Quality Upgrading and the Stages of Diversification

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Preliminary and incomplete

1 Introduction

The goal of the paper is to build a framework that can assess the contribution of different determinants of firms’ activity to the U-shaped evolution of export concentration found in the data by Imbs and Wacziarg (2003) and Cadot et al. (2011), among others. Our main focus is on the importance of quality upgrading at the sectoral level; however, we also consider the effects of trade costs, input costs, and market size.

Figure 1 presents the patterns that drive the paper. The left hand side panel shows the theil index of export concentration for nations that possess different GDP per capita levels. There we see that, on average, this level of concentration falls with income, and when real GDP reaches about 25,000 dollars this tendency is reversed. The right hand side panel, in turn, shows that product unit values – which are mainly driven by product quality – increase as nations become richer in per capita terms, more rapidly at the beginning.

In order to achieve the above goal, we build a model of international trade; which is later calibrated, and its predictions compared to the data.
2 The Model

We present a static framework that considers three main dimensions of product exports and imports: the intensive, the extensive, and the quality margins. The *intensive margin* refers to more units produced of a good. The *quality margin* captures changes in the unit value of a given product. The third dimension, the *extensive margin*, is related to the number of product lines. Trade is formalized following Eaton and Kortum (2002, EK from now on), but extended to include product quality. As driven mechanism behind the process of diversification and posterior concentration, the model proposes the existence of heterogeneity in quality-upgrading potential across export lines.

2.1 Consumers

Consider a nation $n$ populated by $L_n$ individuals. Each agent is endowed with one unit of time that is inelastically allocated to labor. Households have preferences defined over products supplied by $N$ different sectors. Products lines that belong to different activities can have different quality upgrading potential. Each sector, in turn, offers a continuum of mass one of product lines. The flow of utility depends on the amount of the different goods consumed weighed by their quality.

More specifically, at each point of time, a representative agent in nation $n$ solves
the following problem:

$$
\max \{ c_{nv(j)} \} \quad c_n = \left( \sum_{v=1}^{N} \omega_v^{1/\xi} c_{nv(j)}^{1-1/\xi} \right)^{\frac{\xi}{\xi-1}},
$$

with

$$
c_{nv} = \left\{ \int_0^1 \left[ q_{nv}(j) c_{nv}(j) \right]^{1-\frac{1}{\eta}} \, dj \right\}^{\eta},
$$

subject to the budget constraint

$$
w_n = \sum_{v=1}^{N} \left[ \int_0^1 p_{nv}(j) c_{nv}(j) \, dj \right].
$$

Above, $c_{nv}(j)$ is the amount of good $j$ from sector-$v$ consumed by the representative individual in country $n$; the variables $p_{nv}(j)$ and $q_{nv}(j)$ capture the consumer price and the quality of that good in nation $n$, respectively; $w_n$ gives the wage rate; the parameters $\varepsilon > 0$ and $\eta > 0$ represent the elasticity of substitution between sectors and among goods, respectively; and $\omega_v > 0$ weighs the contribution of sector-$v$ consumption in the individual’s utility.

The solution to this problem obtains the following optimality conditions for consumption:

$$
\frac{q_{nv}(j) c_{nv}(j)}{c_{nv}} = \left( \frac{p_{nv}(j)}{q_{nv}(j)} \right)^{-\eta},
$$

and

$$
c_{nv} = \omega_v \left( \frac{P_{nv}}{P_n} \right)^{-\varepsilon},
$$

where the CES exact price indices equal

$$
P_{nv} = \left\{ \int_0^1 \left[ \frac{p_{nv}(j)}{q_{nv}(j)} \right]^{1-\eta} \, dj \right\}^{\frac{1}{1-\eta}},
$$

and

$$
P_n = \left( \sum_{v=1}^{N} \omega_v P_{nv}^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}.
$$

