The couch moving problem

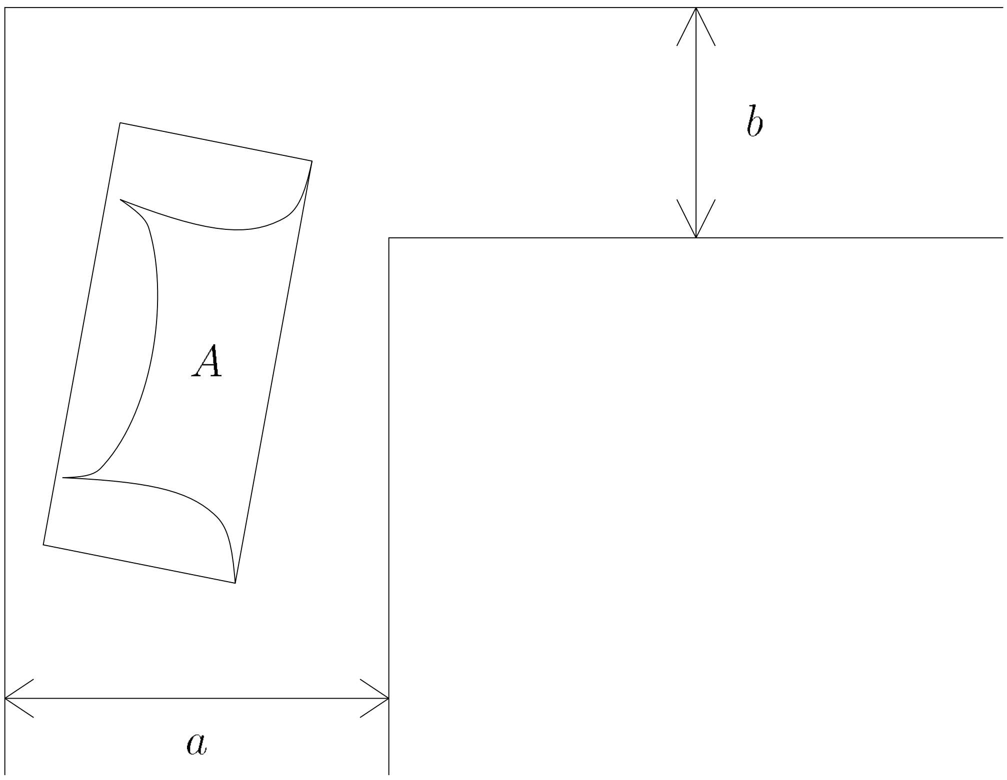

Figure 1:

What is the maximal area A a couch can have, so that one can move it around the corner of the corridors given in the figure 1 ?

