SCCM ADVICE: STANFORD UNIVERSITY'S CONSULTING GROUP FOR APPLIED MATH AND NUMERICAL ANALYSIS

MELISSA ACZON MARTIN GANDER MARGOT GERRITSEN
TONY SHARDLOW RONNIE SIRCAR

1. Advice and SCCM

SCCM Advice is a free consulting service run by the Program in Scientific Computing and Computational Mathematics (SCCM) at Stanford University. Advice was started in the fall of 1995 by Margot Gerritsen, a senior graduate student. She wanted to provide a source of information on numerical analysis and applied mathematics for the Stanford community. Advice is now led by four senior students (listed above) and has consulted on the whole range of scientific computing and applied mathematics. In this article, we describe the format and achievements of Advice, give example problems, and provide advice, as is our tradition, for those interested in developing their own consulting group.

2. Goals

SCCM Advice has three goals. Primarily, we want to help scientists and engineers by sharing our expertise in numerical analysis and mathematical modeling. We have helped with many problems, both by referring to other research groups and by finding our own solutions. We will give many examples later in this article.

A second important goal is training the students. Working as a consultant introduces the students of SCCM to a broad range of applications. SCCM is a multi-disciplinary program, where the students have Ph.D advisors in many different departments. Working for Advice gives students an opportunity to learn about research in other departments and even meet potential advisors. Consulting, moreover, develops the ability to explain concepts and communicate with researchers in different fields: how to get abreast of a problem in a short interview with a customer and collect all the information necessary for discussions with peers and faculty.

Finally, we wanted to develop connections with industry, in order to support the SCCM program. When Advice was first initiated, we made an effort to contact the many scientific and computer based firms of the Bay area. So far, this enterprise has been unsuccessful. We are only just beginning to find opportunities for Advice to consult on real world problems. Such contacts

are going to change the nature of Advice. At present, Advice is very informal and we deal with mostly small problems. Consulting for industry involves larger problems and will demand a larger time commitment.

3. Format

Advice is currently organized by four senior graduate students, with one student *primus inter pares* each quarter. These students are all former consultants and provide the service with continuity. This student leadership distinguishes Advice from other services; faculty are consulted as problems arise in their areas of expertise, but the day to day running is undertaken by students.

At the start of each quarter, the organizers find three consultants, arrange advertising, and organize office hours. The consultants are taken from students, visitors, and postdocs of the SCCM program. Because of its multidisciplinary nature, SCCM has a varied set of interests, which allows Advice to have consultants with different areas of expertise. To encourage students to participate, Advice may be taken for credit and counts as a full-fledged course.

The consultants are responsible for holding office hours for customers, following up questions, and writing up their problems in the Advice files. We feel that recording the progress of each problem in a file is a good way to keep track of our achievements and to make consultants work for their credit. Furthermore, the Advice files offer new students a good overview of research in other departments.

We aim for a one consultant one customer relationship. In some cases, it pays to switch to a consultant with some relevant experience, but for the most part this relationship ensures that each consultant deals with all stages of the consulting process and makes Advice more customer friendly.

Each of the three consultants holds two office hours per week, typically dealing with one or two problems each week. Most problems consist of a short interview between consultant and customer, after which the problem is discussed by the whole team, followed if necessary by a meeting with faculty. The main purpose of the first meeting, and the hardest part of consulting, is to find out what the customer really wants. Often the customers present their problems in an unfamiliar language or with the wrong perspective. We illustrate this in the Worked Example (§5).

Faculty meetings occur about twice every quarter. In the last year, these meetings have been driven by the enthusiasm of SCCM faculty members Joe Keller and George Papanicolaou.

4. Examples

A wide range of problems come to Advice. When we first started, there were many skeptics; some thought that undergraduate homeworks and whole thesis topics would be the only problems. The skeptics were proved wrong;

admittedly there are questions of this nature, but the range of problems is much fuller. We describe some typical problems and how we dealt with them.

There are many simple questions: how do I use Matlab; where can I find software packages; I've never solved a PDE numerically before; please find my bugs. Such questions are dealt with quickly. References are given to the literature, including links, often through our web page, to reputable software packages. One common problem stems from the use of explicit methods: customers come complaining of bad behaviour in their ODE or PDE solvers; we make such customers aware of the stability problem and guide them to implicit methods.

A number of problems are solved by contacting groups within Stanford. One customer from Geology brought a finite element mesh generator to Advice. He wanted help in compiling the Lisp source code. On this point we failed, but we did succeed in introducing him to a group in Electrical Engineering who had developed a different mesh generator, one likely to be more efficient than a Lisp implementation. In another case, we introduced a customer from Statistics with a large nonlinear optimization problem to a professor in Operations Research. Using Advice as a go-between in such situations offers the consultants experience with problems outside their realm of research. We are involved in such meetings, rather than providing a simple referral.

