Solution of Large Transmission Line Type Circuits Using a New Optimized Waveform Relaxation Partitioning

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Abstract

Larger problems can be solved with the ever increasing availability of computing power. However, our ability to solve large circuit and transmission line problems, in the time domain, is still relatively modest. Partitioning into a set of smaller problems is essential for the electrical modeling of very large systems. However, so far the partitioning of strongly coupled circuits like transmission lines has been difficult. We introduce a new approach for coupling partitioned problems in the context of waveform relaxation methods, which leads to the class of optimized waveform relaxation algorithms with greatly enhanced performance. We analyze convergence for the case of a transmission line type circuit and illustrate the method with several quasi TEM mode transmission line problems.

1 Introduction

Presently, the performance of electronic systems is increasing rapidly as the frequencies and cycle times are getting higher and as the rise time of the signals is decreasing. This leads to numerous new challenges for the Electrical Interconnect and Package (EIP) and the EMC electromagnetic modeling of realistic structures. Probably the most challenging aspect, which results from the faster signal rise times and higher frequencies, is the increase in the system or circuit size. It is clear that conventional solution methods will slowly make progress toward larger systems mostly due to the increase in available computing power. However, since the size of a system is limited, it is very important to be able to partition large problems into smaller ones without loss of accuracy.

In general, time domain methods have a great advantage over spectral methods from a partitioning point of view since a natural decoupling occurs due to the spatial time retardation. Unfortunately, coupling extends over the entire solution space in the frequency domain. From an EM solver point of view, we see a gradual shift of the environment and interfaces from purely electromagnetic toward Spice circuit solver interfaces which process circuits as well as the electromagnetic parts. Hence, the solver approaches must be able to handle EM parts as well as Spice type circuits.

Partitioning is essential for the electrical modeling of very large systems. In this paper we introduce a new approach for the coupling of partitioned systems. We illustrate the method with a quasi TEM mode transmission line model problem and then show numerical experiments for several larger test cases. The underlying technique, which makes use of the time domain behavior and its decoupling aspects is called waveform relaxation (WR). It has its origin in the Spice type circuit solver area [1] and it has been applied to many different problems [2]. In this approach, large circuits are subdivided into sub-circuits which are then solved individually, usually for a window in time while the time axis is divided into multiple windows. A relaxation procedure is then used to iterate among the sub-circuits until convergence is achieved. A summary of the application of the classical WR to circuit problems is given in [3]. The new optimized WR approach was first applied to partial differential equation problems in [4]. We recognized that the new optimized WR method is very much suitable for all techniques which can be formulated in the circuit domain, see for example [5] for RC type circuits. One of the well known aspects of the conventional, *classical* WR is that the partitioning of the models has to be done very carefully to avoid an excessive number of WR iterations. This will also be evident from the work in this paper. It is very desirable to keep the number of WR iterations low, even for the largest problems to be solved. Usually, the relative error in time domain EM and Spice type circuit solvers is 10^{-2} to 10^{-3}. We want to note that even the classical WR approach is well suited for parallel computations since it fits into the class of *highly parallelizable* methods. This has been substantiated by partitioning a very large 180k transistor circuit into about 10^4 sub-circuits or subsystems. An average speed up of about 180 was achieved on a system with 265 processors [6]. Hence, the suitability for parallel processing of the new WR algorithm is an aspect which we strongly emphasize.

In this paper we show how the new *optimized* WR technique can be applied to the partitioning of quasi TEM mode transmission line models. We derive the new algorithm for a small model problem in Section 2. In Section 3 we then show that the technique derived for the small model problem also greatly enhances the performance of the algorithm when it is applied to large transmission line circuits also with variable elements. We consider here the improvement for the new *optimized* WR approach over the classical WR for the efficient solution of partitioning transmission lines.

2 The Transmission Line Model

First, the small quasi TEM mode transmission line example given in Figure 1 is analyzed with the new approach. To show the basic idea of the WR partitioning, we divide the example into two sub-circuits which is equivalent to two sections of a lumped quasi TM transmission line model. This corresponds to breaking the line into two parts. We analyze the model using the classical as well as optimized WR algorithm to compare the convergence behavior for the small model lumped cir-

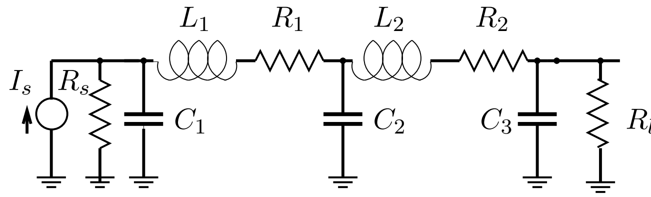

Figure 1: Two sections of a quasi TEM mode lumped circuit.

