SPECTRA OF GRAPHS

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The course will cover several chapters of Algebraic Graph Theory related to the study of the Laplace operator on a finite graph.

It appears that the spectral properties of this operator reflect strongly the geometry of the graph.

For example, a well known theorem asserts that the number of spanning trees in a graph on n vertices is equal to $1/n$ times the product of all non-zero eigenvalues of the Laplacian. First partial results towards this theorem were obtained by English mathematician A. Cayley 110 years ago.

Another example is the "spectral gap", or the smallest non-zero eigenvalue of the Laplacian, which is responsible for the "expanding" property of the graph. Infinite sequences of graphs with uniformly big spectral gaps play important role both in mathematics and computer science, but prove very difficult to construct explicitely. This problem was solved only recently, and graphs with big spectral gap continue to be subject of intensive research nowdays.

If time permits, we will also try to explore some connections between the subject of spectra on graphs and other parts of mathematics, as Riemannian geometry, random walks, and discrete groups.

Recommended literature:

1. N. Biggs. *Algebraic graph theory.* Cambridge University Press, 1974, 1993.
2. D. M. Cvetković, M. Doob, and H. Sachs. *Spectra of graphs.* Academic Press (New York), 1980.
3. A. Lubotzky. *Discrete groups, expanding graphs and invariant measures.* Progress in mathematics, Vol 125, Birkhäuser, 1994. Chapters 1,4,8. (This is more advanced reading.)