Analytical solution

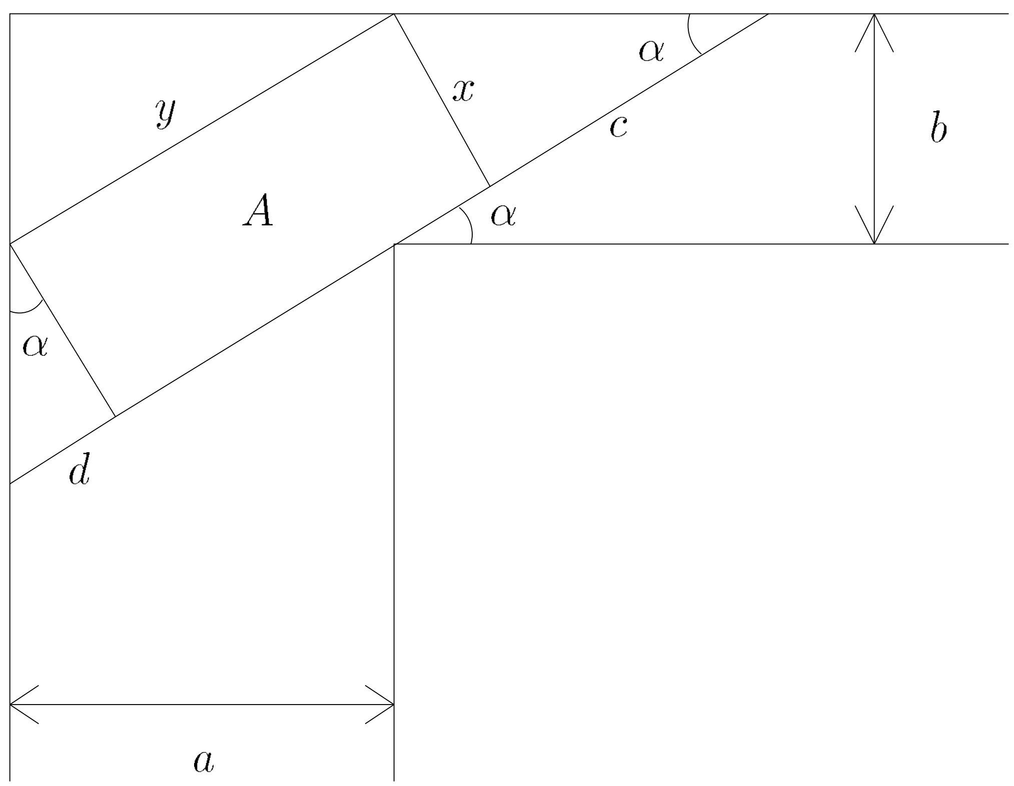

Figure 2:

To find the maximal possible area A of a couch, which fits around the corner, we have to minimize the area of the rectangle A with two corners touching the the wall and one side touching the corner. This area is given by

$$A(x, \alpha) = xy,$$

where

$$y + c + d = \frac{b}{\sin \alpha} + \frac{a}{\cos \alpha}$$

and

$$c = \frac{x}{\tan \alpha}, \quad d = x \tan \alpha.$$

Inserting all these expressions into $A(x,\alpha)$, we get

$$A(x,\alpha) = \frac{x(b\cos\alpha + a\sin\alpha) - x^2}{sin\alpha\cos\alpha}.$$

To find the minimum, we take the gradient of $A(x,\alpha)$ and set it equal to zero. This leads, after some manipulation, to the result

$$\begin{aligned} A &= ab, \\ x &= \frac{ab}{\sqrt{a^2+b^2}}, \\ y &= \sqrt{a^2+b^2}. \end{aligned}$$

Geometrical solution

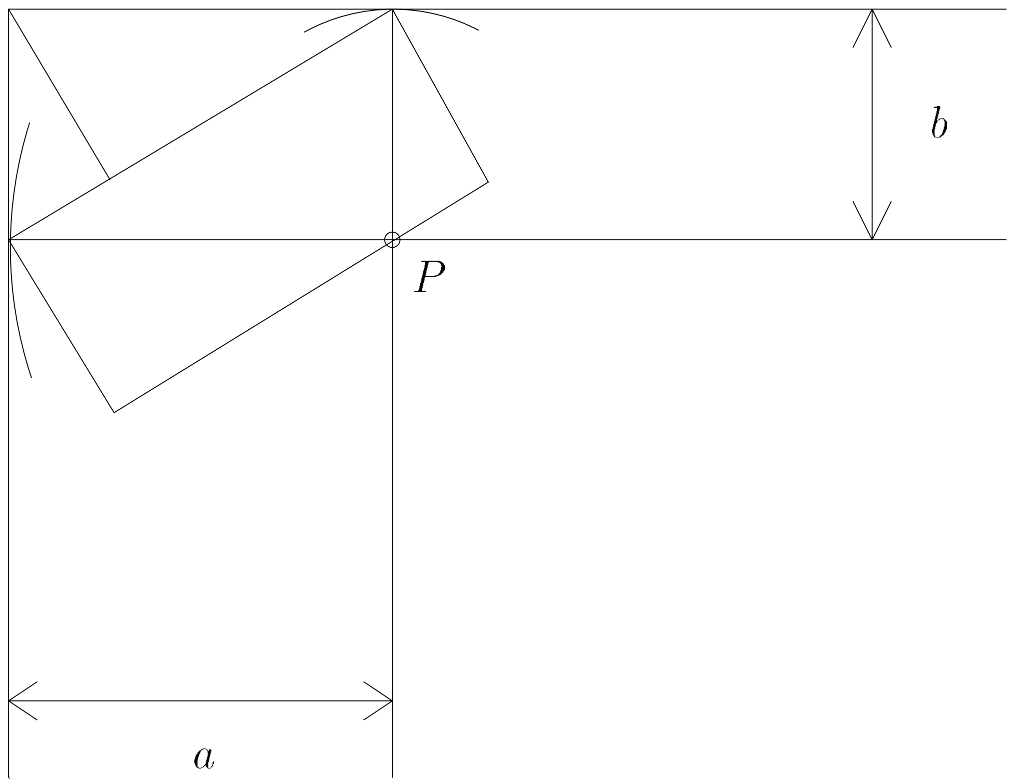

Figure 3:

In the critical moment, the couch is only doing a rotation around the point P. Therefore the size of the couch is limited where the two arcs touch the walls. By similarity of the triangles, the maximal area of the couch is ab. Note that a second solution exists. The couch with sides a and b fits exactly into the corner and can therefore get through without turning.