### 2.2 Producers and quality

Product and input markets are perfectly competitive. The only input of production is labor. We denote by $Y_{nv}(j)$ the amount produced of good $j$ in sector $v$ by country $n$. The production function is the following:

$$
Y_{nv}(j) = \frac{L_{nv}(j)}{a_{nv}(j)};
$$
where $L_{nv}(j)$ represents the amount of labor, and $a_{nv}(j)$ provides the number of units of labor required to produce one unit of output. As a result, the unit cost of output equals

$$p_{nv}(j) = a_{nv}(j)w_n.$$  \hfill (9)

The quality dimension is inspired on Melitz (2003) assuming that product quality requires input quality as in Kugler and Verhoogen (2012). More specifically, we suppose that additional number of workers are required to produce higher quality goods. Following Baldwin and Harrigan (2011):

$$a_{nv}(j) = q_{nv}(j)^{1/\beta}. \hfill (10)$$

By expression (4), goods demand depends on the price-to-quality ratio. From (9) and (10), we can deduce that this ratio equals

$$\frac{p_{nv}(j)}{q_{nv}(j)} = \frac{w_n}{a_{nv}(j)^\beta}. \hfill (11)$$

The last expression implies that the parameter $\beta$ needs to be greater than 0 to guaranty that the price-to-quality ratio falls with quality, and more costly, higher quality versions are preferred.\footnote{Alternatively, we could think that input use also depends on the efficiency level of the economy, and in particular that $a_{nv}(j) = [q_{nv}(j)\varphi_n]^{1/(1+\beta)}$, where $\varphi_n$ is the efficiency of labor in country $n$. This would obtain a price-to-quality ratio equal to $p_{nv}(j)/q_{nv}(j) = \varphi_n w_n / a_{nv}(j)^\beta$. Therefore, in expression (11), we are implicitly assuming that the efficiency level of the economy is subsumed in the salary.}

### 2.3 Trade

Next, we embed the above structure into EK’s model. Compared to the EK setup, the main difference is that we consider product quality. In addition, to simplify the analysis, we follow Uy et al. (2013) and assume that there are only two economies: a small open country $i$; and the rest of the world $n$. We think of this small open country as a developing nation that starts from lower quality levels than the rest of the world.

In order to generate trade flows, suppose that the labor productivity parameter $a_{nv}(j)$ is a draw from a random variable $A_{nv}$ that follows a Fréchet with cumulative distribution function

$$F_{nv}(a) = \Pr[a_{nv}(j) \leq a] = \exp(-T_{nv} a^{-\alpha}). \hfill (12)$$
The parameter \( T_{nv} > 0 \) is country and sector specific. A higher \( T_{nv} \) implies that a higher draw of \( a_{nv}(j) \) is more likely and, therefore, controls for the nation’s absolute advantage. The parameter \( \theta_v > 1 \), on the other hand, can proxy comparative advantage. This coefficient is sector specific and governs the degree of variation within the distribution. A bigger \( \theta_v \) implies less variation.

Products engage in international trade, whereas labor is only supplied domestically. There is an iceberg cost involved in shipping goods from the origin country to the destination nation: for each unit that country \( i \) ships to nation \( n \), only \( 1/d_{ni} \) units arrive. We suppose that \( d_{nn} = 1 \).

Let us denote the free-on-board price (i.e., the producer price) and quality that the origin country offers by \( p_{niv}(j) \) and \( q_{niv}(j) \), respectively. Notice that \( p_{niv}(j) = a_{iv}(j)w_i \). Consumers’ Demand function (4) says that country \( i \) will be able to sell product \( j \) in country \( n \) if it can offer a better price-to-quality ratio in the destination market, that is, a lower \( d_{ni}p_{niv}(j)/q_{niv}(j) \). The link between consumer and producer prices is then given by

\[
p_{nv}(j) = \min \left\{ \frac{d_{ni}w_i}{a_{iv}(j)^{\beta}}, \frac{w_n}{a_{nv}(j)^{\beta}} \right\}.
\]

(13)

A fast implication is the following: advancing along the extensive margin in exports requires increasing quality, lowering shipping costs, and controlling wage increases.

