Waveform Relaxation Technique for Longitudinal Partitioning of Transmission Lines

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Abstract

Recently, the classical Waveform Relaxation (WR) approach was shown to be quite effective in solving the excessive run-time problem for multiple coupled transmission lines by a so-called transverse partitioning technique. The WR approach yields a remarkable linear increase in run time with respect to the number of lines. For some problems with more complex circuit topologies, additional splittings are desirable besides the transverse TL partitioning. This paper presents a new approach for the efficient longitudinal splitting for TEM mode transmission line problems. Previous longitudinal partitioning work using the classical WR for transmission lines led to limited solutions. This resulted in too many WR iterations and challenging partitioning problems. Here, we demonstrate a new optimized longitudinal WR algorithm for a single transmission line and determine an optimal choice of parameters. We give numerical examples to show the drastically improved convergence behavior.

Introduction

The Waveform Relaxation (WR) algorithm was first introduced for VLSI circuit analysis in [1], and has been extended to time dependent Partial Differential Equations (PDEs) in [2]. It was shown that the coupling between subdomains in physical space using Dirichlet transmission conditions at artificial domain interfaces corresponds to using a classical WR algorithm. Recent work in PDEs shows that these transmission conditions are far from optimal [3]. Much better performance can be obtained if additional information is exchanged at the interfaces. In fact, we recently demonstrated that these new techniques can be applied to diffusive circuits in [4], and for a transmission line circuit in [5] without overlap. Several attempts have been made before to improve the subsystem transmission conditions for WR with different types of circuit overlap schemes, e.g., [6], [7] to improve the transmission of information across the interface. Today, WR has a multitude of other applications in the mathematical community. In this paper, we introduce a new WR algorithm similar to the one in [5] with overlap and asymptotically determined optimal parameters.

Recently, the classical waveform relaxation approach was shown to be very effective in solving the excessive run-time problem for multiple coupled transmission lines as compared to the conventional incremental time analysis [8]. This approach utilizes the limited line-to-line coupling present in the transverse coupling of transmission lines. For this case, it was shown that the transverse WR approach leads to a large decrease in the run time with respect to the number of lines rather than $O(L^3)$ to $O(L^4)$ for conventional algorithms where L is the number of coupled lines. Here, we consider the *longitudinal* decoupling of a transmission line, a problem which does not converge for a reasonable number of iterations using classical WR algorithms. We show that using our new optimized WR we also can speed-up convergence for TEM mode type transmission lines used in todays technologies.

Longintudinal WR for TEM mode equivalent circuit model

We use the well know quasi TEM mode equivalent circuit in Figure 1 as the model for the transmission line considered in this work. Other recent macromodel type TEM-TL models could also be used. Here, the MNA circuit matrix is given in (1) in a slightly modified form. First, the solution vector is ordered in the form $\boldsymbol{x} = (v_1, i_1, v_2, i_2, v_3)^T$, which consists of nodal capacitive voltages alternating with inductance currents in the

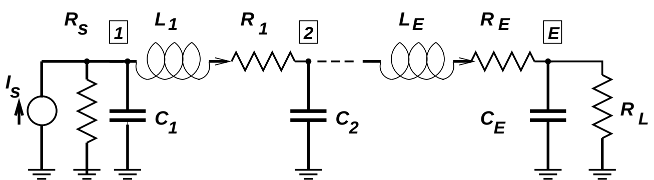

Fig. 1. Conventional quasi TEM mode tranmmission line model.

