A simple Algebra Problem

Given

$$(a+1)*(b+1)*(c+1)=8 \quad \text{with } a,b,c >= 0$$

show that

$$a*b*c <= 1$$

Proof with maple:

```
> f:=(a+1)*(b+1)*(c+1)-8;
```

$$f := (a + 1)\,(b + 1)\,(c + 1) - 8$$

```
> g:=-expand(f)+a*b*c;
```

$$g := -a\,b - a\,c - a - b\,c - b - c + 7$$

```
> a:=solve(f=0,a);
```

$$a := -\frac{b\,c + b + c - 7}{b\,c + b + c + 1}$$

```
> g:=simplify(g);
```

$$g := -\frac{b\,c\,(b\,c + b + c - 7)}{b\,c + b + c + 1}$$

```
> solve({diff(g,b)=0,diff(g,c)=0},{b,c});
```

$$\{b = 0,\ c = 0\},\ \{b = 0,\ c = 7\},\ \{c = 0,\ b = 7\},\ \{c = 1,\ b = 1\},$$

$$\{c = \%1,\ b = -\frac{2\,\%1 - 7 + \%1^2}{(\%1 + 1)^2}\}$$

$$\%1 := \mathrm{RootOf}(_Z^2 + 4\,_Z + 7) \quad \text{\% complex}$$