginal product of labor is proportional to its average product:

$$(24) \qquad y_L(v_{L,t}, v_{K,t}, t) = (1-\beta_K) \frac{e^{\alpha_0} v_{K,t}^{\beta_K} v_{L,t}^{1-\beta_K} e^{\mu t}}{v_{L,t}} = (1-\beta_K) g_L(v_{L,t}, v_{K,t}, t) \ .$$

It follows from (9) and (24) that $1-\beta_K$ can be interpreted as the share of labor in total income, which is thus invariant by construction:

$$(25) \qquad s_{L,t} = 1 - \beta_K \ .$$

To sum up, in the Cobb-Douglas case, the two measures of labor productivity (6) and (7) must give exactly the same result. This is simply due to the fact that in (10) $S_{t,t-1}$ is equal to unity.

5. The Translog functional form

The Cobb-Douglas function forces the Hicksian elasticity of complementarity to be unity. A more general representation of the technology is given by the Translog functional form.[4] Maintaining for the time being the assumption of disembodied, factor-augmenting technological change, we can write it as follows:

$$\ln y_t = \alpha_0 + \beta_K \ln \tilde{v}_{K,t} + (1-\beta_K)\ln \tilde{v}_{L,t} + \frac{1}{2}\phi_{KK}(\ln \tilde{v}_{K,t} - \ln \tilde{v}_{L,t})^2 \ . \tag{26}$$

Making use of the definitions of $\tilde{v}_{L,t}$ and $\tilde{v}_{K,t}$, we get:

$$\begin{aligned} \ln y_t &= \alpha_0 + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 \\ &\quad + \{\mu_L + (\mu_K - \mu_L)[\beta_K + \phi_{KK}(\ln v_{K,t} - \ln v_{L,t})]\}t \\ &\quad + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2 t^2 \end{aligned} \tag{27}$$

The labor share is obtained by logarithmic differentiation:

$$s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln v_{K,t} - \ln v_{L,t}) - \phi_{KK}(\mu_K - \mu_L)t \ . \tag{28}$$

The Hicksian elasticity of complementarity can be obtained as:

$$\psi_{LK} = \frac{-\phi_{KK} + s_{L,t}(1-s_{L,t})}{s_{L,t}(1-s_{L,t})} \ . \tag{29}$$

ψ_{LK} is greater than one if and only if ϕ_{KK} is negative.[5] In that case the share of labor increases with capital intensity. This matches our result of Section 2. However, it is also apparent from (28) that the form of technological change plays a role. If $\mu_L > \mu_K$ and $\phi_{KK} > 0$, or, alternatively, if $\mu_K > \mu_L$ and $\phi_{KK} < 0$, technological change is pro-labor biased in the sense that the share of labor will increase as the result of the passage of time. In that case, the marginal product of labor will tend to increase more rapidly than the average product. Parameter estimates of (27) are reported in Table 1, column 2.[6] These results suggest that $\mu_L > \mu_K$ and $\phi_{KK} < 0$ in the case of the United states. Thus, technological change is labor-augmenting, but anti-labor biased.

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[4] See Christensen, Jorgenson and Lau (1973), and Diewert (1974).

[5] Note that concavity of the production function requires ψ_{LK} to be positive; that is, the following constraint must hold: $\phi_{KK} < s_{L,t}(1-s_{L,t})$.

[6] Equation (27) was estimated jointly with (28) by nonlinear iterative Zellner. The estimate of ψ_{LK} is reported in Table 1 as well.

Function (27) is flexible with respect to the quantities of labor and capital, but not with respect to time.[7] A somewhat more general formulation would be given by:

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\begin{aligned}
(30)\quad \ln y_t = {} & \alpha_0 + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 + \\
& \phi_{KT}(\ln v_{K,t} - \ln v_{L,t})t + \beta_T\, t + \frac{1}{2}\phi_{TT}\, t^2
\end{aligned}
$$

Comparing (27) with (30), one sees that the latter contains one extra parameter. The share of labor is now given by:

$$
(31)\quad s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln v_{K,t} - \ln v_{L,t}) - \phi_{KT}\, t \ .
$$

It is immediately visible that technological change is anti-labor biased, in the sense that it leads to a reduction in the share of labor, if and only if $\phi_{KT} > 0$.

6. On the form of technological change: a digression

When it comes to technological progress and the analysis of its impact on labor and capital, one finds many different competing concepts in the literature. The overall picture can therefore become quite confusing. Thus, does technological progress favor labor or capital? Is technological progress labor saving, labor using, labor augmenting, labor rewarding, or labor penalizing? Is it pro- (or anti-) labor biased, or even ultra pro- (or anti-) labor biased? To some extent, these concepts apply to different situations and they are not mutually exclusive. In the production-function context, where the input quantities are taken as exogenous and their marginal products as endogenous, technological change will tend to impact on these marginal products. Technological progress can be said to favor – or reward – labor and/or capital, in so far it increases their marginal products. In this sense, it enhances their work. Naturally, it may favor one more than the other. It may also penalize one factor by reducing its marginal product, although, other things equal, a technological improvement must necessarily have a favorable impact on at least one factor.

In the production-function context, one can also think of technological change as being factor augmenting, i.e. increasing the endowment of one or both factors in terms of efficiency units, even if the observed quantities have not changed. If

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[7] To use the terminology of Diewert and Wales (1992), (27) is not TP flexible.

technological change is labor augmenting in this sense (i.e. $\mu_L > 0$), it will, other things equal, tend to depress the marginal product of an efficiency unit of labor and enhance the marginal product of capital.[8] Whether the actual marginal product of labor increases or not will ultimately depend on the Hicksian elasticity of complementarity between the two factors. If labor and capital are strong complements, labor might well be penalized and suffer a drop in its wage. Unless labor and capital are indeed rather weak Hicksian complements, the share of labor will tend to decrease. In that sense, technological change can be said to be anti-labor biased. If the share of labor not only falls, but the wage rate actually declines, one could talk about an ultra anti-labor bias. Table 2-A gives an overview of the cases that might occur in the two-input state, assuming that technological change is disembodied and factor augmenting. For simplicity, we only consider the polar cases of Harrod-, Hicks- and Solow-neutrality, but intermediate situations can obviously arise as well.[9] Based on the estimates of the Translog function discussed in Section 5 and reported in Table 1, column 2, technological change is nearly Harrod-neutral. It is thus labor-augmenting. The elasticity of complementarity is greater than one, but less than the inverse of the capital share. The case described in the second column of Table 2-A is therefore the one that is relevant for the United States. Although technological change is labor (and capital) rewarding, it is nevertheless anti-labor (and pro-capital) biased.

The terms labor (or capital) using and saving are relevant when the aggregate technology is described by way of a cost function.[10] In the aggregate, this would be appropriate in a Keynesian setting, where output and factor rental prices can be viewed as predetermined, and where the model yields the demand for labor and capital services. For a given level of output, technological progress will lead to a reduction in the demand for one or both inputs. In that sense, technological progress can be labor and/or capital saving, just like it could be labor or capital using (but not

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[8] The return of labor per unit of efficiency can be defined as $\widetilde{w}_{L,t} \equiv w_{L,t} e^{-\mu_L t}$.

[9] The ε_{jT}'s *(j=L,K)* are the semi-elasticities of factor rewards with respect to time; see Diewert and Wales (1987). The κ_j's ($\kappa_j \equiv \varepsilon_{jT} - \mu$) measure the bias; see Kohli (1994) for details. The hats indicate relative changes. The changes in factor rental prices are derived under the assumption that the price of output remains constant.

[10] See Jorgenson and Fraumeni (1981), for instance.

both). Since, in this context, factor rental prices are assumed to be given, the share of labor can change either way, depending on how strongly technological change impacts on labor relative to capital. If the labor share increases, one might say that technological progress is pro-labor biased, although this outcome is possible whether technological progress is labor using or labor saving. If the labor share falls, technological change would necessarily have to be labor saving, but at the same time, it can be either capital using or capital saving. In this context, we can also think of technological change as modifying the effective rental price of one or both inputs. That is, technological progress could lead to the lowering the rental price of labor per unit of efficiency. Other things equal, this will favor the demand for labor at the expense of capital in terms of efficiency units, but whether or not the measured demand for labor increases or not depends on the size of the Allen-Uzawa elasticity of substitution between labor and capital. If that elasticity is close to zero, the actual demand for labor might well fall. It is easy to see that the share of labor could in general go in either direction. Table 2-B summarizes the possible outcomes in the cost-function setting.[11] Given the empirical results to which we alluded earlier, we can conclude that in the U.S. case technological change is labor- (and capital-) saving, and anti-labor biased.

In the two input case, there is a simple correspondence between the cost-function setting and the production-function setting, since the elasticity of substitution is then equal to the inverse of the elasticity of complementarity. This is no longer the case if the number of inputs exceeds two, since the passage of one type of elasticity to the other requires the inversion of a bordered Hessian matrix.[12] It is no longer true, then, that an elasticity of complementarity between a pair of inputs greater than one necessarily implies that the corresponding elasticity of substitution is less than unity. In fact, the two elasticities need not even have the same sign. This makes any characterization of technological progress without reference to the analytical setting at best ambiguous, and at worst useless.

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[11] The function $c(\cdot)$'s *(j=L,K)* is the unit cost function, and σ_{LK} is the Allen-Uzawa elasticity of substitution. The ε_{jT} 's *(j=L,K)* now designate the semi-elasticities of input demands with respect to time. In deriving these results, we have assumed that output remains constant.

[12] See Kohli (1991).

7. Accounting for labor productivity

We now turn to the task of accounting for the changes over time in the average and the marginal products of labor. Using (6) as a starting point, we can define the following index that isolates the effect of changes in factor endowments over consecutive periods of time:

$$(32) \qquad A_{V,t,t-1}^{L} \equiv \frac{g_L\left(v_{L,t}, v_{K,t}, t-1\right)}{g_L\left(v_{L,t-1}, v_{K,t-1}, t-1\right)} .$$

When defining $A_{V,t,t-1}^{L}$ we have held the technology constant at its initial (period *t-1*) state. $A_{V,t,t-1}^{L}$ has thus the Laspeyres form, so to speak. Alternatively, we could adopt the technology of period *t* as a reference. We would then get the following Paasche-like index:

$$(33) \qquad A_{V,t,t-1}^{P} \equiv \frac{g_L\left(v_{L,t}, v_{K,t}, t\right)}{g_L\left(v_{L,t-1}, v_{K,t-1}, t\right)} .$$

Since there is no reason *a priori* to prefer one measure over the other, we can following Diewert and Morrison's (1986) example and take the geometric mean of the two indexes just defined. We thus get:

$$(34) \qquad A_{V,t,t-1} \equiv \sqrt{\frac{g_L\left(v_{L,t}, v_{K,t}, t-1\right)}{g_L\left(v_{L,t-1}, v_{K,t-1}, t-1\right)} \cdot \frac{g_L\left(v_{L,t}, v_{K,t}, t\right)}{g_L\left(v_{L,t-1}, v_{K,t-1}, t\right)}} .$$

Note that if capital deepening takes place, both $A_{V,t,t-1}^{L}$ and $A_{V,t,t-1}^{P}$ are greater than one, in which case $A_{V,t,t-1}$ must exceed one as well.