cuit. The circuit equations are given in the form of (2.1) which is convenient for the convergence analysis,

$$\dot{\boldsymbol{x}} = \begin{bmatrix} b_1 & c_1 & & & \\ a_1 & b_2 & c_2 & & \\ & a_2 & b_3 & c_3 & \\ & & a_3 & b_4 & c_4 \\ & & & a_4 & b_5 \end{bmatrix} \boldsymbol{x} + \boldsymbol{f}. \tag{2.1}$$

The vector of unknown waveforms $\boldsymbol{x} = (v_1, i_1, v_2, i_2, v_3)^T$ which consists of nodal voltages and inductance currents is the same we use in the Modified Nodal Analysis (MNA) technique. The elements of the matrix are given by

$$a_i = \frac{1}{L_{(i+1)/2}},\ (i \text{ odd}), \quad a_i = \frac{1}{C_{(i+1)/2}},\ (i \text{ even}),$$

$$b_1 = \frac{-1}{R_s\,C_1}, \quad b_i = 0,\ (i > 1,\ \text{odd}),$$

$$b_i = -\frac{R_{i/2}}{L_{i/2}}, (i \text{ even}), \quad b_5 = \frac{-1}{R_L\,C_3},$$

$$c_i = \frac{-1}{C_{(i+1)/2}},\ (i \text{ odd}), \quad c_i = \frac{-1}{L_{i/2}},\ (i \text{ even}),$$

and the source on the right hand side is given by $\boldsymbol{f}(t) = (I_s/C_1\,, 0\,, 0\,, 0\,, 0)^T$. We also need the initial values $\boldsymbol{x}(0) = (v_1^0, i_1^0, v_2^0, i_2^0, v_3^0)^T$ to start the transient analysis.

2.1 The Classical WR Algorithm

We partition the circuit into two sub-circuits or subsystems and we call the unknown values in subsystem 1 $\boldsymbol{u}(t)$ while the unknowns in subsystem 2 are called $\boldsymbol{w}(t)$. Then the classical WR algorithm applied to (2.1) with two sub-circuits is given by

$$\begin{aligned} \dot{\boldsymbol{u}}^{k+1} &= \begin{bmatrix} b_1 & c_1 & \\ a_1 & b_2 & c_2 \\ & a_2 & b_3 \end{bmatrix} \boldsymbol{u}^{k+1} + \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_3 w_1^k \end{pmatrix}, \\ \dot{\boldsymbol{w}}^{k+1} &= \begin{bmatrix} b_4 & c_4 \\ a_4 & b_5 \end{bmatrix} \boldsymbol{w}^{k+1} + \begin{pmatrix} f_4 \\ f_5 \end{pmatrix} + \begin{pmatrix} a_3 u_3^k \\ 0 \end{pmatrix}, \end{aligned} \tag{2.2}$$

with corresponding initial conditions $\boldsymbol{u}^{k+1}(0) = (v_1^0, i_1^0, v_2^0)^T$ and $\boldsymbol{w}^{k+1}(0) = (i_2^0, v_3^0)^T$. To start the WR iteration, we need to specify five initial waveforms $\boldsymbol{u}^0(t) = (u_1^0(t), u_2^0(t), u_3^0(t))^T$ and $\boldsymbol{w}^0(t) = (w_1^0(t), w_2^0(t))^T$ for $t \in [0, T]$ where T is the end of the transient analysis interval. It is well known that the classical WR algorithm does not work well for transmission line problems, convergence is very slow and initially the algorithm often seems to diverge, as we will also see in the numerical experiments in Section 3. We introduce in the next Subsection a remedy for this problem.

2.2 The Optimized WR Algorithm

The key improvements in our new WR algorithm are better transmission conditions than the ones used for the classical WR algorithm. It is evident that the WR convergence will be slow if the information exchange at the interface between two subsystems is ineffective. We first emphasize that the classical transmission conditions employed in (2.2) can be written explicitly as

$$u_4^{k+1} = w_1^k, \quad w_0^{k+1} = u_3^k. \tag{2.3}$$

From this we see that in the first sub-circuit the current u_4 is directly replaced in (2.2) by a current source, whereas in the second sub-circuit the voltage w_0 is directly replaced by a voltage source. Hence sub-circuit 1 passes only voltage information to sub-circuit 2, while sub-circuit 2 only passes current information to sub-circuit 1. At convergence we obtain with the classical transmission conditions

$$u_4^\infty = w_1^\infty, \quad w_0^\infty = u_3^\infty. \tag{2.4}$$

Under these conditions, the nodes at the subsystem boundaries assume the converged current and voltage respectively, as expected.