Another brand of problem, broad questions concerning an entire research project, are harder. One customer wanted help in developing equations to model the movement of sediments in a fluid in several dimensions, incorporating both sediments of various sizes and turbulence in the underlying fluid. Naturally, such broad questions, on the scale of NSF Grand Challenge projects, are out of the scope of Advice; but here conversations succeeded in putting this project into perspective, which in this case meant the fundamental difficulty in numerically solving an elaborate model.

We have dealt with many small scale problems that have nice answers. For example, one customer wanted to understand the breaking of seaweed under hydrodynamic forces in the ocean. We helped this customer understand the pros and cons of ODE and PDE models. In his case, a PDE involved writing a large code and collecting empirical data; but his original ODE model could not incorporate all the stress–strain characteristics of the seaweed. Another customer, concerned with storing data in holograms, wanted analytical solutions. We found and corrected mistakes in his model: originally he had looked for solutions in terms of the electrostatic field, which gave difficult equations; we found a solvable PDE by working with the free carrier density.

5. Worked Example

The Biomechanics group in Mechanical Engineering is concerned with tracking the movements of the knee joint using sensors attached to various

parts of the leg. This group came to Advice wanting the rotation and translation matrices that describe the motion of the knee over a certain time interval. With the help of faculty member Gene Golub, Advice found a good solution: the problem can be reduced to the *orthogonal Procrustes problem*. If $B, A \in \mathbb{R}^{n\times 3}$ are the matrices of co-ordinates of the sensors before and after a particular time interval, then the problem is to find the orthogonal (rotation) matrix $Q \in \mathbb{R}^{3\times 3}$ that "best" rotates B into A. The problem has a neat solution if "best" is measured with the Frobenius norm $\|\cdot\|_F$ and we look for a solution to

$$\min_Q \|A - BQ\|_F, \text{ subject to } Q^TQ = I_3.$$

The minimizer is $Q = UV^T$, where $U\Sigma V^T$ is the Singular Value Decomposition of B^TA. The calculation in Golub & Van Loan must be modified to incorporate translations of the co-ordinate frame. The problem becomes

$$\min_{Q,d} \|B - AQ + ed^T\|_F, \text{ subject to } Q^TQ = I_3,$$

where $e = [1, \ldots, 1]^T \in \mathbb{R}^n$, and d is the translation vector in $\mathbb{R}^3$. It reduces to the pure rotation problem by replacing B with $(I_n - \frac{1}{n}ee^T)B$, and replacing A with $(I_n - \frac{1}{n}ee^T)A$. The translation vector $d = -\frac{1}{n}[B^Te - Q^TA^Te]$.

This problem illustrates a common feature of consulting: often customers are focusing on details and cannot see the "forest through the trees". In this case, the customer asked the wrong question. The knee problem can be solved by selecting three sensors, and computing rotation/translation matrices from only those readings. This customer posed her question as how to select three good sensors out of the ones available. However, the question of which sensors to choose does not arise if the problem is formulated as a minimization problem involving all sensors.

6. Pitfalls

Advice is not without its pitfalls. Many questions should be turned away. For example, detailed questions about statistics are out of scope, and are referred to a consulting service of the Statistics department. Similarly, detailed computer questions are turned away: we cannot compile Lisp source codes and will not write PDE solvers.

Many customers have high expectations. Often customers look for a short cut to coding and come with hopes of the "Black Box", the simple easy to use package lurking on someone's web page, ready to solve their exact problem. However, any interesting problem in PDEs quickly escapes the capabilities of existing packages. We suggest algorithms and related work, but customers unwilling to code our advice will make little headway.

Because of the demand for help with coding, the SCCM program has developed a short course in the programming language Fortran 77. This course is supported by the Mechanical Engineering department and now taught annually by students from the SCCM program.

7. Conclusion

Advice has succeeded in helping many people around the Stanford community, as well as giving the students of SCCM an opportunity to interact with a wide applied math community. We believe such a program could benefit many universities, and encourage Applied Math programs to consider the benefit of consulting for their students. We are happy to supply you with more information, and answer any questions you might have; in short, we are here to give advice on Advice. Please do not hesitate to contact us.

Advice, SCCM Program, Gates Building 2B, M/C 9025, Stanford University, Stanford CA 94305

E-mail address: advice@sccm.stanford.edu, www-sccm.stanford.edu/∼Advice