Similarly, we can define the following index that isolates the impact of technological change. That is, we compute the index of average labor productivity, allowing for the passage of time, but holding factor endowments fixed, first at their level of period *t-1*, and then at their level of period *t*:

$$(35) \qquad A_{T,t,t-1}^{L} \equiv \frac{g_L\left(v_{L,t-1}, v_{K,t-1}, t\right)}{g_L\left(v_{L,t-1}, v_{K,t-1}, t-1\right)}$$

(36) $$A^{P}_{T,t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t-1)} .$$

Taking the geometric mean of these two indexes, we get:

(37) $$A_{T,t,t-1} \equiv \sqrt{\frac{g_L(v_{L,t-1}, v_{K,t-1}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t-1)}} .$$

It can easily be seen that $A_{V,t,t-1}$ and $A_{T,t,t-1}$ together yield a complete decomposition of the index of average labor productivity:

(38) $$A_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1} .$$

We can proceed along exactly the same lines with the marginal productivity index. We thus get the two following components:

(39) $$M_{V,t,t-1} \equiv \sqrt{\frac{y_L(v_{L,t}, v_{K,t}, t-1)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t)}}$$

(40) $$M_{T,t,t-1} \equiv \sqrt{\frac{y_L(v_{L,t-1}, v_{K,t-1}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t}, v_{K,t}, t-1)}} .$$

Together these two partial indexes provide a complete decomposition of $M_{t,t-1}$:

(41) $$M_{t,t-1} = M_{V,t,t-1} \cdot M_{T,t,t-1} .$$

An alternative way of tackling the decomposition of $M_{t,t-1}$ would be on the basis of (9). Indeed, since $y_L(\cdot) = s_L(\cdot) g_L(\cdot)$, $M_{V,t,t-1}$ could also be expressed as:

(42) $$M_{V,t,t-1} = S_{V,t,t-1} \cdot A_{V,t,t-1}$$

where

(43) $$S_{V,t,t-1} \equiv \sqrt{\frac{s_L(v_{L,t}, v_{K,t}, t-1)}{s_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{s_L(v_{L,t}, v_{K,t}, t)}{s_L(v_{L,t-1}, v_{K,t-1}, t)}}$$

measures the contribution of changes in factor endowments on the share of labor. Similarly, it can be seen that:

(44) $$M_{T,t,t-1} = S_{T,t,t-1} \cdot A_{T,t,t-1}$$

where

(45) $$S_{T,t,t-1} \equiv \sqrt{\frac{s_L(v_{L,t-1},v_{K,t-1},t)}{s_L(v_{L,t-1},v_{K,t-1},t-1)} \cdot \frac{s_L(v_{L,t},v_{K,t},t)}{s_L(v_{L,t},v_{K,t},t-1)}} \ .$$

$S_{T,t,t-1}$ measures the contribution of technological progress to changes in the share of labor; it will be greater than one if technological change is pro-labor biased, and less than one otherwise.

Note that (38) and (41) only hold as long as $A_{t,t-1}$ and $M_{t,t-1}$ are indeed given by (6) and (7). If one uses instead actual data and if the average product of labor is measured as output per unit of labor and its marginal product is measured by its real wage rate, then one cannot expect expressions such as (38) and (41) to hold exactly, since production function (1) itself is only an approximation of reality, and the same is true for first-order condition (2). Let $AA_{t,t-1}$ and $MM_{t,t-1}$ be the *observed* values of the average and marginal productivities of labor, respectively:

(46) $$AA_{t,t-1} \equiv \frac{y_t / v_{L,t}}{y_{t-1} / v_{L,t-1}}$$

(47) $$MM_{t,t-1} \equiv \frac{w_{L,t} / p_t}{w_{L,t-1} / p_{t-1}} \ .$$

The full decomposition of both indexes will then read as follows:

(48) $$AA_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1} \cdot A_{U,t,t-1}$$

(49) $$MM_{t,t-1} = M_{V,t,t-1} \cdot M_{T,t,t-1} \cdot M_{U,t,t-1} \ ,$$

where $A_{U,t,t-1}$ and $M_{U,t,t-1}$ are the two error (or unexplained) components:

(50) $$A_{U,t,t-1} \equiv \frac{AA_{t,t-1}}{A_{t,t-1}}$$

(51) $$M_{U,t,t-1} \equiv \frac{MM_{t,t-1}}{M_{t,t-1}} \ .$$

8. Labor productivity vs. total factor productivity

While labor productivity remains the concept of choice when it comes to the public debate, most economists prefer to think in terms of total factor productivity. The measure of total factor productivity treats all inputs symmetrically. In the production function context, it can be defined as the increase in output that is not explained by increases in input quantities. Put differently, it is the increase in output made possible by technological change, holding all inputs constant. One state-of-the art definition of total factor productivity, $Y_{T,t,t-1}$, is drawn from the work of Diewert and Morrison (1986). It too can be thought of as the geometric average of Laspeyres-like and Paasche-like measures:

$$(52) \qquad Y_{T,t,t-1} \equiv \sqrt{\frac{y(v_{L,t-1}, v_{K,t-1}, t)}{y(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y(v_{L,t}, v_{K,t}, t)}{y(v_{L,t}, v_{K,t}, t-1)}} \ .$$

In view of the definition of $g_L(\cdot)$, it is immediately clear that $Y_{T,t,t-1}$ as given by (52) is in fact identical to $A_{T,t,t-1}$ as defined by (37). That is, total factor productivity in this model is equal to the contribution of technological change when explaining the average productivity of labor. The average productivity of labor will exceed total factor productivity to the extent that capital deepening occurs ($A_{V,t,t-1} > 1$).

9. Measurement

Consider first the case of the Cobb-Douglas production function. In that case, it is straightforward to show that:

$$(53) \qquad A_{X,t,t-1} = M_{X,t,t-1} = \left(\frac{v_{K,t}}{v_{L,t}} \Big/ \frac{v_{K,t-1}}{v_{L,t-1}} \right)^{\beta_K}$$

$$(54) \qquad A_{T,t,t-1} = M_{T,t,t-1} = e^{\mu} \ .$$

It is interesting to note that, since $e^{\mu} = e^{(1-\beta_K)\mu_L} = e^{\beta_K \mu_K}$, it does not matter for (54) to hold whether technological change is Hicks-neutral, Harrod-neutral, or Solow-neutral, or more general yet.[13] We report in Table 1, first column, parameter estimates of the Cobb-Douglas production function,[14] and in Tables 3 and 4 annual estimates of the decomposition of the average and marginal productivity of labor The factor

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[13] See (21)–(23) above.

[14] See the Appendix for a description of the data. To get estimates of the parameters of the Cobb-Douglas function we jointly estimated equations (21) (in logarithmic form) and (25). The estimation method is iterative Zellner as implemented in TSP.

endowments and the technological change components are the same in both tables, but the observed values of average and marginal differ, so that the corresponding error terms differ as well. Labor productivity has increased by close to 1.3% per annum over the sample period. Technological progress accounted for the bulk of the increase, with a contribution of about one percentage point. Capital deepening added about a quarter of a percentage point on average.

Consider next the Translog functional form. Introducing (30) into (34) and (37), we find that:

$$\text{(55)}\quad \begin{aligned}\ln A_{V,t,t-1} = {} & \left(\beta_K + \frac{1}{2}\phi_{KT}(2t-1)\right)\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right) \\ & + \frac{1}{2}\phi_{KK}\left[\left(\ln\frac{v_{K,t}}{v_{L,t}}\right)^2 - \left(\ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)^2\right]\end{aligned}$$

$$\text{(56)}\quad \ln A_{T,t,t-1} = \beta_T + \frac{1}{2}\phi_{KT}\left(\ln\frac{v_{K,t}}{v_{L,t}} + \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right) + \frac{1}{2}\phi_{TT}(2t-1) \ .$$

For the marginal productivity indexes, we can apply (42) and (44) after having introduced (31) into (43) and (45) to get:

$$\text{(57)}\quad S_{V,t,t-1} \equiv \sqrt{\frac{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}}-\phi_{KT}(t-1)}{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}}-\phi_{KT}(t-1)}\cdot\frac{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}}-\phi_{KT}t}{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}}-\phi_{KT}t}}$$

$$\text{(58)}\quad S_{T,t,t-1} \equiv \sqrt{\frac{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}}-\phi_{KT}t}{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}}-\phi_{KT}(t-1)}\cdot\frac{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}}-\phi_{KT}t}{1-\beta_K-\phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}}-\phi_{KT}(t-1)}} \ .$$

Parameter estimates of the Translog production function are reported in Table 1, column 3.[15] A decomposition of the average and marginal productivity indexes

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[15] Estimates of the parameters of the Translog function were obtained by jointly estimating equations (30) and (31); the estimation method again being iterative Zellner.

based on the Translog functional form is provided in Tables 5 and 6. Remember that $A_{T,t,t-1}$ in Table 5 can also be interpreted as a model-based measure of total factor productivity. The decomposition of the average productivity index is quite similar to the one obtained with the Cobb-Douglas, with total factor productivity accounting for about four fifths of the increase in average labor productivity. The decomposition of the marginal productivity index, on the other hand, shows a somewhat different picture: technological progress accounts for less than two thirds of real wage increases with capital deepening now playing a larger role. The reason for this difference has to do with the estimate of the elasticity of complementarity, which is found to be significantly larger than one. By restricting this elasticity to be unity, the Cobb-Douglas functional form leads to an underestimation of the impact of capital deepening on the marginal product of labor.

10. The average productivity of labor: an index number approach

To make the decomposition (55)–(58) operational one needs econometric estimates of the parameters of the Translog production function. This is indeed how we were able to construct the figures reported in Tables 5 and 6. It turns out, however, that, as long as the true production function is Translog, the decomposition of the average productivity of labor can also be obtained on the basis of knowledge of the data alone; that is, without needing to know the individual parameters of the production function.

Following Diewert and Morrison (1986), one can show that, as long as the true production function is given by (30), $A_{T,t,t-1}$ defined by (37) – or, equivalently, $Y_{T,t,t-1}$ defined by (52) – can be computed as:

$$(59) \qquad A_{T,t,t-1} = \frac{Y_{t,t-1}}{V_{t,t-1}} \ ,$$

where $Y_{t,t-1}$ is the index of real GDP:

$$(60) \qquad Y_{t,t-1} \equiv \frac{y_t}{y_{t-1}} \ ,$$

and $V_{t,t-1}$ is a Tornqvist index of input quantities:

$$(61) \qquad V_{t,t-1} \equiv \exp\left[\sum_{i\in\{L,K\}} \frac{1}{2}(s_{i,t}+s_{i,t-1})\ln\frac{v_{i,t}}{v_{i,t-1}}\right],$$

where $s_{K,t}$ $(=1-s_{L,t})$ is the income share of capital. Hence the following gives a complete decomposition of real GDP growth:

$$(62) \qquad Y_{t,t-1} = Y_{L,t,t-1}\cdot Y_{K,t,t-1}\cdot A_{T,t,t-1}$$

where:

$$(63) \qquad Y_{L,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{L,t}+s_{L,t-1})\ln\frac{v_{L,t}}{v_{L,t-1}}\right]$$

$$(64) \qquad Y_{K,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t}+s_{K,t-1})\ln\frac{v_{K,t}}{v_{K,t-1}}\right].$$

$Y_{L,t,t-1}$ and $Y_{K,t,t-1}$ can be interpreted as the contributions of labor and capital to real GDP growth. Next, let $L_{t,t-1}$ be the labor input index:

$$(65) \qquad L_{t,t-1} \equiv \frac{v_{L,t}}{v_{L,t-1}}.$$

It follows from (46) that:

$$(66) \qquad AA_{t,t-1} \equiv \frac{Y_{t,t-1}}{L_{t,t-1}}.$$

Making use of (62) – (64), we get:

$$(67) \qquad AA_{t,t-1} = A_{V,t,t-1}\cdot A_{T,t,t-1},$$

where:

$$(68) \qquad A_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t}+s_{K,t-1})\left(\ln\frac{v_{K,t}}{v_{L,t}}-\ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)\right].$$

We show in Table 7 the decomposition of the average productivity of labor based on (67). This decomposition does not require knowledge of the parameters of the

Translog function.[16] This is obviously very convenient. On the other hand, as indicated by (59), the total factor productivity term ($A_{T,t,t-1}$) is obtained as a Solow residual. Hence, unlike what is done in (48), it is not possible to split it up into a secular component and an error term.[17] The estimates shown in Table 7 are very similar to those shown in Table 5, except obviously for the total factor productivity term which now incorporates the unexplained component.