We do not know the exact price for each good in each country. However, as EK show, we can obtain their distribution. In particular, from expressions (10) and (13), the probability that the price-to-quality ratio for product \( j \) originated in country \( i \) and sold in nation \( n \) is less than or equal to an arbitrary number \( \rho \) equals:

\[
G_{niv}(\rho) = \Pr \left[ \frac{d_{ni}p_{niv}(j)}{q_{niv}(j)} \leq \rho \right] = 1 - F_{iv} \left[ \left( \frac{d_{ni} w_i}{\rho} \right)^{1/\beta} \right] .
\]

(14)

And from (13) and (14), the distribution of the price-to-quality ratio for what country \( n \) actually buys sector-\( v \) goods is given by

\[
G_{nv}(\rho) = \Pr \left[ \frac{p_{nv}(j)}{q_{nv}(j)} \leq \rho \right] = 1 - \exp(-\Phi_{nv}\rho^{\theta_v/\beta}),
\]

where \( \Phi_{nv} = T_{iv} (d_{ni}w_i)^{-\theta_v/\beta} + T_{nv}w_n^{-\theta_v/\beta} \).

The last expression differs from EK in that the quality-input-requirement parameter \( \beta \) weights the effect of \( \theta_v \).
EK proves that this distribution implies that the probability that country \( i \) provides the best price-to-quality ratio in sector \( v \) to nation \( n \) is

\[
\pi_{niv} = \frac{T_{iv} (d_{ni} w_i)^{-\theta_v/\beta}}{\Phi_{nv}}. \tag{16}
\]

Because the continuum of goods is of mass one in each sector, this number also represents the fraction of sector-\( v \) goods that country \( n \) buys from country \( i \), and the fraction of country’s \( n \) expenditure that goes to products exported by nation \( i \).²

An implication of this property is that the value of sector-\( v \) exports from nation \( i \) to \( n \) equals:

\[
X_{niv} = \pi_{niv} (P_{nv} C_{nv} L_n); \tag{17}
\]

where the terms in parenthesis provide total \( v \)-goods expenditure in \( n \); and the price index is given by

\[
P_{nv} = \gamma \Phi_{nv}^{-\beta/\theta_v}, \quad \text{with } \gamma = \Gamma \left[ \left( \frac{\theta_v/\beta + 1 - \eta}{\theta_v/\beta} \right)^{\frac{1}{\eta}} \right]; \tag{18}
\]

where \( \Gamma \) represents the gamma function that requires \( \eta < 1 + \theta_v \).

Another useful expression is the one for net exports \( (NX) \). Country \( i \)'s net exports in sector \( v \) equal:

\[
NX_{iv} = \pi_{niv} \left( \frac{P_{nv}}{P_n} \right)^{1-\varepsilon} w_n L_n - \pi_{inv} \left( \frac{P_{iv}}{P_i} \right)^{1-\varepsilon} w_i L_i \tag{19}
\]

If we imposed balanced trade, this would imply that

\[
\sum_{v=1}^{N} NX_{iv} = 0. \tag{20}
\]

### 3 A Simplified Version of the Model

In this section, we simplify the framework to illustrate how quality may help to explain the hump-shaped evolution of diversification observed in the data. Suppose that the economy is closed, that there only two sectors in the economy – \( m \) and \( h \) – and that quality upgrading is the result of learning-by-doing; more specifically,

\[
q_{nv}(j) = \phi_v q_{nie}^{\alpha} Y_{nv}(j); \tag{21}
\]

²In our version of the EK model, this is as well true because demand depends on the price-to-quality ratio.
where $q_n$ is the average quality level across goods in country $n$; $0 < \phi_h < \phi_m$; $0 < \mu_m < \mu_h$; and $\varphi \in (0, 1)$. The restrictions in the parameters imply that sector $h$ can start with relatively less quality, but has more quality upgrading potential. The expression also says that leaning-by-doing arises as a result of the production activity, and is subject to diminishing returns each period.

