ENUMERATIVE COMBINATORICS

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It is difficult to define exactly the subject of Combinatorics. One thing is certain, the problem of Enumerating occupies an important place in this branch of Mathematics. This course will concentrate on Enumerative Combinatorics, and our goal will be to get acquainted with its main notions and tools. The latter often come from different areas of Mathematics, as for example, Theory of Finite Groups. Rather than following the subject from beginning to end, we shall choose to get a feeling of it by considering as many examples as possible. The examples are likely to come from such areas of Combinatorics as Graph Theory, Linear Codes, Cryptography.

Deep knowledge of Generating Functions is central in solving enumerating problems. We shall spend some time on studying properties of generating functions and consider in details examples in which they are of use. Operations on generating functions will be discussed, as Composition Formula and Lagrange Inversion Formula. These will be applied to such famous problems as

- enumerating graphs with a fixed number of connected components
- enumerating rooted trees
- enumerating Eulerian paths in a directed graph
- enumerating de Brujin sequences, etc.

We shall eventually pay some attention to two fundamental properties of generating functions which are rationality and algebraicity. A natural and powerful framework to study these properties is the theory of non-commutative generating functions and formal power series.

An important place in the subject is taken by the problem of enumerating partitions. Most powerful tools to solve it belong to the theory of Symmetric Functions. Though developped for combinatorial purposes, it has also found applications in Representation Theory and Algebraic Geometry. In particular, symmetric functions can be used to describe all irreducible characters of a symmetric group.

Recommended literature:

1. R. Stanley. *Enumerative Combinatorics*, Vol. 1 & 2, Cambridge Studies in Advanced Mathematics 49 & 62.
2. J. H. van Lint and R. M. Wilson. *A Course In Combinatorics*, Cambridge University Press.