TL circuit to get the banding of the circuit matrix. Further, to simplify the mathematical analysis we also divide by the diagonal elements which leads to

$$\dot{\boldsymbol{x}} = \begin{bmatrix} b_1 & c_1 & & & \\ a_1 & b_2 & c_2 & & \\ & a_2 & b_3 & c_3 & \\ & & a_3 & b_4 & c_4 \\ & & & a_4 & b_5 \end{bmatrix} \boldsymbol{x} + \boldsymbol{f}, \tag{1}$$

where the number of nodes E is three.. The entries of the matrix are given by

$$a_i = \begin{cases} \frac{1}{L_{(i+1)/2}}, & i \quad \text{odd} \\ \frac{1}{C_{(i/2)+1}}, & i \quad \text{even} \end{cases}; \quad c_i = \begin{cases} -\frac{1}{C_{(i+1)/2}}, & i \quad \text{odd} \\ -\frac{1}{L_{i/2}}, & i \quad \text{even} \end{cases}; \quad b_i = \begin{cases} -\frac{1}{R_s C_1}, & i = 1 \\ -\frac{R_{i/2}}{L_{i/2}}, & i \quad \text{even} \\ 0, & i > 1 \quad \text{odd} \\ -\frac{1}{R_L C_3}, & i = 5 \end{cases},$$

and the source on the right hand side is given by $\boldsymbol{f}(t) = (I_s/C_1, 0, 0, 0, 0)^T$. We also need the initial values $\boldsymbol{x}(0) = (v_1^0, i_1^0, v_2^0, i_2^0, v_3^0)^T$ to start the transient solutions by setting the initial values to zero. The classical WR algorithm is given by

$$\begin{aligned} \dot{\boldsymbol{u}}^{k+1} &= \begin{bmatrix} b_1 & c_1 & \\ a_1 & b_2 & c_2 \\ & a_2 & b_3 \end{bmatrix} \boldsymbol{u}^{k+1} + \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_3 w_2^k \end{pmatrix}, \\ \dot{\boldsymbol{w}}^{k+1} &= \begin{bmatrix} b_3 & c_3 & \\ a_3 & b_4 & c_4 \\ & a_4 & b_5 \end{bmatrix} \boldsymbol{w}^{k+1} + \begin{pmatrix} f_3 \\ f_4 \\ f_5 \end{pmatrix} + \begin{pmatrix} a_2 u_2^k \\ 0 \\ 0 \end{pmatrix}, \end{aligned} \tag{2}$$

where u belongs to subsystem 1 and w belongs to subsystem 2. We used the classical transmission conditions with overlap,

$$u_4^{k+1} = w_2^k, \quad w_0^{k+1} = u_2^k. \tag{3}$$

The corresponding initial conditions are $\boldsymbol{u}^{k+1}(0) = (v_1^0, i_1^0, v_2^0)^T$, and $\boldsymbol{w}^{k+1}(0) = (v_2^0, i_2^0, v_3^0)^T$. To start the WR iteration, we need to specify initial waveforms $\boldsymbol{u}^0(t)$, and $\boldsymbol{w}^0(t)$ for $t \in [0, T]$. The optimized WR algorithm uses the new transmission conditions

$$u_4^{k+1} + \alpha u_3^{k+1} = w_2^k + \alpha w_1^k, \quad w_1^{k+1} + \beta w_0^{k+1} = u_3^k + \beta u_2^k, \tag{4}$$

which leads to

$$\begin{aligned} \dot{\boldsymbol{u}}^{k+1} &= \begin{bmatrix} b_1 & c_1 & \\ a_1 & b_2 & c_2 \\ & a_2 & b_3 - c_3\alpha \end{bmatrix} \boldsymbol{u}^{k+1} + \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_3(w_2^k + \alpha w_1^k) \end{pmatrix}, \\ \dot{\boldsymbol{w}}^{k+1} &= \begin{bmatrix} b_3 - \frac{a_2}{\beta} & c_3 & \\ a_3 & b_4 & c_4 \\ & a_4 & b_5 \end{bmatrix} \boldsymbol{w}^{k+1} + \begin{pmatrix} f_3 \\ f_4 \\ f_5 \end{pmatrix} + \begin{pmatrix} \frac{a_2}{\beta}(u_3^k + \beta u_2^k) \\ 0 \\ 0 \end{pmatrix}. \end{aligned} \tag{5}$$