11. Domestic real value added

The model of the production function is rather limiting since it imposes the number of outputs to be one.[18] Moreover, the production function approach does not make it possible to take into account imports and exports. In what follows, we therefore opt for the description of the aggregate technology by a real value added (or real income) function, such as the one proposed by Kohli (2004a). It is based on the GDP function approach to modeling the production sector of an open economy.[19] We assume that the technology counts two outputs, domestic (nontraded) goods (*D*) and exports (*X*), as well as three inputs, labor (*L*), capital (*K*), and imports (*M*). We will treat imports as a variable input, i.e. as a negative output. We denote output (including import) quantities by y_i and their prices by p_i, $i \in \{D, X, M\}$. Furthermore, we denote the inverse of the terms of trade by q ($q \equiv p_M / p_X$) and the relative price of tradables vs. nontradables by e ($e \equiv p_X / p_D$). Note that for given terms of trade, a change in e can be interpreted as a change in the real exchange rate, an increase in e being equivalent to a real depreciation of the home currency. Let π_t be nominal GDP:

$$(69) \qquad \pi_t \equiv p_{D,t} y_{D,t} + p_{X,t} y_{X,t} - p_{M,t} y_{M,t} = p_t y_t \ .$$

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[16] This index-number approach essentially boils down to using the observed share of labor (8) instead of the fitted one as given by (31). See Kohli (1990) for a further discussion of the differences between the two approaches.

[17] An index-number approach is not feasible for the marginal productivity index, for even if the true production function is Translog, the first-order condition is not: as shown by (31), it is linear in logarithms, rather than quadratic.

[18] Alternatively, one must assume that outputs are globally separable from domestic inputs.

[19] See Kohli (1978), Woodland (1982).

Domestic real value added (z_t) – or real domestic income – is defined as nominal GDP deflated by the price of domestic output:

$$(70) \qquad z_t \equiv \frac{\pi_t}{p_{D,t}} = y_{D,t} + e_t y_{X,t} - e_t q_t y_{M,t} \ .$$

Let T_t be the production possibilities set at time *t*. We assume that T_t is a convex cone. The aggregate technology can be described by a real valued added function defined as follows:

$$(71) \qquad z(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \max_{y_D, y_X, y_M} \left\{ \begin{array}{l} y_{D,t} + e_t y_{X,t} - e_t\, q_t y_{M,t} : \\ \qquad (y_{D,t}, y_{X,t}, y_{M,t}, v_{K,t}, v_{L,t}) \in T_t \end{array} \right\} .$$

In this context, the *average* real value added of labor ($h_{L,t}$) can be expressed as follows:

$$(72) \qquad h_{L,t} = h_L(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \frac{z(q_t, e_t, v_{K,t}, v_{L,t}, t)}{v_{L,t}} \ ,$$

whereas as the *marginal* real value added of labor ($z_{L,t}$) is given by:

$$(73) \qquad z_{L,t} = z_L(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \frac{\partial z(q_t, e_t, v_{K,t}, v_{L,t}, t)}{\partial v_{L,t}} \ .$$

The average and marginal productivity indexes are now as follows:

$$(74) \qquad A_{t,t-1} \equiv \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}$$

$$(75) \qquad M_{t,t-1} \equiv \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \ .$$

The Translog functional form is well suited to give a flexible representation of the real value added function. It is as follows:

$$
\begin{aligned}
\ln z_t = {} & \alpha_0 + \alpha_Q \ln q_t + \alpha_E \ln e_t + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} \\
& + \frac{1}{2}\gamma_{QQ}(\ln q_t)^2 + \gamma_{QE} \ln q_t \ln e_t + \frac{1}{2}\gamma_{EE}(\ln e_t)^2 \\
& + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 + (\delta_{QK} \ln q_t + \delta_{EK} \ln e_t)(\ln v_{K,t} - \ln v_{L,t}) \\
& + (\delta_{QT} \ln q_t + \delta_{ET} \ln e_t)\, t + \phi_{KT}(\ln v_{K,t} - \ln v_{L,t})\, t + \beta_T\, t + \frac{1}{2}\phi_{TT}\, t^2
\end{aligned}
\tag{76}
$$

Logarithmic differentiation yields the following system of equations:[20]

$$\frac{\partial \ln z(\cdot)}{\partial \ln q_t} = -s_{M,t} = \alpha_Q + \gamma_{QQ} \ln q_t + \gamma_{QE} \ln e_t + \delta_{QK}(\ln v_{K,t} - \ln v_{L,t}) + \delta_{QT}\, t \tag{77}$$

$$\frac{\partial \ln z(\cdot)}{\partial \ln e_t} = s_{B,t} = \alpha_E + \gamma_{QE} \ln q_t + \gamma_{EE} \ln e_t + \delta_{EK}(\ln v_{K,t} - \ln v_{L,t}) + \delta_{ET}\, t \tag{78}$$

$$\frac{\partial \ln z(\cdot)}{\partial \ln v_{L,t}} = s_{L,t} = 1 - \beta_K - \delta_{QK} \ln q_t - \delta_{EK} \ln e_t - \phi_{KK}(\ln v_{K,t} - \ln v_{L,t}) - \phi_{KT}\, t \tag{79}$$

$$\frac{\partial \ln z(\cdot)}{\partial t} \equiv \mu_t = \beta_T + \delta_{QT} \ln q_t + \delta_{ET} \ln e_t + \phi_{KT}(\ln v_{K,t} - \ln v_{L,t}) + \phi_{TT}\, t \ , \tag{80}$$

where s_M is the GDP share of imports ($s_M \equiv p_M y_M / \pi$), s_B is the trade balance relative to GDP ($s_B \equiv (p_X y_X - p_M y_M)/\pi$), s_L is, as before, the GDP share of labor, and μ is again the instantaneous rate of technological change. Parameter estimates, obtained from the joint estimation of equations (76)–(80), are reported in the last column of Table 1.[21] It is noteworthy that the labor share now depends on four items. Besides relative factor endowments and the passage of time, the terms of trade and the real exchange rate may influence the share of labor as well. A deterioration in the terms of trade (an increase in *q*) will tend to lower the share of labor as indicated by the positive estimate of δ_{QK}. Similarly, a real appreciation of the home currency (a fall in *e*) will tend to reduce s_L in view of the negative estimate of δ_{EK}. In both these cases, the marginal product of labor would, *ceteris paribus*, increase less rapidly than its average product.

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[20] See Kohli (2004a).

[21] The estimation method was once again nonlinear iterative Zellner.

12. Average productivity in the open economy

Proceeding along the same lines as in Section 7, we can define the following index to capture the contribution of changes in the terms of trade to the average productivity of labor:

$$(81) \qquad A_{Q,t,t-1} \equiv \sqrt{\frac{h_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)}} \; .$$

Similarly, we can identify the contribution of changes in the real exchange rate as:

$$(82) \qquad A_{E,t,t-1} \equiv \sqrt{\frac{h_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)}} \; ,$$

the contribution of changes in domestic factor endowments:

$$(83) \qquad A_{V,t,t-1} \equiv \sqrt{\frac{h_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)}} \; ,$$

and, finally, the contribution of technological progress:

$$(84) \qquad A_{T,t,t-1} \equiv \sqrt{\frac{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)}} \; .$$

Assuming that the real value added function is given by (76) and that its parameters are known, it is straightforward to compute the values of (81)–(84). Moreover, it can easily be shown that these four effects together give a complete decomposition of the average productivity of labor as defined by (74):

$$(85) \qquad A_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1} \; .$$

Finally, if we seek to explain the *observed* increase in average labor productivity, we get:

$$(86) \qquad AA_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1} \cdot A_{U,t,t-1} \; ,$$

where $AA_{t,t-1}$ is now defined as:

$$(87) \qquad AA_{t,t-1} \equiv \frac{z_t / v_{L,t}}{z_{t-1} / v_{L,t-1}} \; ,$$

and $A_{U,t,t-1}$ is the unexplained component of $AA_{t,t-1}$:

$$(88)\qquad A_{U,t,t-1} = \frac{AA_{t,t-1}}{A_{t,t-1}} \ .$$

If the true real value added function is indeed Translog it is possible to compute (81)–(84) based on the data alone, that is without knowledge of the parameters of (76). Indeed, one can show that:[22]

$$(89)\qquad A_{Q,t,t-1} = \exp\left[\frac{1}{2}(-s_{M,t} - s_{M,t-1})\ \ln\frac{q_t}{q_{t-1}}\right]$$

$$(90)\qquad A_{E,t,t-1} = \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1})\ \ln\frac{e_t}{e_{t-1}}\right]$$

$$(91)\qquad A_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)\right]$$

$$(92)\qquad A_{T,t,t-1} \equiv \frac{Y_{t,t-1}}{V_{t,t-1}} \ ,$$

so that:

$$(93)\qquad AA_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1} \ .$$

A decomposition the average productivity of labor according to (86) and (93) is reported in Tables 8 and 9, respectively. Our results are illustrated by Figures 3-A and 3-B. The upper panel shows the annual contributions of changes in the terms of trade and in the real exchange rate, together with the fitted value of the average productivity index. The lower panel focuses on capital intensity and total factor productivity, again with the average productivity index as reference. It appears that the dynamics of the fitted value of the average labor productivity index is dominated by movements in the contribution of capital deepening, although changes in the terms of trade contribute to the overall profile as well.