For the optimized WR algorithm we propose transmission conditions which exchange both current and voltage information in both directions,

$$\begin{aligned} u_4^{k+1} + \alpha u_3^{k+1} &= w_1^k + \alpha w_0^k, \\ w_1^{k+1} + \beta w_0^{k+1} &= u_4^k + \beta u_3^k. \end{aligned} \tag{2.5}$$

As can be ascertained by comparing (2.3) with the new transmission conditions (2.5) we now exchange a combination of voltage and current in both directions, and we introduced weighting factors α and β which can be used to optimize the performance of the new waveform relaxation algorithm. Note that the new transmission conditions lead to the correct solution of the underlying TEM circuit equations if the new WR algorithm converges and if $\alpha \neq \beta$, because then at convergence we have from (2.5)

$$\begin{aligned} (u_4^\infty - w_1^\infty) \; + \; \alpha \; (u_3^\infty - w_0^\infty) \; &= \; 0, \\ (u_4^\infty - w_1^\infty) \; + \; \beta \; (u_3^\infty - w_0^\infty) \; &= \; 0 \end{aligned}$$

and the determinant of this system is different from zero, if $\alpha \neq \beta$. Hence the old transmission conditions (2.3) are implied by the new ones.

For the optimized WR algorithm, the equivalent to the classical WR algorithm (2.2) is

$$\begin{aligned} \dot{\boldsymbol{u}}^{k+1} &= \begin{bmatrix} b_1 & c_1 & \\ a_1 & b_2 & c_2 \\ & a_2 & b_3 - c_3\alpha \end{bmatrix} \boldsymbol{u}^{k+1} + \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_3(w_1^k + \alpha w_0^k) \end{pmatrix}, \\ \dot{\boldsymbol{w}}^{k+1} &= \begin{bmatrix} b_4 - \frac{a_3}{\beta} & c_4 \\ a_4 & b_5 \end{bmatrix} \boldsymbol{w}^{k+1} + \begin{pmatrix} f_3 \\ f_4 \end{pmatrix} + \begin{pmatrix} \frac{a_3}{\beta}(\beta u_3^k + u_4^k). \\ 0 \end{pmatrix}, \end{aligned} \tag{2.6}$$

where the values u_4^k and w_0^k are determined by the transmission conditions (2.5). Hence, the parameters α and β will be part of the WR equations. The key question is how to choose α and β to obtain a WR algorithm with excellent performance for transmission line problems. We have developed a theory which allows us to determine the optimal parameters, but this theory is beyond the scope of this short paper. It leads to a small optimization problem to be solved for the optimal parameters. With these parameters, the algorithm becomes what we call an optimized waveform relaxation algorithm.

3 Numerical Experiments

We first perform numerical experiments on the small model transmission line given in Figure 1. We use the circuit parameters $C_i = 0.63$ pF, for $i = 1, 2, 3$, $R_i = 0.5e - 3$ kOhms, $i = 1, 2$, $L_i = 4.95e - 3\mu$H, $i = 1, 2$ with $R_s = 0.05$ kOhms and $R_l = 0.05$ kOhms. We use the source $I_s = 10t$ for $0 < t < 0.1$ and $I_s = 1$ mA for $t \geq 0.1$ and the analysis time interval is $[0, T]$ with $T = 1$ ns. To illustrate the main difference between the classical and optimized WR algorithm we first show iterations 2, 4 and 6 for the classical WR and the optimized WR in Figure 2. The much faster and more