From the classical transmission conditions we see that only current information is exchanged between the subcircuits. In the new transmission conditions we exchange a combination of voltage and current in both directions, and we introduce weighting factors α and β which can be used to optimize the new WR algorithm. The analysis for choosing the parameters α and β that lead to a WR algorithm with good performance for transmission line problems is based on the Laplace transform. This analysis allows us to determine the optimal α and β, as we now describe briefly. A Laplace transform analysis of problem (5) with parameter $s \in \mathbb{C}$ leads, after some computations, to the convergence factor of the new algorithm,

$$\rho_{opt}(s, \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \alpha, \beta) = \frac{a_2 a_3 (s-b_5) + \alpha a_2 ((s-b_4)(s-b_5) - a_4 c_4)}{(s - b_3 + c_3\alpha)((s-b_1)(s-b_2) - a_1 c_1) - a_2 c_2 (s - b_1)} \cdot \frac{\beta c_2 c_3 (s-b_1) + c_3((s-b_1)(s-b_2) - a_1 c_1)}{(\beta(s-b_3) + a_2)((s-b_4)(s-b_5) - a_4 c_4) - \beta a_3 c_3 (s - b_5)}. \tag{6}$$

The convergence factor of the classical WR can be obtained from the optimal convergence factor (6) by taking $\alpha = 0$ and $\beta \to \infty$. The convergence factor (6) vanishes for the choice

$$\alpha = -\frac{a_3(s-b_5)}{(s-b_4)(s-b_5) - a_4 c_4}, \qquad \beta = -\frac{(s-b_1)(s-b_2) - a_1 c_1}{c_2(s-b_1)}, \tag{7}$$

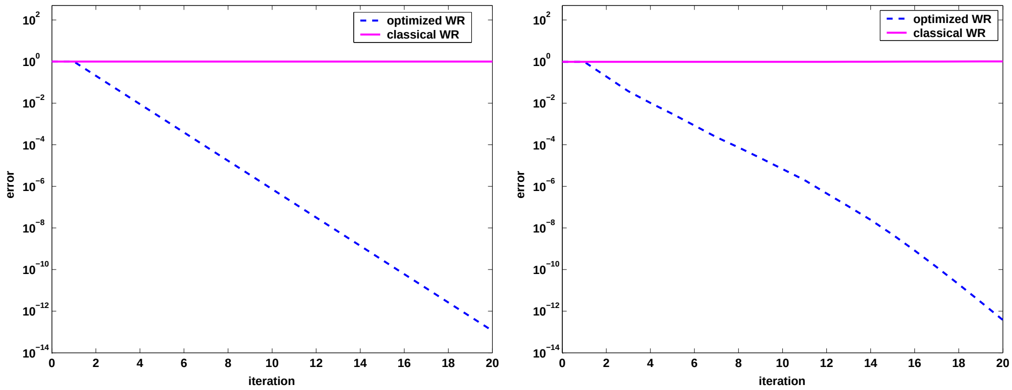

Fig. 2. Left: Convergence factors of the classical versus the optimized WR algorithms for a small transmission line circuit, Right: Convergence for a 5 cm quasi TEM mode transmission line model.

which is optimal. From the s^{-1} type frequency domain behavior, we see that the optimal choice corresponds to an integral operator in time. This operator would be expensive to implement, since it would require a convolution in the transmission condition. Hence, we approximate the optimal choice (7) by constants, which leads to a very practical algorithm. For good performance, we want $|\rho_{opt}| \ll 1$, which leads to the min-max problem

$$\min_{\alpha<0,\beta>0}\left(\max_{\Re(s)\geq 0}|\rho_{opt}(s,\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\alpha,\beta)|\right). \tag{8}$$

Since, from the optimal choice, with $c_2 = c_4 = -a_3 = -a_1$, $c_1 = c_3 = -a_4 = -a_2$, $b_3 = 0$, $b_5 = b_1$, and $b_4 = b_2$, we have $\beta_{opt} = -\frac{1}{\alpha_{opt}}$, and the circuit considered here behaves identically on both sides of the cut, we choose $\beta = -\frac{1}{\alpha}$ to simplify the optimization process. Hence, α is the only optimization parameter left. To solve the min-max problem (8), we use a two scale expansion motivated by typical small transmission line circuit elements. This leads to

$$a_1 = \frac{30}{4.95e-3} = \mathscr{O}\left(\frac{1}{\epsilon}\right),\; b_1 = -\frac{30}{0.0315} = \mathscr{O}\left(\frac{1}{\sqrt{\epsilon}}\right),\; a_2 = \frac{30}{0.63} = \mathscr{O}(1),\; b_2 = -\frac{0.5e-3}{4.95e-3} = \mathscr{O}(1). \tag{9}$$