13. Accounting for changes in the share of labor

In the next section, we will focus on the decomposition of the *marginal* productivity index, but beforehand we will briefly turn our attention to the behavior of the labor

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[22] See Kohli (2004a).

share. Indeed, we will follow here essentially the same route as in the production function context; that is, we will exploit the link between the marginal and the average productivity measures by way of the labor share index. There will be one important difference compared to our treatment in Section 7, however. Thus, a decomposition such as (41), which is exact independently of the underlying functional form, only holds if the number of elements on the right-hand side is two. If the underlying functional form is Translog, however, the decomposition is exact even if the number of components is larger than two; see (85) above, for instance. Unfortunately, even if the underlying function is Translog, as is the case here, the first-order conditions are not. Indeed, as shown by (77)–(80), the share equations are linear in logarithms. Hence the best we can hope for is a linear approximation of the decomposition of the marginal productivity and the labor share indices. With this caveat in mind, we will proceed as in Sections 7 and 9.[23]

In the context of the real value-added function, the labor share index can be defined as follows:

$$(94)\qquad S_{t,t-1} \equiv \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}\ ,$$

where $s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)$ is given by the right-hand side of (79). This index can easily be calculated once the parameters of the real value-added function are known. The same holds true for the following four indices that identify the contributions of the terms of trade, the real exchange rate, the relative factor endowments and the passage of time:

$$(95)\qquad S_{Q,t,t-1} \equiv \sqrt{\frac{s_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)}}$$

$$(96)\qquad S_{E,t,t-1} \equiv \sqrt{\frac{s_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)}}$$

$$(97)\qquad S_{V,t,t-1} \equiv \sqrt{\frac{s_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)}}$$

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[23] See Sfreddo (2001) for a further discussion and for three alternative decompositions of the first-order conditions.

$$(98) \quad S_{T,t,t-1} \equiv \sqrt{\frac{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)}} \ .$$

An approximation to $S_{t,t-1}$ is given by the following:[24]

$$(99) \quad S_{t,t-1} \cong S_{Q,t,t-1} \cdot S_{E,t,t-1} \cdot S_{V,t,t-1} \cdot S_{T,t,t-1} \ .$$

We next define the *observed* labor share index:

$$(100) \quad SS_{t,t-1} \equiv \frac{x_{L,t} w_{L,t} / \pi_t}{x_{L,t-1} w_{L,t-1} / \pi_{t-1}} \ .$$

A complete decomposition of the change in the labor share is hence given by:

$$(101) \quad SS_{t,t-1} \cong S_{Q,t,t-1} \cdot S_{E,t,t-1} \cdot S_{V,t,t-1} \cdot S_{T,t,t-1} \cdot S_{U,t,t-1} \ ,$$

where $S_{U,t,t-1}$ is the unexplained component:

$$(102) \quad S_{U,t,t-1} = \frac{SS_{t,t-1}}{S_{t,t-1}} \ .$$

We show in Table 10 the decomposition of the labor share index according to (101). Geometric averages for the entire period are shown at the bottom of the table. One can see that the labor share has slightly decreased over time (by about 0.06% annually; this is also visible in Figure 1). The worsening of the terms of trade and the real appreciation of the dollar that the United States has experienced over the past forty years have contributed somewhat to the decrease in the labor share (given the positive estimate of δ_{QK} and the negative estimate of δ_{EK}).[25] More significant, though, have been the impact of capital deepening – which has increased the labor share by 0.35% per year on average – and the offsetting effect of technological change – which has reduced the share by about 0.22% per annum. For some years, the individual effects have been rather large. Thus, in 1974 the adverse terms-of-trade effect has reduced the labor share by about 1.3%, whereas the simultaneous real

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[24] See Sfreddo (2001); we have verified that the residual is essentially nil.

[25] The assertion that the United States has experienced a real appreciation of its currency over the past four decades may come as a surprise to some readers. The drop in *e* simply reflects the fact that the price of domestic goods has increased more rapidly than the price of exports. This increase in the price of nontradables relative to the price of tradables is characteristic of the large U.S. current account deficit.

depreciation of the dollar has had a reverse effect of close to 0.6%. In 1975, capital deepening has raised the labor share by over 2%. The impact of technological change has been very steady over time, whereas the unexplained component – which can be interpreted as a productivity shock – has been in excess of 2% on a few occasions, while being close to zero on average.

14. Accounting for changes in real wages

We are now in a position to account for the marginal productivity index and the changes in real wages. The marginal productivity index is defined by (75). We can also identify the following partial effects:

$$(103)\quad M_{Q,t,t-1} \equiv \sqrt{\frac{z_L(q_t,e_{t-1},v_{K,t-1},v_{L,t-1},t-1)}{z_L(q_{t-1},e_{t-1},v_{K,t-1},v_{L,t-1},t-1)} \cdot \frac{z_L(q_t,e_t,v_{K,t},v_{L,t},t)}{z_L(q_{t-1},e_t,v_{K,t},v_{L,t},t)}}$$

$$(104)\quad M_{E,t,t-1} \equiv \sqrt{\frac{z_L(q_{t-1},e_t,v_{K,t-1},v_{L,t-1},t-1)}{z_L(q_{t-1},e_{t-1},v_{K,t-1},v_{L,t-1},t-1)} \cdot \frac{z_L(q_t,e_t,v_{K,t},v_{L,t},t)}{z_L(q_t,e_{t-1},v_{K,t},v_{L,t},t)}}$$

$$(105)\quad M_{V,t,t-1} \equiv \sqrt{\frac{z_L(q_{t-1},e_{t-1},v_{K,t},v_{L,t},t-1)}{z_L(q_{t-1},e_{t-1},v_{K,t-1},v_{L,t-1},t-1)} \cdot \frac{z_L(q_t,e_t,v_{K,t},v_{L,t},t)}{z_L(q_t,e_t,v_{K,t-1},v_{L,t-1},t)}}$$

$$(106)\quad M_{T,t,t-1} \equiv \sqrt{\frac{z_L(q_{t-1},e_{t-1},v_{K,t-1},v_{L,t-1},t)}{z_L(q_{t-1},e_{t-1},v_{K,t-1},v_{L,t-1},t-1)} \cdot \frac{z_L(q_t,e_t,v_{K,t},v_{L,t},t)}{z_L(q_t,e_t,v_{K,t},v_{L,t},t-1)}}\ .$$

Since $z_L(\cdot) = h_L(\cdot) \cdot s_L(\cdot)$ under profit maximization (as long as the timing of the arguments is the same in all three functions), it immediately follows that:

$$(107)\quad M_{t,t-1} = A_{t,t-1} \cdot S_{t,t-1}$$

$$(108)\quad M_{Q,t,t-1} = A_{Q,t,t-1} \cdot S_{Q,t,t-1}$$

$$(109)\quad M_{E,t,t-1} = A_{E,t,t-1} \cdot S_{E,t,t-1}$$

$$(110)\quad M_{V,t,t-1} = A_{V,t,t-1} \cdot S_{V,t,t-1}$$

$$(111)\quad M_{T,t,t-1} = A_{T,t,t-1} \cdot S_{T,t,t-1}\ .$$

Furthermore, it follows from (85) and (99) that:

(112) $$M_{t,t-1} \cong M_{Q,t,t-1} \cdot M_{E,t,t-1} \cdot M_{V,t,t-1} \cdot M_{T,t,t-1} \ .$$

We finally consider the *observed* marginal productivity of labor index. It is now as follows:

(113) $$MM_{t,t-1} \equiv \frac{w_{L,t} / p_{D,t}}{w_{L,t-1} / p_{D,t-1}} \ .$$

A complete decomposition of the progression in real wages is therefore given by:

(114) $$MM_{t,t-1} \cong M_{Q,t,t-1} \cdot M_{E,t,t-1} \cdot M_{V,t,t-1} \cdot M_{T,t,t-1} \cdot M_{U,t,t-1} \ ,$$

where $M_{U,t,t-1}$ is as usual the unexplained component:

(115) $$M_{U,t,t-1} = \frac{MM_{t,t-1}}{M_{t,t-1}} \ .$$

We show in Table 11 the decomposition of the marginal productivity of labor based on (114). Real wages increased by just over 1.2% per year over the sample period. This increase is dominated by technological progress, although capital deepening played an important role too. In fact, comparing these results with those in Table 8, we again find that capital deepening has a relatively larger impact on marginal productivity than on average productivity. Terms-of-trade changes have reduced real wages by approximately 0.1% per annum on average. Changes in the real exchange rate have only had a negligible effect on average, although the impact has been noticeable in some years, such as 1974 when it added about 0.6% to real wages. The annual contribution of the four forces that we have identified can best be assessed with the help of Figures 4-A and 4-B. The first panel shows the terms-of-trade and the real-exchange-rate effects, together with the fitted change in real wages. The second panel shows the contribution of capital deepening and of technological change, again with estimated marginal productivity in the background. The dominating influence of capital deepening in the short run is clearly documented.

15. About unit labor costs

Many economic analysts attach much importance to the development of unit labor costs. An increase in unit labor costs – that is, an increase in nominal wages that is not matched by an increase in average productivity – is often viewed as being a threat to price stability. This concern as to the inflationary consequences of an increase in unit labor costs can be understood if one considers that, in most industries and for the economy as a whole, labor costs make up the largest component of total costs. This might explain why increases in unit labor costs are sometimes thought of as being the prime source of inflation, even though a theory of inflation that leaves no place for money may sound somewhat suspect. In any case, it may be useful to investigate what role unit labor costs play in our analysis.

Unit labor costs ($\omega_{L,t}$) can be defined as follows:

$$(116) \quad \omega_{L,t} \equiv \frac{w_{L,t}}{z_t / v_{L,t}} \; .$$

In view of our earlier definitions, this can also be expressed as:

$$(117) \quad \omega_{L,t} = \frac{w_{L,t}}{h_{L,t}} = \frac{z_{L,t} \cdot p_{D,t}}{h_{L,t}} = s_{L,t} \cdot p_{D,t} \, .$$

In terms of change factors we get:

$$(118) \quad \Omega_{L,t,t-1} = SS_{L,t,t-1} \cdot P_{D,t,t-1} \; ,$$

where $\Omega_{L,t,t-1}$ is the unit labor cost index and $P_{D,t,t-1}$ is the (one plus) the rate of domestic price inflation:

$$(119) \quad \Omega_{L,t,t-1} \equiv \frac{\omega_{L,t}}{\omega_{L,t-1}}$$

$$(120) \quad P_{D,t,t-1} \equiv \frac{p_{D,t}}{p_{D,t-1}} \; .$$

Looking at (118), the link between increases in unit labor costs and inflation is evident. In fact, if the share of labor is constant ($SS_{t,t-1} = 1$ in that case), the correlation is perfect. An increase in unit labor costs, be it as the result of an increase in nominal wages or a reduction in average productivity, would necessarily go hand in hand with an increase in the price of output. Correlation is not causation, however. Nominal wages need not be exogenous, no more than average productivity. In fact, it is reasonable to assume that both are endogenous for the economy as a whole, and this is indeed how they have been treated in the model developed in this paper. Similarly, as we have stressed throughout the paper, the share of labor is endogenous as well. Rather than viewing changes in unit labor costs as an exogenous factor impacting on prices, it might be more useful to explain the changes in unit labor costs as a function of the factors that we have identified earlier on.

In the context of our model, it is immediately clear from (118) that changes in unit labor costs reflect changes in (i) the share of labor and (ii) the price of output. As to the second item, it could reasonably argue that unit labor costs mirror changes in the general price level, rather than cause them. Regarding the impact of changes in the labor share, we may refer the reader to our discussion of Section 13 as summarized in Table 10. Thus, in the U.S. case, a worsening in the terms of trade and/or a real appreciation of the currency, *ceteris paribus*, reduce unit labor costs. The same is true for technological change, whereas capital deepening acts to increase unit labor costs. Some of these results may sound counter-intuitive. Thus, an increase in the stock of capital, which, for a given labor endowment, must unambiguously increase output and average labor productivity, might yet increase unit labor costs if the marginal product of labor (i.e. real wages) increases by relatively more. If the Hicksian elasticity of complementarity is greater than one, this will indeed be the case.

16. Conclusions

Productivity is an important, yet elusive concept. In this paper we tried to sort out some of the ideas linked to this concept and to identify the main components of labor productivity. The distinction between the marginal and the average productivity of labor induced us to focus on the GDP share of labor. This in turn led us to identify the main forces at work: technological progress, capital deepening, terms-of-trade changes, and changes in the real exchange rate. These last two factors, although statistically significant, were found to play a minor role. This has no doubt to do with the fact that the United States is a relatively closed economy. It is well possible that changes in the terms of trade and in the real exchange rate play a more important role for labor productivity in more open economies.