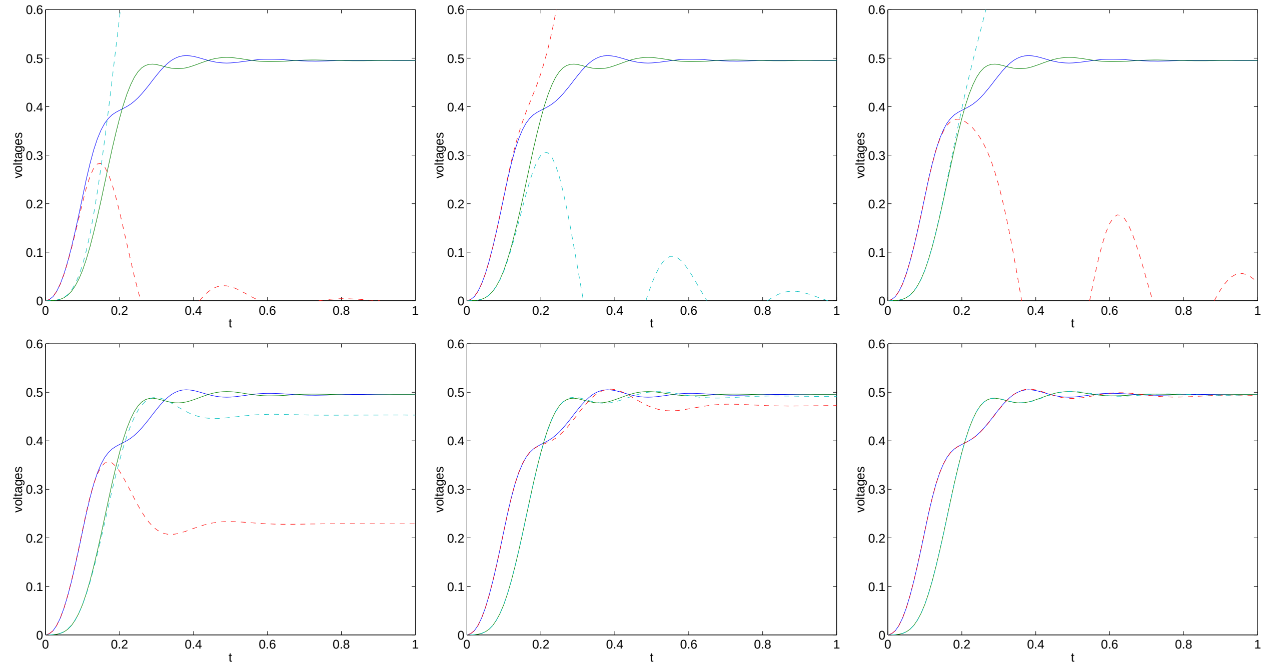

Figure 2: Exact Voltages (solid lines), WR solutions (dashed lines). Top three figures: classical WR iterations 2, 4 and 6. Bottom three figures: optimized WR iterations 2, 4 and 6.

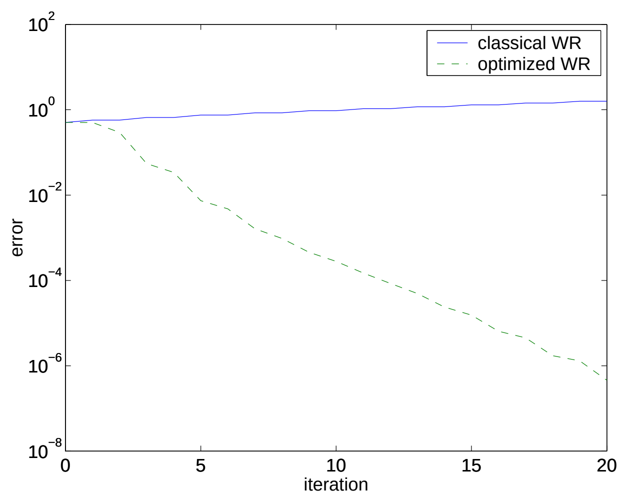

Figure 3: Convergence rates of the classical versus the optimized waveform relaxation algorithms for a small transmission line circuit.

uniform convergence of the optimized WR algorithm is evident from this computer experiment. The optimal parameters we used were $\alpha = -5.955$ and $\beta = 14.411$. In Figure 3 we show the error as a function of the WR iterations. The remarkable improvement of the optimized WR algorithm over the classical one is evident from this comparison where the classical WR is shown not to converge in less than 20 iterations in the 1 ns time window, while the optimized WR decreases the error to 10^{-6} in the same number of iterations.