An asymptotic analysis of (8) with the two scale expansion (9) leads to the asymptotically optimal choice

$$\alpha = \alpha^* = \left(-0.228 + 0.002a_2 + 0.004\frac{b_1}{\sqrt{a_1}}\right)\sqrt{a_1}, \tag{10}$$

valid in the neighborhood of the typical transmission line circuit elements given in (9). Note that b_2 does not appear in the asymptotic result for α^*, b_2 only appears in higher order terms in the asymptotic expansion.

Numerical Experiments

We choose a single line example to show the convergence for longitudinal partitioning for the classical and the optimized WR. The examples are a two section and a 5cm TEM mode transmission line model with typical per unit length parameters of 4.95 nH/cm, C=0.63 pF/cm and R=0.5 Ohms/cm. The load and termination resistance are 50 Ohms. The 2 mA current source has a linear ramp to 0.1ns and the analysis time interval is $[0, T]$ with $T = 2.5$ ns. The time step is chosen to be sufficiently small, $h = 5ps$, to get an accurate answer with the backward Euler time integration method.

First, we give results for the rate of convergence for the optimized WR in comparison to the classical algorithm. Figure 2 shows how the classical WR does not converge in the interval with 20 iterations while the optimized convergence exceeds Spice accuracy in 4 iterations for both a two section and a 5 cm TEM mode transmission line circuits. In the past, clever partitioning algorithms were used to avoid strong coupling problems in the classical WR environment [9], [6]. In general, this had the disadvantage of leading in some cases to large subsystems since strongly coupled systems could not be partitioned.

In the second sequence of numerical experiments, we choose the 5 cm TEM mode transmission line with 150 sections. Note that all the results given in Fig. 3 are compared with the solid line solution obtained by a conventional Spice type approach. In the two top graphs in Fig. 3, we show WR iterations 3 and 5 for the propagated waveforms at the beginning (Node 1) and end (Node 150) with $R_L = 50$ Ohms, and in the bottom,