Our analysis also led us to emphasize the role played by the Hicksian elasticity of complementarity. It turns out that this elasticity is significantly greater than unity. This explains to a large extent why the share of labor has been fairly steady over time, and thus why the marginal and the average measures of labor productivity have moved in unison. Capital deepening tends to increase the marginal product of labor, and given the large elasticity of complementarity this tends to increase the share of labor. Technological progress, on the other hand, by being mainly labor augmenting can be thought of as anti-labor biased (although not ultra anti-labor biased). This tends to reduce the share of labor, largely offsetting the impact of capital deepening. The slight deterioration in the terms of trade and the small real appreciation of the U.S. dollar that took place over the sample period have further contained the increase in the labor share.

This paper has also documented the relationship between total factor productivity and the average productivity of labor. Although total factor productivity is the main driving force in the increase in output and average productivity, expression (93) shows that there are other forces at work as well.

The growth in U.S. labor productivity in recent years has been truly impressive. It is often considered as a tribute and a testimony of the remarkable performance of American workers. However, it is useful to keep a couple of things in mind. First, the headline figures typically relate to the nonfarm business sector only. That is, the farming sector, the government sector and the household sector – that is, probably close to half the economy – are left out of the calculation. Second, productivity is not always the outcome of hard work. It can also be the outcome of a conjunction of favorable events. Thus, capital deepening will unavoidably increase the average and the marginal productivity of labor. A technological progress will necessarily increase average productivity as well, but it may impact either way on real wages, although in the U.S. case, the effect is positive. An improvement in the terms of trade and a depreciation of the home currency also lead to increases in average labor productivity, and, in the U.S. case, the impact on real wages is even magnified through the increase in the GDP share of labor.

Appendix: Description of the data

All data are annual for the period 1970 to 2001. We require the prices and quantities of all inputs and outputs. The data for GDP and its components, in nominal and in real terms, are taken from the *Bureau of Economic Analysis* website. Prices are then obtained by deflation. Data on the capital stock, labor compensation, and national income are also retrieved from the *BEA* website. The quantity of capital services is assumed to be proportional to the stock. Capital income is defined as national income minus labor compensation. The quantity of labor services is computed by multiplying the total number of employees on nonfarm payrolls by an index of the average number of weekly hours worked in the nonfarm business sector. Both these series are taken from the *Bureau of Labor Statistics* website. The user costs of labor and capital are then obtained by dividing labor and capital income by the corresponding quantity series. For the purpose of Sections 9 and 10, output is expressed as an implicit Törnqvist index of real GDP; see Kohli (2004b) for details. In Sections 11 to 15, the price of nontraded goods is computed as a Törnqvist price index of the deflators of consumption, investment and government purchases.

References

Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau (1973) "Transcendental Logarithmic Production Frontiers", *Review of Economics and Statistics* 55, 28-45.

Diewert, W. Erwin (1974) "Applications of Duality Theory", in Michael D. Intriligator and David A. Kendrick (eds.) *Frontiers of Quantitative Economics*, Vol. 2 (Amsterdam: North-Holland).

Diewert, W. Erwin (1976) "Exact and Superlative Index Numbers", *Journal of Econometrics* 4, 115-145.

Diewert, W. Erwin and Catherine J. Morrison (1986) "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *Economic Journal* 96, 659-679.

Diewert, W. Erwin and Terence J. Wales (1987) "Flexible Functional Forms and Global Curvature Conditions", *Econometrica* 55, 43-68.

Diewert, W. Erwin and Terence J. Wales (1992) "Quadratic Spline Models for Producer's Supply and Demand Functions", *International Economic Review* 33, 705-722.

Jorgenson, Dale W. and Barbara M. Fraumeni (1981) "Relative Prices and Technical Change", in Ernst R. Berndt and Barry C. Fields (eds.) *Modeling and Measuring Natural Resource Substitution* (Cambridge, MA: MIT Press).

Kohli, Ulrich (1978) "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports", *Canadian Journal of Economics* 11, 167-182.

Kohli, Ulrich (1990) "Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates", *Journal of Economic and Social Measurement* 16, 125-136.

Kohli, Ulrich (1991) *Technology, Duality, and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports* (Ann Arbor, MI: University of Michigan Press).

Kohli, Ulrich (1994) "Technological Biases in U.S. Aggregate Production", *Journal of Productivity Analysis* 5, 5-22.

Kohli, Ulrich (2004a) "Real GDP, Real Domestic Income, and Terms-of-Trade Changes", *Journal of International Economics* 62, 83-106.

Kohli, Ulrich (2004b) "An Implicit Törnqvist Index of Real GDP", *Journal of Productivity Analysis* 21, 337-353.

Sfreddo, Claudio (2001) *Trade, Technology, and Factor Prices: A GDP Function Approach for the European Union*, doctoral thesis, University of Geneva.

United States Department of Commerce, Bureau of Economic Analysis, *http://www.bea.doc.gov*.

United States Department of Labor, Bureau of Labor Statistics, *http://www.bls.gov*.

Woodland, Alan D. (1982) *International Trade and Resource Allocation* (Amsterdam: North-Holland).