Next, we show the analysis of larger circuits. The first experiment is to show that the technique developed for the small model circuit also works for large transmission lines. We chose a transmission line which consists of one hundred elements with the same topology as the small circuit in Figure 1. We partition the circuit into two sub-circuits of equal size and compare again the classical waveform relaxation algorithm to the new, optimized one, which uses the transmission conditions (2.5). We use the same circuit parameters and the same source term as for the small circuit, but for a longer transient analysis time interval $[0, T]$, $T = 5$ ns, to give the signal time to travel along the line. The performance of the classical and optimized waveform relaxation algorithm on this larger circuit is shown in Figure 4. Again the optimized WR algorithm converges very rapidly, while the classical WR does not converge in 20 iterations on the 5 ns time window. It is important to note that we have used here the same optimization parameter values $\alpha = -5.955$ and $\beta = 14.411$ that were found for the small circuit, the optimization is a local one and the resulting

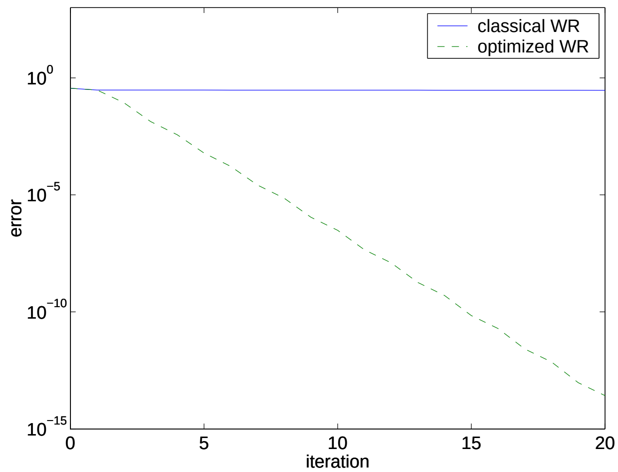

Figure 4: Convergence rates of the classical versus the optimized waveform relaxation algorithms for the large transmission line circuit.

parameters work equally well on the larger circuit.

In the last sequence of numerical experiments we show that the optimized waveform relaxation algorithm also works well if the transmission line components are not uniform, which shows its robustness. We chose a transmission line with eight elements similar to the two used to build the small circuit in Figure 1, partition the circuit again into two circuits of equal size and compare the classical waveform relaxation algorithm to the new, optimized one, which uses the transmission conditions (2.5). As a reference, we first use the same constant circuit parameters as in the earlier experiments and integrate on the time window $[0, T]$ with $T = 2$ ns, which leads to the convergence behavior of the classical and optimized WR shown in Figure 5. We now repeat the same experiment for a circuit with moderately varying parameters (up to 50 %), as shown graphically in Figure 6 on top. Again we use the same values for the optimization parameters α and β obtained for the small model transmission line circuit with constant circuit elements. As one can see from the convergence behavior in Figure 6 at the bottom, the classical WR is very much affected by the varying circuit elements, it diverges even more rapidly on the first 2 ns time window than in the constant element case shown in Figure 5, whereas the optimized WR converges to the same tolerance as before for the constant element reference case. We finally performed an experiment for strongly vary-

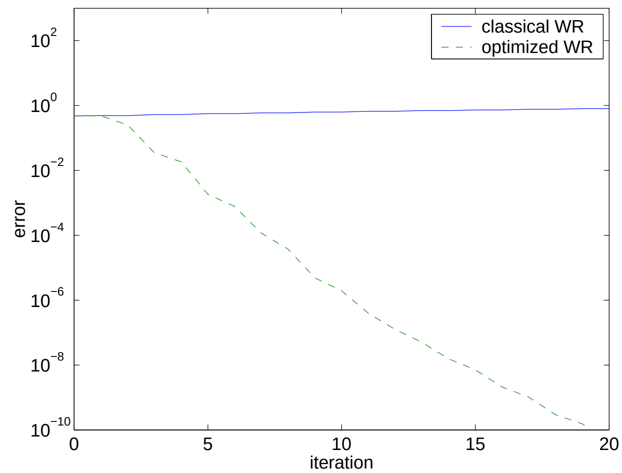

Figure 5: Reference performance of the classical and optimized waveform relaxation algorithm for a medium size transmission line circuit with constant circuit elements.