Furthermore, assume that $a_{nv}(j)$ is the same for all $j$ in sector $v$. Hence, without loss of generality, we can think that there is only a product line in each sector, and eliminate the good index $j$ from the notation for simplicity. Equations (2) and (4) to (7) reduce to

\begin{equation}
\frac{c_{nv}}{c_n} = q_{nv} c_{nv},
\end{equation}

and

\begin{equation}
\frac{q_{nv} c_{nv}}{c_n} = \omega_v \left( \frac{p_{nv}/q_{nv}}{P_n} \right)^{-\varepsilon};
\end{equation}

where the price index is given by

\begin{equation}
P_n = \left[ \sum_{v=m,h} \omega_v \left( \frac{p_{nv}}{q_{nv}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.
\end{equation}

The evolution of the country’s level of diversification can be traced looking at the relative weights or the two sectors $m$ and $h$. In a closed economy, the clearing condition that requires supply equals demand for output $j$ in sector $v$ is

\begin{equation}
Y_{nv}(j) = c_{nv}(j) L_n;
\end{equation}

and for labor, equilibrium requires

\begin{equation}
L_n = \int_0^1 L_{nv}(j) \, dj.
\end{equation}

Using conditions (25) and (22), the relative weight of the two sectors in consumption expenditure can be written as

\begin{equation}
\frac{p_{nh} c_{nh}}{p_{nm} c_{nm}} = \frac{p_{nh} Y_{nh}}{p_{nm} Y_{nm}} = \frac{\omega_h}{\omega_m} \left( \frac{p_{nm}/q_{nm}}{p_{nh}/q_{nh}} \right)^{\varepsilon-1};
\end{equation}

which adding (9) and (10) delivers

\begin{equation}
\frac{p_{nh} c_{nh}}{p_{nm} c_{nm}} = \frac{\omega_h}{\omega_m} \left( \frac{q_{nh}}{q_{nm}} \right)^{(\varepsilon-1)\beta/(1+\beta)}.
\end{equation}
Relative expenditure in $h$-goods increases in their quality level when the two sectors are relative substitutes.

The next step is taking into account the way quality upgrading is generated in the economy. Technology (21) along with the same expressions that we employed to obtain (28) imply that

$$q_{nm} = \left[ \frac{\phi_m \nu^{-\mu_m}}{\omega_m} \right]^{1+\frac{1}{1+\gamma}}. \tag{29}$$

Finally, putting (28) and (29) together obtains

$$p_{nm}c_{nh} = \left[ \frac{\omega_h}{\phi_m} \right]^{\frac{\beta(\varepsilon-1)}{1+\beta+\varphi[1-\beta(\varepsilon-1)]}}. \tag{30}$$

Equality (30) says that, as average quality in the economy increases, the relative share of expenditure in higher quality-potential goods will go up if and only if the two sectors are relative substitutes but not too much; in particular, if and only if $1 < \varepsilon < 1 + [(1 + \beta)/\varphi + 1]/\beta$. Otherwise, the share of high quality-upgrading potential goods will go down with average quality.

Therefore, quality upgrading can contribute to generate the stages of diversification observed in the data. The first step to see this is to restrict the set of possible values of $\varepsilon$. Across big activity sectors, goods a complementary; for example, Herrendorf, Rogerson and Valentinyi (2013) estimate a value of $\varepsilon$ close to zero across agriculture, manufacturing and services. Within big sectors, however, products seem to be relative substitutes. Ilyina and Samaniego (2012), for instance, estimate a value for $\varepsilon$ of 3.75 employing manufacturing data; an estimate that is consistent with alternative figures offered in the international trade literature surveyed, among others, by Anderson and Van Wincoop (2004).