Figure 1

GDP share of labor, 1971-2001

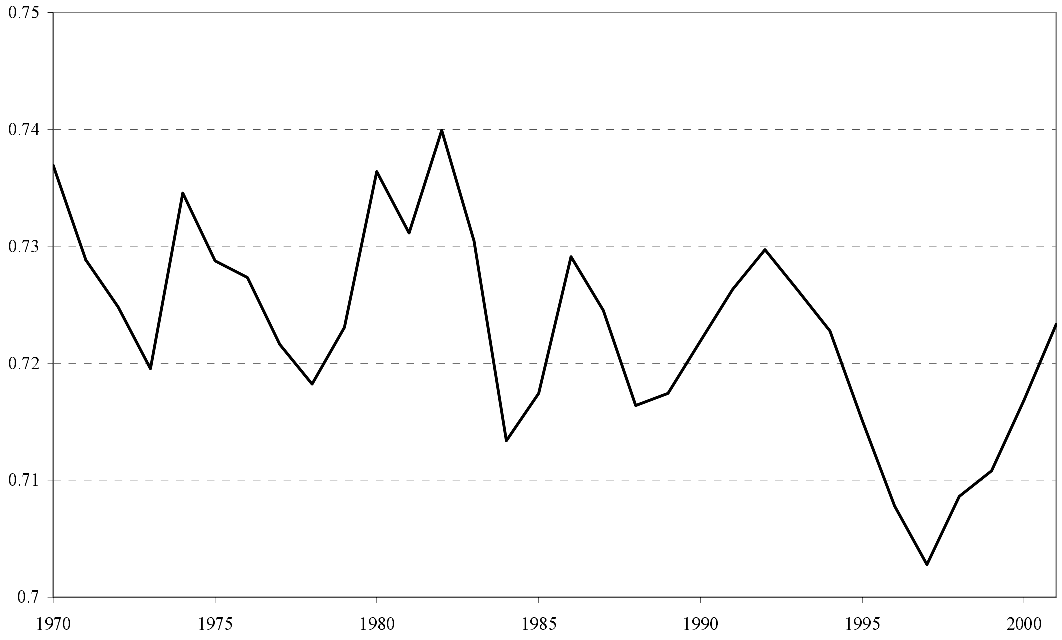

Figure 2

Labor productivity, 1970-2001

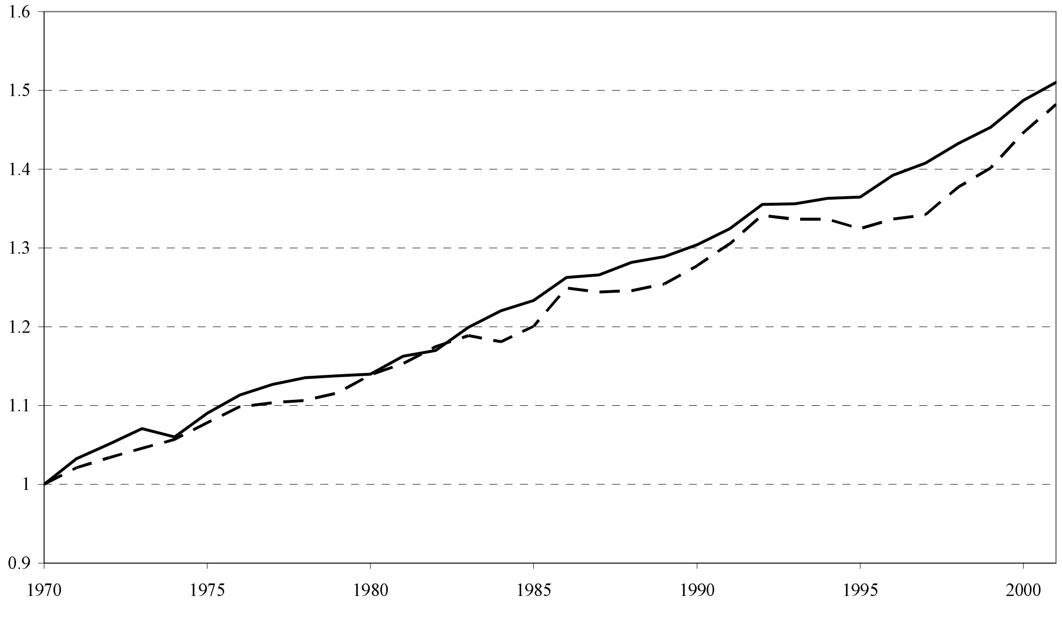

Figure 3-A

Terms of trade, real exchange rate, and average productivity of labor

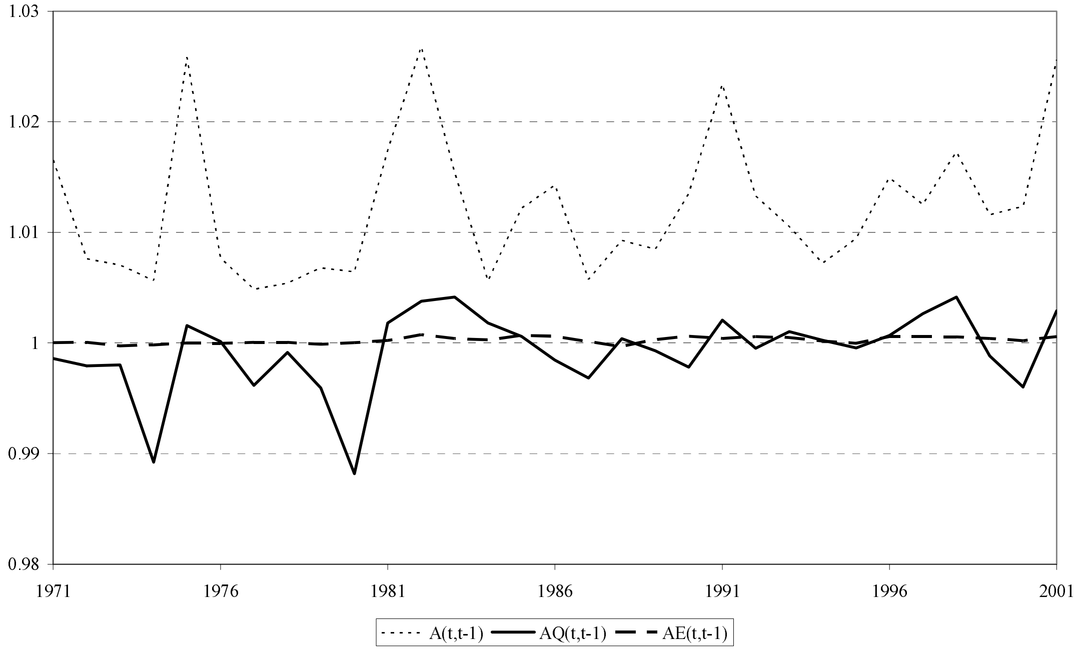

Figure 3-B

Capital intensity, total factor productivity, and average productivity of labor

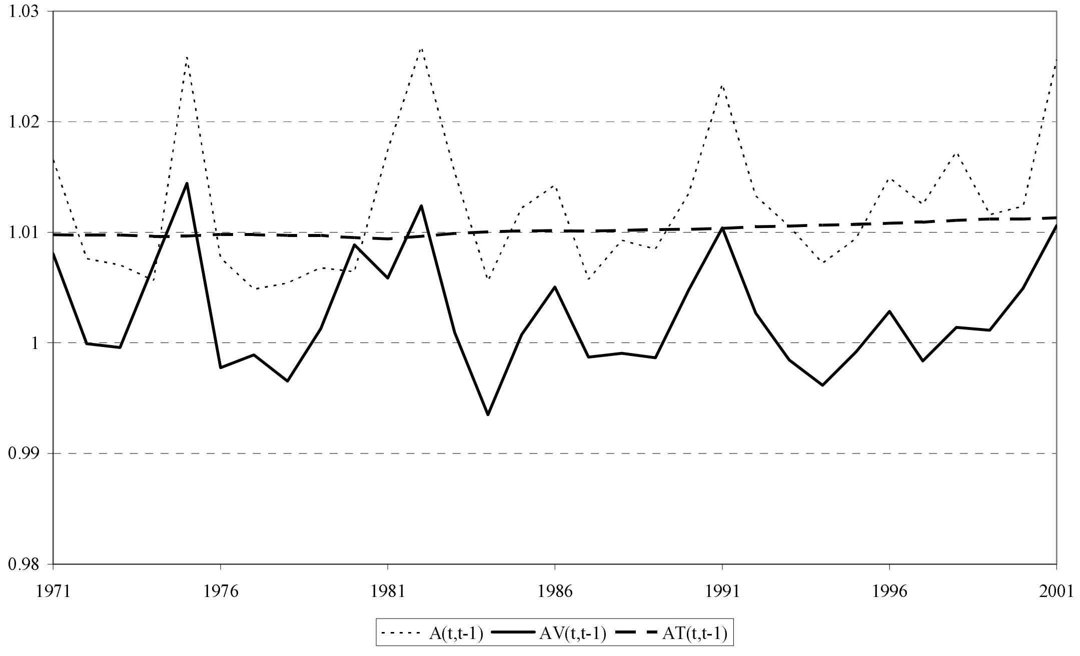

Figure 4-A

Terms of trade, real exchange rate, and marginal productivity of labor

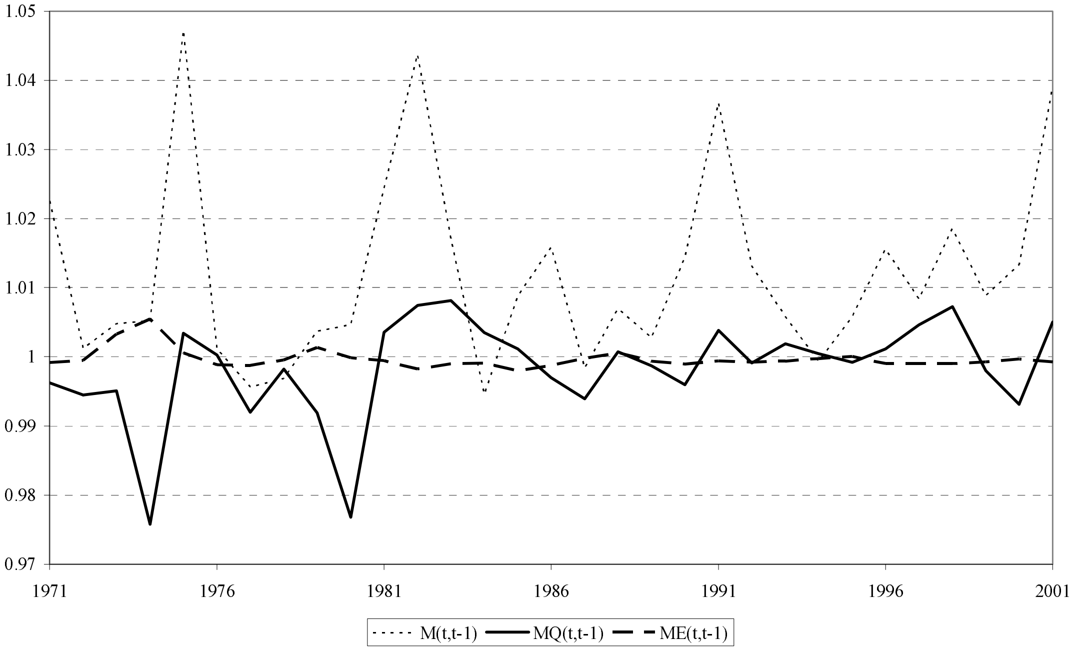

Figure 4-B

Capital intensity, technological progress, and marginal productivity of labor

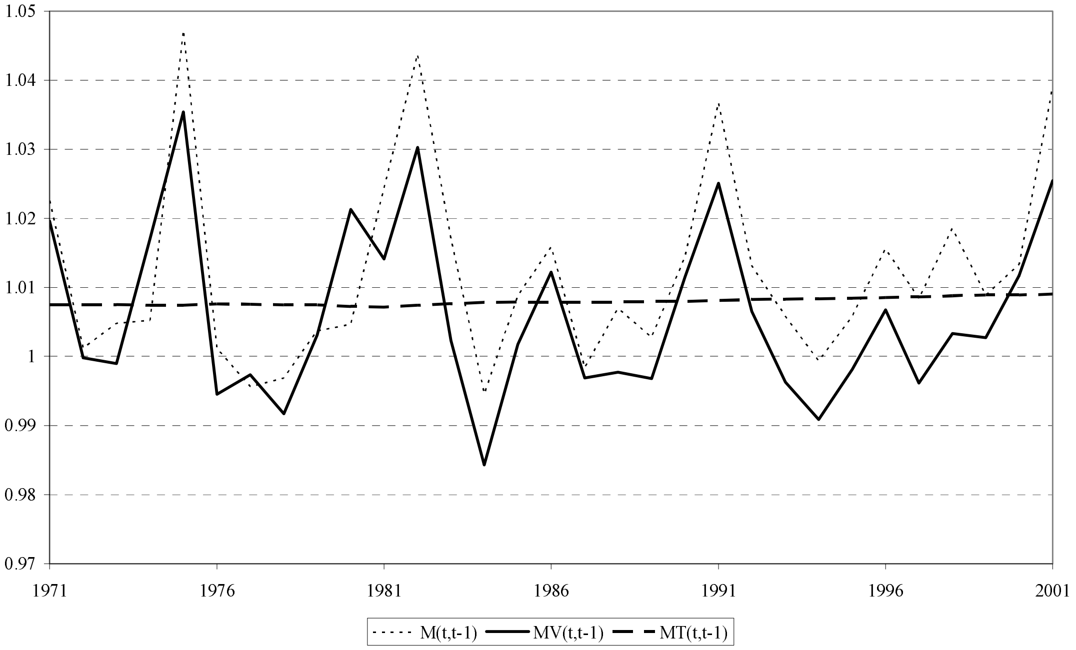

Table 1

Parameter estimates

	(21)	*(27)*	*(30)*	*(76)*
α_0	8.96585	8.96960	8.97085	8.97034
	(3624.2)	(3475.5)	(3537.4)	(3546.0)
α_Q				-0.12534
				(-91.51)
α_E				-0.01913
				(-7.75)
β_K	0.27709	0.28448	0.28477	0.28537
	(179.4)	(171.2)	(172.4)	(146.4)
γ_{QQ}				0.01086
				(0.95)
γ_{QE}				-0.09404
				(-7.07)
γ_{EE}				0.09973
				(4.89)
φ_{KK}		-0.13597	-0.15322	-0.28064
		(-3.31)	(-3.75)	(-5.49)
δ_{QK}				0.06788
				(3.85)
δ_{EK}				-0.03545
				(-1.49)
δ_{QT}				-0.00430
				(-13.14)
δ_{ET}				0.00177
				(3.55)
φ_{KT}			0.00171	0.00162
			(5.19)	(2.17)
β_T			0.01109	0.01080
			(24.25)	(23.72)
φ_{TT}			0.00008	0.00010
			(1.93)	(2.46)
μ	0.00980			
	(60.46)			
μ_K		0.00182		
		(1.75)		
μ_L		0.01337		
		(27.75)		
R^2_p	213.63	224.09	226.70	469.78
ψ_{KL}	1.00	1.67	1.75	2.39

Table 2-A
Neutral, disembodied, factor-augmenting technological change in the two-input case: production-function setting

$$y_t = f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})$$

$\mu_L > 0, \mu_K = 0$ **Harrod-neutral** *(L-augmenting)*				$\mu_L = \mu_K (\equiv \mu) > 0$ **Hicks-neutral** *(L & K-augmenting)*	$\mu_L = 0, \mu_K > 0$ **Solow-neutral** *(K-augmenting)*			
$\hat{\tilde{w}}_L < 0, \hat{\tilde{w}}_K > 0$				$\hat{\tilde{w}}_L = \hat{\tilde{w}}_K = 0$	$\hat{\tilde{w}}_L > 0, \hat{\tilde{w}}_K < 0$			
$\psi_{LK} > 1$		$\psi_{LK} = 1$	$\psi_{LK} < 1$		$\psi_{LK} > 1$		$\psi_{LK} = 1$	$\psi_{LK} < 1$
$\psi_{LK} > 1/s_K$	$1 < \psi_{LK} < 1/s_K$				$\psi_{LK} > 1/s_L$	$1 < \psi_{LK} < 1/s_L$		
$\hat{w}_L < 0, \hat{w}_K > 0$ *L-penalizing* *K-rewarding*	$0 < \hat{w}_L < \hat{w}_K$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L > \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K = \mu > 0$ *L- & K-rewarding*	$\hat{w}_L > 0, \hat{w}_K < 0$ *L-rewarding* *K-penalizing*	$\hat{w}_L > \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K > 0$ *L- & K-rewarding*	$0 < \hat{w}_L < \hat{w}_K$ *L- & K-rewarding*
$\varepsilon_{LT} < 0, \varepsilon_{KT} > 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *ultra anti-labor, pro-capital biased*	$0 < \varepsilon_{LT} < \varepsilon_{KT}$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor, pro-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} > \varepsilon_{KT} > 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor, anti-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} = \mu > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} > 0, \varepsilon_{KT} < 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor, ultra anti-capital biased*	$\varepsilon_{LT} > \varepsilon_{KT} > 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor, anti-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$0 < \varepsilon_{LT} < \varepsilon_{KT}$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor, pro-capital biased*