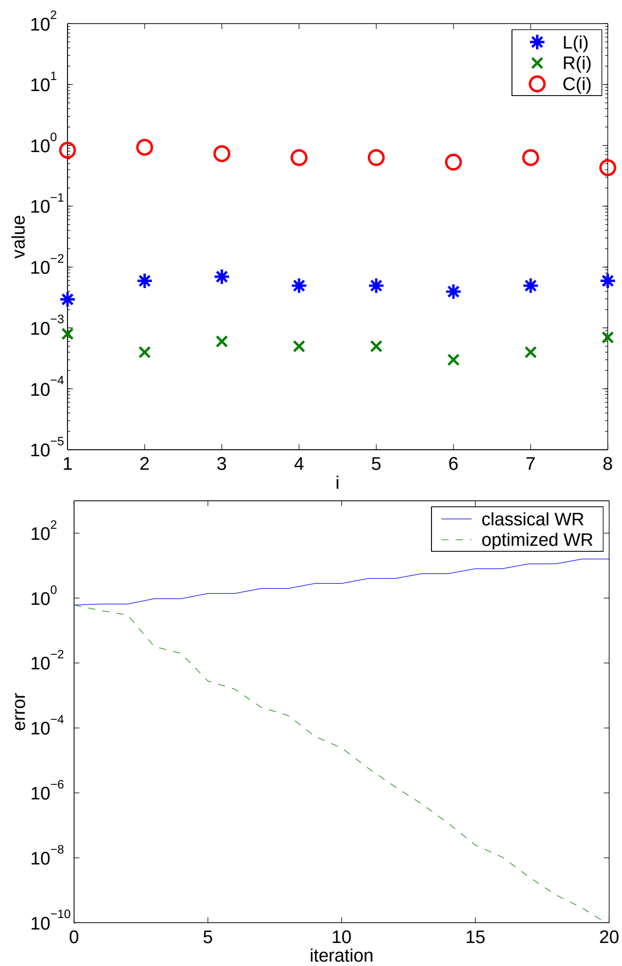

Figure 6: Moderate variation in the circuit parameters for a medium size TL shown graphically on the top, and the performance of classical and optimized WR for this problem below.

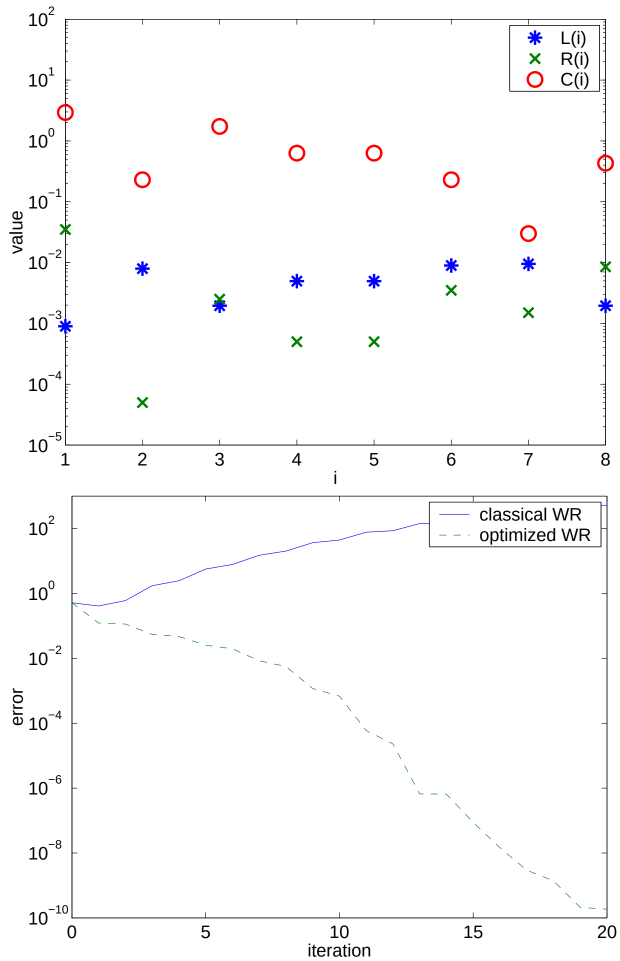

Figure 7: Large variation in the circuit parameters for a medium size TL shown graphically on top, and performance of the classical and optimized WR for this problem below.

ing parameters with different orders of magnitude, as shown graphically in Figure 7 on top. Again we use the same optimization parameters α and β found for the small model circuit with constant elements in the optimized WR. As is evident from Figure 7 at the bottom, the optimized waveform relaxation algorithm converges still to the same level of accuracy also when the circuit parameters are highly varying, while the classical WR is now rapidly diverging on the first 2 ns time window.

4 Conclusions

We have introduced an optimized WR algorithm for the partitioned time domain analysis of transmission line problems. The approach uses new transmission conditions which are adapted to the circuit and exchange both current and voltage information. We have shown numerical experiments that illustrate the large improvement in the convergence behavior of the new algorithm over classical WR. In fact, it would not be practical to use the classical WR for the important strongly connected structure of a transmission line. We also showed that the convergence rate of the optimized WR is rather insensitive for a nonuniform transmission line.

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