Suppose then that $m$ and $h$ goods belong to manufacturing, that the above inequality restrictions on the elasticity of substitution is fulfilled, $\alpha_m \omega^0_m > \phi_h \omega^0_h$, and that $q_n$ initially equals one. As a consequence, the share of $m$-products in consumption expenditure is initially larger. As quality rises though learning-by-doing, the weight of $h$-goods in total expenditure will rise, and an increasing diversification will eventually be reversed and make way for increasing concentration.

Can the mechanism described above be as well relevant to explain the U-shaped evolution of trade concentration? The answer is yes. To see this, we can focus on the
Theil index (denoted by $Th$) as measure of concentration. For country $i$, it can be written as:

$$Th_i = \frac{1}{\pi_{nim} + \sum_{v=m,h} \pi_{niv}} \int_{0}^{\pi_{niv}} X_{niv}(j) \ln \left( \frac{X_{niv}(j)}{\mu_i} \right) dj; \quad (31)$$

where $X_{niv}(j)$ represents the value of country $i$’s good-$j$ exports; and $\mu_i$ is the average level of exports from nation $i$ across goods.

The fraction $\pi_{niv}$ captures the extensive margin, and increases in nation $i$’s quality relative to the rest of the world always contribute to diversify exports. However, as equality (17) suggests, the effect of the evolution of the $X_{niv}(j)$’s in the concentration index are driven by the changing pattern of cross-sector consumption expenditure in country $n$ described by (30); the mechanism explained above is therefore also at work with exports. Furthermore, this second effect will eventually dominate – for example, once convergence in quality with the rest of the world is complete – leading to increasing concentration.

4 Future Work: Quantitative Predictions

4.1 The Theil Index and the Equation System

Given that we have data for the different variables at the sectoral level for $N$ sectors, let us assume that all products within the same sector are exported in the same amount. Hence, expression (31) becomes:

$$Th_i = \frac{1}{\sum_{v=1}^{N} \pi_{niv}} \sum_{v=1}^{N} X_{niv} \ln \left( \frac{X_{niv}}{\mu_i} \right); \quad (32)$$

where

$$\mu_i = \frac{1}{\sum_{v=1}^{N} \pi_{niv}} \sum_{v=1}^{N} X_{niv}. \quad (33)$$

Notice as well that combining expressions (5), (3) and (17), the value of sector-$v$ exports from country $i$ to nation $n$ can be written as

$$X_{niv} = \pi_{niv} \omega_v \left( \frac{P_{nv}}{P_n} \right)^{1-\epsilon} w_n L_n. \quad (34)$$

Therefore, in order to study the evolution of the Theil index, it is necessary to get the endogenous values of input and output prices. Output prices in country $n$ are given.
by expression (18). In turn, wages are determined by the clearing conditions in output and labor markets, which for example for country $i$ are the following:

$$w_i L_{iv} = \pi_{ivi} P_{iiv} c_{ivi} L_i + X_{niv}$$

$$= \omega_v \left[ \pi_{ivi} \left( \frac{P_{iiv}}{P_i} \right)^{1-\varepsilon} w_i L_i + \pi_{niv} \left( \frac{P_{niv}}{P_n} \right)^{1-\varepsilon} w_n L_n \right],$$

and

$$\sum_{v=1}^{N} L_{iv} = L_i.$$  

For nation $n$, the above expressions are symmetric.

There are some simplifications that we can perform to make it easy finding solutions. The first one is related to the equations: 35) and (36) reduce to

$$w_i = \sum_{v=1}^{N} \omega_v \left[ \pi_{ivi} \left( \frac{P_{iiv}}{P_i} \right)^{1-\varepsilon} w_i L_i + \pi_{niv} \left( \frac{P_{niv}}{P_n} \right)^{1-\varepsilon} w_n L_n \right].$$

The second has to do with the variables: one of the prices is redundant, because all prices can be written in terms of one of the salaries; then we normalize $w_n$ to one.