Table 2-B

Neutral, disembodied, factor-augmenting technological change in the two-input case: cost-function setting

$$p_t = c(w_{L,t}e^{-\mu_L t}, w_{K,t}e^{-\mu_K t})$$

$\mu_L > 0, \mu_K = 0$ **Harrod-neutral** *(L-augmenting)*				$\mu_L = \mu_K (\equiv \mu) > 0$ **Hicks-neutral** *(L & K-augmenting)*	$\mu_L = 0, \mu_K > 0$ **Solow-neutral** *(K-augmenting)*			
$\hat{\hat{v}}_L > 0, \hat{\hat{v}}_K < 0$				$\hat{\hat{v}}_L = \hat{\hat{v}}_K = 0$	$\hat{\hat{v}}_L < 0, \hat{\hat{v}}_K > 0$			
$\sigma_{LK} > 1$		$\sigma_{LK} = 1$	$\sigma_{LK} < 1$		$\sigma_{LK} > 1$		$\sigma_{LK} = 1$	$\sigma_{LK} < 1$
$\sigma_{LK} > 1/s_K$	$1 < \sigma_{LK} < 1/s_K$				$\sigma_{LK} > 1/s_L$	$1 < \sigma_{LK} < 1/s_L$		
$\hat{v}_L > 0, \hat{v}_K < 0$ *L-using* *K-saving*	$0 > \hat{v}_L > \hat{v}_K$ *L- & K-* *saving*	$\hat{v}_L = \hat{v}_K < 0$ *L- & K-* *saving*	$\hat{v}_L < \hat{v}_K < 0$ *L- & K-* *saving*	$\hat{v}_L = \hat{v}_K = -\mu < 0$ *L- & K-* *saving*	$\hat{v}_L < 0, \hat{v}_K > 0$ *L-saving* *K-using*	$\hat{v}_L < \hat{v}_K < 0$ *L- & K-* *saving*	$\hat{v}_L = \hat{v}_K < 0$ *L- & K-* *saving*	$0 > \hat{v}_L > \hat{v}_K$ *L- & K-* *saving*
$\varepsilon_{LT} > 0, \varepsilon_{KT} < 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *ultra pro-labor,* *anti-capital* *biased*	$0 > \varepsilon_{LT} > \varepsilon_{KT}$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor,* *anti-capital* *biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} < \varepsilon_{KT} < 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor,* *pro-capital* *biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} < 0, \varepsilon_{KT} > 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor,* *ultra pro-capital* *biased*	$\varepsilon_{LT} < \varepsilon_{KT} < 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor,* *pro-capital* *biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$0 > \varepsilon_{LT} > \varepsilon_{KT}$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor,* *anti-capital* *biased*

Table 3

Decomposition of the average productivity of labor: 2-input Cobb-Douglas production function

	$AA_{t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$	$A_{U,t,t-1}$
1971	1.0325	1.0082	1.0099	1.0140
1972	1.0182	0.9999	1.0099	1.0083
1973	1.0186	0.9996	1.0099	1.0091
1974	0.9901	1.0071	1.0099	0.9736
1975	1.0281	1.0149	1.0099	1.0031
1976	1.0214	0.9976	1.0099	1.0138
1977	1.0121	0.9989	1.0099	1.0033
1978	1.0076	0.9965	1.0099	1.0013
1979	1.0020	1.0013	1.0099	0.9910
1980	1.0018	1.0087	1.0099	0.9835
1981	1.0199	1.0058	1.0099	1.0041
1982	1.0064	1.0127	1.0099	0.9841
1983	1.0251	1.0010	1.0099	1.0141
1984	1.0174	0.9933	1.0099	1.0143
1985	1.0107	1.0007	1.0099	1.0001
1986	1.0238	1.0051	1.0099	1.0087
1987	1.0025	0.9987	1.0099	0.9940
1988	1.0125	0.9991	1.0099	1.0036
1989	1.0055	0.9987	1.0099	0.9970
1990	1.0119	1.0047	1.0099	0.9973
1991	1.0156	1.0103	1.0099	0.9954
1992	1.0230	1.0027	1.0099	1.0103
1993	1.0009	0.9985	1.0099	0.9927
1994	1.0050	0.9962	1.0099	0.9990
1995	1.0014	0.9992	1.0099	0.9924
1996	1.0199	1.0028	1.0099	1.0072
1997	1.0112	0.9984	1.0099	1.0029
1998	1.0177	1.0014	1.0099	1.0064
1999	1.0144	1.0011	1.0099	1.0034
2000	1.0236	1.0047	1.0099	1.0089
2001	1.0154	1.0104	1.0099	0.9951
1971-2001	**1.0134**	**1.0025**	**1.0099**	**1.0010**

Table 4

Decomposition of the marginal productivity of labor: 2-input Cobb-Douglas production function

	$MM_{t,t-1}$	$M_{V,t,t-1}$	$M_{T,t,t-1}$	$M_{U,t,t-1}$
1971	1.0210	1.0082	1.0099	1.0028
1972	1.0126	0.9999	1.0099	1.0028
1973	1.0112	0.9996	1.0099	1.0018
1974	1.0108	1.0071	1.0099	0.9939
1975	1.0200	1.0149	1.0099	0.9952
1976	1.0193	0.9976	1.0099	1.0117
1977	1.0042	0.9989	1.0099	0.9955
1978	1.0029	0.9965	1.0099	0.9966
1979	1.0088	1.0013	1.0099	0.9977
1980	1.0203	1.0087	1.0099	1.0016
1981	1.0126	1.0058	1.0099	0.9969
1982	1.0185	1.0127	1.0099	0.9959
1983	1.0120	1.0010	1.0099	1.0011
1984	0.9936	0.9933	1.0099	0.9906
1985	1.0165	1.0007	1.0099	1.0058
1986	1.0405	1.0051	1.0099	1.0251
1987	0.9962	0.9987	1.0099	0.9878
1988	1.0011	0.9991	1.0099	0.9923
1989	1.0070	0.9987	1.0099	0.9985
1990	1.0182	1.0047	1.0099	1.0035
1991	1.0219	1.0103	1.0099	1.0016
1992	1.0278	1.0027	1.0099	1.0150
1993	0.9963	0.9985	1.0099	0.9881
1994	1.0001	0.9962	1.0099	0.9941
1995	0.9908	0.9992	1.0099	0.9819
1996	1.0095	1.0028	1.0099	0.9969
1997	1.0041	0.9984	1.0099	0.9958
1998	1.0261	1.0014	1.0099	1.0147
1999	1.0175	1.0011	1.0099	1.0065
2000	1.0323	1.0047	1.0099	1.0174
2001	1.0246	1.0104	1.0099	1.0042
1971-2001	**1.0128**	**1.0025**	**1.0099**	**1.0004**

Table 5

Decomposition of the average productivity of labor: 2-input Translog production function

	$AA_{t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$	$A_{U,t,t-1}$
1971	1.0325	1.0081	1.0088	1.0152
1972	1.0182	0.9999	1.0089	1.0093
1973	1.0186	0.9996	1.0090	1.0100
1974	0.9901	1.0070	1.0091	0.9744
1975	1.0281	1.0145	1.0092	1.0041
1976	1.0214	0.9977	1.0094	1.0142
1977	1.0121	0.9989	1.0094	1.0037
1978	1.0076	0.9966	1.0095	1.0016
1979	1.0020	1.0013	1.0096	0.9913
1980	1.0018	1.0087	1.0097	0.9837
1981	1.0199	1.0057	1.0098	1.0043
1982	1.0064	1.0123	1.0099	0.9843
1983	1.0251	1.0010	1.0100	1.0139
1984	1.0174	0.9934	1.0101	1.0139
1985	1.0107	1.0007	1.0102	0.9998
1986	1.0238	1.0051	1.0103	1.0083
1987	1.0025	0.9987	1.0104	0.9935
1988	1.0125	0.9991	1.0104	1.0030
1989	1.0055	0.9987	1.0105	0.9964
1990	1.0119	1.0047	1.0106	0.9966
1991	1.0156	1.0104	1.0107	0.9946
1992	1.0230	1.0027	1.0108	1.0094
1993	1.0009	0.9985	1.0109	0.9917
1994	1.0050	0.9962	1.0110	0.9979
1995	1.0014	0.9992	1.0110	0.9912
1996	1.0199	1.0028	1.0111	1.0059
1997	1.0112	0.9984	1.0112	1.0016
1998	1.0177	1.0014	1.0113	1.0049
1999	1.0144	1.0012	1.0114	1.0019
2000	1.0236	1.0049	1.0115	1.0071
2001	1.0154	1.0107	1.0116	0.9931
1971-2001	**1.0134**	**1.0025**	**1.0102**	**1.0006**

Table 6

Decomposition of the marginal productivity of labor: 2-input Translog production function

	$MM_{t,t-1}$	$M_{V,t,t-1}$	$M_{T,t,t-1}$	$M_{U,t,t-1}$
1971	1.0210	1.0144	1.0065	1.0000
1972	1.0126	0.9999	1.0066	1.0061
1973	1.0112	0.9993	1.0066	1.0053
1974	1.0108	1.0125	1.0067	0.9917
1975	1.0200	1.0260	1.0069	0.9874
1976	1.0193	0.9959	1.0070	1.0163
1977	1.0042	0.9981	1.0071	0.9991
1978	1.0029	0.9939	1.0071	1.0019
1979	1.0088	1.0023	1.0072	0.9993
1980	1.0203	1.0154	1.0073	0.9975
1981	1.0126	1.0102	1.0074	0.9950
1982	1.0185	1.0221	1.0076	0.9890
1983	1.0120	1.0017	1.0077	1.0026
1984	0.9936	0.9884	1.0077	0.9976
1985	1.0165	1.0013	1.0078	1.0073
1986	1.0405	1.0090	1.0079	1.0232
1987	0.9962	0.9977	1.0080	0.9906
1988	1.0011	0.9984	1.0080	0.9948
1989	1.0070	0.9977	1.0081	1.0012
1990	1.0182	1.0084	1.0082	1.0015
1991	1.0219	1.0184	1.0083	0.9952
1992	1.0278	1.0048	1.0084	1.0143
1993	0.9963	0.9973	1.0085	0.9906
1994	1.0001	0.9933	1.0086	0.9983
1995	0.9908	0.9986	1.0086	0.9837
1996	1.0095	1.0050	1.0087	0.9958
1997	1.0041	0.9972	1.0088	0.9982
1998	1.0261	1.0025	1.0089	1.0146
1999	1.0175	1.0020	1.0089	1.0065
2000	1.0323	1.0086	1.0090	1.0143
2001	1.0246	1.0188	1.0092	0.9965
1971-2001	**1.0128**	**1.0044**	**1.0078**	**1.0005**

Table 7

Decomposition of the average productivity of labor: 2-input Translog production function, index-number approach

	$AA_{t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$
1971	1.0325	1.0081	1.0243
1972	1.0182	0.9999	1.0183
1973	1.0186	0.9996	1.0191
1974	0.9901	1.0070	0.9833
1975	1.0281	1.0145	1.0135
1976	1.0214	0.9977	1.0238
1977	1.0121	0.9989	1.0132
1978	1.0076	0.9966	1.0112
1979	1.0020	1.0013	1.0007
1980	1.0018	1.0087	0.9934
1981	1.0199	1.0057	1.0142
1982	1.0064	1.0123	0.9943
1983	1.0251	1.0010	1.0241
1984	1.0174	0.9934	1.0243
1985	1.0107	1.0007	1.0099
1986	1.0238	1.0051	1.0186
1987	1.0025	0.9987	1.0038
1988	1.0125	0.9991	1.0135
1989	1.0055	0.9987	1.0069
1990	1.0119	1.0047	1.0071
1991	1.0156	1.0104	1.0053
1992	1.0230	1.0027	1.0203
1993	1.0009	0.9984	1.0024
1994	1.0050	0.9962	1.0088
1995	1.0014	0.9992	1.0021
1996	1.0199	1.0028	1.0170
1997	1.0112	0.9984	1.0129
1998	1.0177	1.0014	1.0162
1999	1.0144	1.0012	1.0132
2000	1.0236	1.0049	1.0187
2001	1.0153	1.0107	1.0048
1971-2001	**1.0134**	**1.0025**	**1.0109**