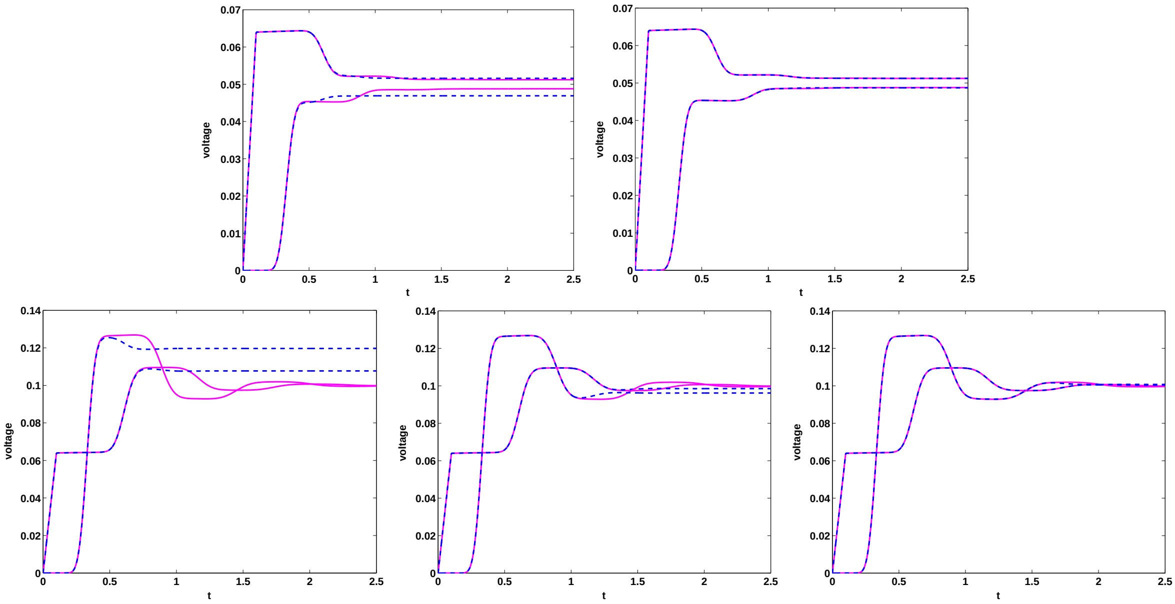

Fig. 3. Time domain waveforms for 5 cm transmission line with 150 sections. Exact solutions (solid lines), optimized WR solutions (dashed lines). Top 2 figures: optimized WR iterations 3 and 5 for transmission line with $R_L = 50$ Ohms. Bottom 3 figures: optimized WR iterations 3, 5 and 7 for open 5 cm transmission line, $R_L \to \infty$.

we show iterations 3, 5 and 7 for the propagated waveforms at the beginning (Node 1) and end (Node 150) with an open transmission line, i.e., $R_L \to \infty$.

Conclusions

The classical WR was recently shown to yield a large speedup for the transverse WR transient analysis for multiple transmission lines while the approach presented in this paper is shown to work well for the very strongly coupled case of the longitudional partitioning of a single line which could also have application to multiple lines in conjunction with the transverse partitioning approach. The optimized WR which is based on new, optimized transmission conditions given in this paper shows much potential for the the longitudinal decoupling of transmission lines. Here, we show that Spice type accuracy can be obtained in about 5 iterations for a 5 cm transmission line with simple partitioning. We also show that under the same conditions, the classical WR algorithm cannot converge in 20 iterations due to its non-uniform convergence properties. A key application of the optimized WR algorithms is parallel processing due to the independent transient analyse of the resultant subcircuits.

References

[1] E. Lelarasmee, A. E. Ruehli, and A. L. Sangiovanni-Vincentelli. The waveform relaxation method for time-domain anaylsis of large-scale integrated circuits. *IEEE Trans. on CAD of Integrated Circuits and Systems*, CAD-1(3):131–145, July 1982.

[2] M. J. Gander and A.M. Stuart. Space-time continuous analysis of waveform relaxation for the heat equation. *SIAM Journal on Scientific Computing*, 19(6):2014–2031, 1998.

[3] M. J. Gander, L. Halpern, and F. Nataf. Optimal Schwarz waveform relaxation for the one dimensional wave equation. *SIAM Journal on Numerical Analysis*, 41(5):1643–1681, 2003.

[4] M. J. Gander and A. Ruehli. Optimized waveform relaxation methods for RC type circuits. *IEEE Transaction on Circuits and Systems*, 51:755–768, April 2004.

[5] M. J. Gander and A. Ruehli. Solution of large transmission line type circuits using a new optimized waveform relaxation partitioning. *IEEE International Symposium on Electromagnetic Compatibility*, 2:636–641, August 2003.

[6] P. Debefve and F. Odeh and A. Ruehli. *in Circuit Analysis, Simulation and Design, Ruehli Ed.* Part II, Elseviers, North-Holland, New York, Amsterdam, 1987.

[7] V. B. Dmitriev-Zdorov and B. Klaassen. An improved relaxation approach for mixed system analysis with several simulation tools. In *Proc. of EURO-DAC'95, IEEE Comp. Soc. Press*, 1995.

[8] N.M. Nakhla, A.E. Ruehli, and R. Achar M.S. Nakhla. Simulation of coupled interconnects using waveform relaxation and tranverse partitioning. *IEEE Transactions on Advanced Packaging*, 29:78–87, Feb 2006.

[9] J. White and A. L. Sangiovanni-Vincentelli. Partitioning algorithms and parallel implementations of waveform relaxation algorithms for circuit simulation. *IEEE Proc. Int. Symp. Circuits and Systems, (ISCAS-85)*, pages 1069–1072, 1985.