Therefore, taking as given the labor supply in each country, we are left with a system of one equation, given by expression (37), and one unknown, $w_i$; notice that output prices and export shares are functions of the salaries and exogenously given parameters. Once we know $w_i$, the other endogenous values can be recovered.

### 4.2 Proposed Exercises

1. Check whether the model can generate an U-shaped evolution of the Theil index with some made up parameter values. In principle, the hypothesis requires that the economy moves towards products that have more quality upgrading potential, and those new products added to the export basket are substitutes.

2. Calibrate the model to a country such as Vietnam and see how far the evolution of the quality index can go in explaining the evolution of the diversification index. Assess also the contribution of the other components.

### 4.3 Calibration of the Parameters

We have data for the following sectors: SITC1: 0 – Food and Life animals; SITC1: 1 – Beverages and tobacco; SITC1: 2 – Crude materials, inedible, except fuels; SITC1: 3 –
Mineral fuels, lubricants and related materials; SITC1: 4 – Animal and vegetable oils and fats; SITC1: 5 – Chemicals; SITC1: 6 – Manufacturing goods classified chiefly by material; SITC1: 7 – Machinery and transport equipment; SITC1: 8 – Miscellaneous manufactured articles; and SITC1: 9 – Commod. & transactions not classified according to kind. Are these subcategories appropriate for the analysis? The reason is that big groups tend to be complements.

**Parameters:**

1. $L_k$ – Directed from the data.

2. $\theta_v$ – We can consider the same value for all sectors. Consistent with recent estimates by Simonovska and Waugh (2011): $\theta = 4$. This is a benchmark value.

3. Uy et al. (2013) take a within-sector elasticity of $\eta = 4$. This one should not matter much. Do sensitivity.

4. $\varepsilon, \omega_k (N + 1$ unknowns) – Following Herrendorf et al. (AER 2012). From sectoral price-to-quality ratios, aggregate consumption expenditure, and sectoral-consumption expenditure-shares for ROW. Minimizing the sum of squared deviations between the actual shares and the model-implied sectoral expenditure shares given the observed sectoral prices-to-quality ratios and consumption expenditures in the ROW.

5. $\beta$ – It can be probably estimated directly from the above data.

6. $T_{iv}, T_{nv}, d_{ni}, d_{in}, w_i (2N + 3$ unknowns) – These are time varying. The model-implied trade costs capture transportation costs, tariffs, and any other costs that impede international trade. They can be chosen to match the relative price-to-quality ratios in country $i$ (relative to ROW: $P_{iv}/P_{nv}$; it provides $N$ equations), $i$’s sectoral trade shares (import and export ones separately: $\pi_{nv}, \pi_{inv}$; they give $N$ equations each), and the price-to-quality ratios in ROW ($P_{nv}$ provides $N$ equations). Questions: Is the unit price index relative in the same way as the quality index? Do we take all imports or the ones coming from frontier-quality nations?

(a) Alternative 1: imposing a balanced budget; then I do not need imports data.
(b) Alternative 2: considering that \( d_{ni} = d_{in} \).

(c) Estimation 1: Minimizing the sum of squared deviations because the number of unknowns is smaller than the number of equations.

(d) Estimation 2: Look more carefully in each equation the variables taken from the data. There should be a few one equation one unknown systems.

7. (This may be useful !?) After the initial period, productivity growth is estimated using Finicelli et al.’s (JIE 2011) method for open economies, using sectoral output data (I guess), and the following unconditional mean of the productivity parameter:

\[
E[A_{kv}] = T_{kv}^{1/\theta} \Gamma \left( \frac{\theta - 1}{\theta} \right).
\]

This gives an estimate of aggregate \( a_{kv} \) that needs to be posteriorly corrected to get the right number in the open economy. The method is explained in Uy et al. (JME 2013).
References