Table 8

Decomposition of the average productivity of labor: 2-input, 3-output Translog real domestic value added function

	$AA_{t,t-1}$	$A_{Q,t,t-1}$	$A_{E,t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$	$A_{U,t,t-1}$
1971	1.0310	0.9986	1.0000	1.0081	1.0098	1.0143
1972	1.0161	0.9979	1.0001	0.9999	1.0097	1.0084
1973	1.0164	0.9980	0.9997	0.9996	1.0097	1.0093
1974	0.9792	0.9892	0.9998	1.0071	1.0096	0.9737
1975	1.0298	1.0016	1.0000	1.0144	1.0097	1.0038
1976	1.0214	1.0001	1.0000	0.9978	1.0098	1.0136
1977	1.0083	0.9962	1.0001	0.9989	1.0098	1.0034
1978	1.0068	0.9991	1.0001	0.9966	1.0097	1.0014
1979	0.9976	0.9959	0.9999	1.0013	1.0097	0.9908
1980	0.9894	0.9882	1.0000	1.0089	1.0095	0.9831
1981	1.0219	1.0018	1.0002	1.0059	1.0094	1.0044
1982	1.0105	1.0038	1.0008	1.0124	1.0096	0.9841
1983	1.0295	1.0042	1.0004	1.0009	1.0099	1.0139
1984	1.0198	1.0018	1.0003	0.9935	1.0101	1.0141
1985	1.0128	1.0006	1.0007	1.0007	1.0101	1.0007
1986	1.0233	0.9985	1.0006	1.0050	1.0101	1.0089
1987	0.9995	0.9968	1.0001	0.9987	1.0101	0.9938
1988	1.0124	1.0004	0.9997	0.9991	1.0102	1.0031
1989	1.0052	0.9993	1.0003	0.9987	1.0103	0.9967
1990	1.0102	0.9978	1.0006	1.0048	1.0103	0.9968
1991	1.0178	1.0021	1.0004	1.0104	1.0104	0.9945
1992	1.0227	0.9995	1.0006	1.0027	1.0105	1.0093
1993	1.0020	1.0010	1.0005	0.9985	1.0106	0.9916
1994	1.0053	1.0003	1.0002	0.9962	1.0107	0.9981
1995	1.0009	0.9996	1.0000	0.9992	1.0107	0.9915
1996	1.0210	1.0007	1.0006	1.0028	1.0108	1.0059
1997	1.0142	1.0026	1.0006	0.9984	1.0109	1.0016
1998	1.0223	1.0042	1.0005	1.0014	1.0111	1.0050
1999	1.0137	0.9988	1.0004	1.0012	1.0112	1.0021
2000	1.0196	0.9960	1.0002	1.0049	1.0112	1.0072
2001	1.0195	1.0029	1.0006	1.0106	1.0113	0.9941
1971-2001	**1.0128**	**0.9993**	**1.0003**	**1.0025**	**1.0102**	**1.0006**

Table 9

Decomposition of the average productivity of labor: 2-input, 3-output Translog real domestic value added function, index-number approach

	$AA_{t,t-1}$	$A_{Q,t,t-1}$	$A_{E,t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$
1971	1.0310	0.9986	1.0000	1.0079	1.0229
1972	1.0161	0.9979	1.0001	0.9999	1.0162
1973	1.0164	0.9980	0.9998	0.9996	1.0168
1974	0.9792	0.9890	0.9999	1.0070	0.9724
1975	1.0298	1.0016	1.0000	1.0144	1.0151
1976	1.0214	1.0001	0.9999	0.9977	1.0238
1977	1.0083	0.9961	1.0002	0.9989	1.0094
1978	1.0068	0.9991	1.0001	0.9965	1.0104
1979	0.9976	0.9959	0.9997	1.0013	0.9963
1980	0.9894	0.9876	1.0000	1.0085	0.9810
1981	1.0219	1.0019	1.0001	1.0056	1.0163
1982	1.0105	1.0038	1.0003	1.0121	0.9984
1983	1.0295	1.0040	1.0003	1.0009	1.0286
1984	1.0197	1.0018	1.0005	0.9933	1.0267
1985	1.0128	1.0006	1.0015	1.0008	1.0121
1986	1.0233	0.9984	1.0011	1.0051	1.0181
1987	0.9995	0.9967	1.0002	0.9987	1.0007
1988	1.0124	1.0004	0.9995	0.9991	1.0134
1989	1.0052	0.9993	1.0003	0.9987	1.0065
1990	1.0102	0.9979	1.0004	1.0047	1.0054
1991	1.0178	1.0020	1.0002	1.0103	1.0074
1992	1.0227	0.9995	1.0001	1.0027	1.0200
1993	1.0020	1.0010	1.0002	0.9985	1.0036
1994	1.0053	1.0002	1.0001	0.9962	1.0091
1995	1.0009	0.9996	1.0000	0.9992	1.0017
1996	1.0210	1.0007	1.0004	1.0029	1.0180
1997	1.0142	1.0026	1.0004	0.9983	1.0159
1998	1.0223	1.0041	1.0004	1.0014	1.0208
1999	1.0137	0.9988	1.0005	1.0012	1.0126
2000	1.0196	0.9957	1.0003	1.0049	1.0147
2001	1.0195	1.0032	1.0010	1.0105	1.0090
1971-2001	**1.0128**	**0.9992**	**1.0002**	**1.0025**	**1.0104**

Table 10

Decomposition of the changes in the share of labor: 2-input, 3-output Translog real domestic value added function

	$SS_{t,t-1}$	$S_{Q,t,t-1}$	$S_{E,t,t-1}$	$S_{V,t,t-1}$	$S_{T,t,t-1}$	$S_{U,t,t-1}$
1971	0.9889	0.9976	0.9991	1.0115	0.9978	0.9831
1972	0.9945	0.9965	0.9994	0.9999	0.9978	1.0008
1973	0.9927	0.9970	1.0036	0.9994	0.9978	0.9949
1974	1.0209	0.9864	1.0056	1.0099	0.9978	1.0213
1975	0.9921	1.0018	1.0006	1.0207	0.9978	0.9719
1976	0.9979	1.0001	0.9988	0.9967	0.9978	1.0044
1977	0.9922	0.9958	0.9987	0.9984	0.9978	1.0014
1978	0.9953	0.9991	0.9995	0.9951	0.9978	1.0039
1979	1.0067	0.9959	1.0015	1.0018	0.9978	1.0098
1980	1.0184	0.9885	0.9998	1.0123	0.9977	1.0202
1981	0.9928	1.0017	0.9992	1.0082	0.9978	0.9861
1982	1.0121	1.0036	0.9975	1.0177	0.9978	0.9956
1983	0.9872	1.0040	0.9986	1.0014	0.9978	0.9855
1984	0.9766	1.0017	0.9988	0.9907	0.9978	0.9875
1985	1.0057	1.0006	0.9973	1.0010	0.9978	1.0091
1986	1.0163	0.9985	0.9982	1.0071	0.9978	1.0147
1987	0.9937	0.9971	0.9997	0.9982	0.9978	1.0010
1988	0.9888	1.0003	1.0009	0.9987	0.9978	0.9911
1989	1.0014	0.9994	0.9990	0.9981	0.9977	1.0072
1990	1.0062	0.9982	0.9984	1.0066	0.9977	1.0053
1991	1.0062	1.0017	0.9990	1.0145	0.9978	0.9932
1992	1.0046	0.9996	0.9987	1.0038	0.9978	1.0048
1993	0.9953	1.0008	0.9989	0.9978	0.9978	1.0000
1994	0.9951	1.0002	0.9996	0.9947	0.9978	1.0029
1995	0.9894	0.9996	1.0001	0.9989	0.9977	0.9930
1996	0.9898	1.0005	0.9985	1.0039	0.9977	0.9892
1997	0.9930	1.0020	0.9984	0.9978	0.9977	0.9971
1998	1.0083	1.0031	0.9985	1.0019	0.9977	1.0070
1999	1.0031	0.9991	0.9988	1.0016	0.9977	1.0058
2000	1.0085	0.9971	0.9995	1.0067	0.9977	1.0075
2001	1.0091	1.0021	0.9987	1.0147	0.9977	0.9960
1971-2001	**0.9994**	**0.9990**	**0.9994**	**1.0035**	**0.9978**	**0.9997**

Table 11

Decomposition of the marginal productivity of labor: 2-input, 3-output Translog real domestic value added function

	$MM_{t,t-1}$	$M_{Q,t,t-1}$	$M_{E,t,t-1}$	$M_{V,t,t-1}$	$M_{T,t,t-1}$	$M_{U,t,t-1}$
1971	1.0196	0.9962	0.9992	1.0196	1.0075	0.9971
1972	1.0105	0.9945	0.9995	0.9998	1.0075	1.0092
1973	1.0090	0.9951	1.0033	0.9990	1.0075	1.0042
1974	0.9996	0.9758	1.0055	1.0171	1.0074	0.9945
1975	1.0217	1.0034	1.0006	1.0354	1.0074	0.9756
1976	1.0193	1.0003	0.9988	0.9945	1.0076	1.0181
1977	1.0005	0.9920	0.9988	0.9974	1.0075	1.0049
1978	1.0021	0.9982	0.9995	0.9917	1.0075	1.0053
1979	1.0043	0.9919	1.0014	1.0031	1.0074	1.0006
1980	1.0076	0.9768	0.9999	1.0213	1.0072	1.0030
1981	1.0146	1.0036	0.9994	1.0141	1.0071	0.9904
1982	1.0226	1.0074	0.9982	1.0303	1.0074	0.9798
1983	1.0164	1.0082	0.9990	1.0023	1.0077	0.9992
1984	0.9959	1.0035	0.9991	0.9843	1.0078	1.0013
1985	1.0186	1.0012	0.9980	1.0018	1.0079	1.0098
1986	1.0400	0.9970	0.9988	1.0122	1.0079	1.0238
1987	0.9931	0.9939	0.9998	0.9969	1.0079	0.9947
1988	1.0011	1.0007	1.0006	0.9977	1.0079	0.9942
1989	1.0066	0.9987	0.9994	0.9968	1.0080	1.0038
1990	1.0165	0.9960	0.9990	1.0115	1.0080	1.0021
1991	1.0241	1.0038	0.9994	1.0251	1.0081	0.9878
1992	1.0274	0.9991	0.9992	1.0065	1.0082	1.0141
1993	0.9974	1.0019	0.9994	0.9963	1.0083	0.9916
1994	1.0004	1.0005	0.9997	0.9908	1.0084	1.0011
1995	0.9903	0.9992	1.0000	0.9981	1.0084	0.9846
1996	1.0105	1.0012	0.9990	1.0068	1.0085	0.9950
1997	1.0070	1.0046	0.9990	0.9961	1.0086	0.9987
1998	1.0308	1.0073	0.9990	1.0033	1.0088	1.0120
1999	1.0169	0.9980	0.9993	1.0027	1.0089	1.0079
2000	1.0282	0.9932	0.9997	1.0117	1.0089	1.0147
2001	1.0288	1.0050	0.9992	1.0254	1.0090	0.9901
1971-2001	**1.0122**	**0.9983**	**0.9997**	**1.0060**	**1.0079**	**1.0002**